Key Concepts and the Mastery Test Process
(AGEC 642 - Dynamic Optimization)

The list on the following pages covers basic, intermediate and advanced skills that you should learn during AGEC 642.

- Your grade is earned by demonstrating mastery of the skills and concepts listed in this document by passing Mastery Tests (MT) with a score of 100%.
- You are allowed as many chances as you need to pass a MT for a given concept, but each MT will be different.
- Some MTs evaluate concepts that are not completely covered in the notes or problem sets – the MTs themselves are part of the learning process.
- Some MTs are administered electronically.

Scheduling a Mastery Test

- MT requests must be made with at least 24 hours' notice.
- A student who fails an MT must review their previous attempt and wait at least 2 days after receiving their grade before they are allowed to retake another MT over that concept.
- MTs are divided into three groups, I, II and III, and three levels, A, B and C.
- On a single day, you may schedule up to 4 Basic (A-level), 2 Intermediate (B-level), 1 Advanced (C-level) or 2 A's and 1 B.
- Requests are processed by a simple program. Hence requests **<u>must</u>** be made in a specific form: name, day & time, and the MTs that you want to take. The request must be on a separate line of your e-mail message formatted as in this example:

  Rich**,** Tuesday 4/20 8:00 a.m.**,** II.B.1.1**,** II.B.2.1**,** …

- Follow this structure **<u>EXACTLY</u>**.
  - As indicated by the arrows, commas are important. A comma and a space must divide the pieces of the request; there must be no other commas or the program will crash.
  - MT numbers must follow the structure used above. For example, your first attempt at demonstrating mastery of the 4th concept in the optimal control portion of the course (II), in the basic skills category (A), you would write II.A.4.1 while your 2nd attempt would be II.A.4.2. Periods must separate each part of an MT number. <u>There must be no spaces.</u>
- With very few exceptions, the MTs draw on material in the lecture notes. Sometimes this is noted in the MT list, but not always. Always review the lecture notes before scheduling an MT.

Prerequisites

- Prerequisites must be completed before scheduling an MT and you must initial at the appropriate places confirming that prerequisites have been completed. In separate lines of your e-mail MT request, list all the prerequisites for each MT requested, e.g.,
  "I.A.1.1: I have completed PS1.1 and reviewed Lecture 1.
  I.B.1.1: I have completed all of II.A & III.A and reviewed Lecture 1"
- Written or electronic proof that problem-set prerequisites have been completed must be sent by email.

	Prerequisites (target class # by which MT should be attempted)	1	2	3	4
I. General issues in dynamic optimization					
A. Basic skills					
1. Identify the key variables in a general problem statement. That is, given a general description of an optimization problem, identify the state and control as distinct from parameters or intermediate variables.	Lecture 1 PS1.1__ (#3)				
2. Given a general problem statement, identify and define key functions that would be used to specify the problem mathematically, i.e. the benefit function, state equation, objective function, and salvage value.	Lecture 1 PS1.1__ (#3)				
3. Identify a published paper related to you areas of interest that uses dynamic optimization. State the state variable(s) control variable(s), the benefit function, state equation(s), objective function, and salvage value (if any).	Lecture 1 PS1.1__ (#3)				
B. Intermediate skills					
1. Show the equivalence of optimal control and dynamic programming for solving a deterministic dynamic optimization problem. (Lecture 1)	Lecture 8 II.A__, III.A__				
2. Convert a problem from a DP to OC specification and vice versa and explain the relative advantages of one or the other approach.	Lecture 8 II.A__, III.A__				
C. Advanced skills					
1. Independently develop a well-defined dynamic optimization problem with reasonable sets of key variables and functions.	A grade of B in the course__				
II. Optimal control (continuous time & analytical solutions)					
A. Basic skills					
1. Solve a very simple (constant or single variable) first-order differential equation.	Lecture 2 PS1.3__ (#3)				
2. Explain in words what a differential equation means (of any order).	Lecture 2 PS1__ (#3)				
3. Solve differential equations using a symbolic algebra program.	Matlab Tutorial PS1.3__ (#4)				
4. Take a clearly defined dynamic opt. problem and write down the Hamiltonian and FOCs.	Lectures 3-5 PS2.1, PS2.2__(#5)				

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	Prerequisites (target class # by which MT should be attempted)	1	2	3	4
5. *Derive* the transversality condition for a well-defined problem.	Lecture 4 PS2.3__ (#7)				
6. Reformulate a present-value OC problem to a current-value specification and vice versa.	Lecture 5 PS2.4__ (#9)				
B. Intermediate skills					
1. Solve a linear first-order differential equation.	Lecture 2 PS1.3__ II.A.1__				
2. Correctly develop a phase diagram given the appropriate differential equations, including identifying the equilibrium.	Lecture 2 PS1.4__, II.A.2__				
3. Describe a phase diagram or set of differential equations in terms of stability and trajectories based only on the system dynamics (arrows in a phase diagram).	Lecture 2 II.B.2__				
4. Evaluate analytically and/or numerically the stability of a system of nonlinear differential equations.	Lecture 2 PS1.4__, II.B.3__				
5. Take a general problem and write down the Hamiltonian and FOCs.	Lecture 3 PS2.3__, II.A.4__				
6. Given a general problem statement, identify the type of end-point problem that it is and correctly state the transversality condition.	Lecture 4 PS2.3__, II.A.4__				
7. Provide economic interpretation of the FOC w.r.t. the control variable (present & current value).	Lecture 5 PS2.5__, II.B.5__				
8. Identify a published paper related to you areas of interest that uses optimal control. State one of the optimization problems used in the paper and the first order conditions used in the paper. Briefly state the economic intuition behind the FOC's.	Lecture 6 II.B.7				
9. Set up a constrained dynamic optimization problem and derive the FOCs.	Lecture 14 II.B.5__				
C. Advanced skills					
1. Solve a relatively simple constrained optimal control problem (Lecture 14, possibly from the bang-bang or MRAP section of these notes)	Lecture 14				
2. Given a general problem statement, provide the correct formal statement of the problem, solve the problem (to the extent that a solution is possible), and use appropriate mathematical and graphical skills to describe the solution to the problem. (**Taken with II.C.3)**	Lecture 6 PS2__, II.B.5__, II.B.6__, II.B.7__				

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	Prerequisites (target class # by which MT should be attempted)	1	2	3	4
3. Provide strong economic intuition for optimal trajectories using the FOCs.	Lecture 6 PS2__, II.B.3__				
4. Derive the transversality condition for any general problem using both a present- and current-value specification, including problems with salvage value and infinite-horizon problems.	Lecture 4 PS2__, II.B.6__				
5. Provide economic interpretation of the FOC w.r.t. the state variable for both present & current value specifications.	Lecture 6 PS2__, II.B.7__				
6. Following the approach of Hartwick (1990) provide intuitive economic interpretation of a Hamiltonian.	Lecture 6 PS2__				
7. Given a general problem statement, solve a constrained dynamic optimization problem including MRAP and bang-bang type problems. (Lecture 14)	Lecture 14 II.C.1__				
8. Set up and solve a relatively simple stochastic control problem. (Lecture 17)	Lecture 15				
III. Numerical dynamic programming (discrete time & numerical solutions)					
A. Basic skills					
1. Set up and solve a well-defined deterministic DD-DP problem using a circle-and-arrow approach.	Lecture 7 PS3.2__ (#14)				
2. Write down the Bellman's equation for a well-specified dynamic optimization problem.	Lecture 8 PS3.2__ (#14)				
3. Write pseudo code that would be used to solve a finite-horizon one-dimensional DD-DP problem.	Lecture 8 PS3.3__ (#14)				
4. Identify a published paper related to you areas of interest that uses dynamic programming. State the Bellman's equation used in the paper and discuss the methods used by the authors to analyze the problem.	Lecture 10 PS3.1__ (#14)				
5. Write a computer program to numerically solve a deterministic finite-horizon one-dimensional DD-DP problem.	Lecture 8 PS3.3__, **Error! Reference source not found.**__, III.A.1__ (#15)				

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	Prerequisites (target class # by which MT should be attempted)	1	2	3	4
6. Given a solution to a deterministic dynamic programming problem, write computer code that simulates the optimal path over a finite or infinite horizon and economically discuss the solution.	Lecture 8 **Error! Reference source not found.**__, III.A.1__, III.A.4 __ (#15)				
7. Understand the meaning of a Markov transition matrix. Convert a general description of state transitions to a Markov matrix, find transition probabilities over *n* periods, and calculate the limiting probability distribution as $n \rightarrow \infty$.	Lecture 9 (#16)				
8. Identify how changes in computer code or pseudo-code would be required for different types of stochastic dynamic programming problems: DD, CD and CC problems.	Lecture 11 PS3.3__, PS3.5__ (#17)				
B. ***Intermediate skills***					
1. Set up and solve a well-defined stochastic DD-DP problem by hand using a circle-and-arrow type approach. (Lectures 7 & 9)	Lecture 9 III.A.1__				
2. Write down the Bellman's equation for a general problem statement for a deterministic or stochastic DD problem.	Lecture 9 PS3.1__				
3. Given the solution to a dynamic programming problem (policy function and value function), economically interpret the solution making use of graphical tools as appropriate.	Lecture 9 PS3.3__				
4. Identify a published paper related to you areas of interest that uses dynamic programming. State the Bellman's equation used in the paper and discuss the methods used by the authors to analyze the problem.	Lecture 10 III.B.3				
5. Given a general problem statement, write a computer program that solves a deterministic infinite horizon DD-DP. You should be able to present and economically interpret the policy and value function and the simulated optimal paths. (2 hour time limit)	Lecture 10 PS3.3__, PS3.4__, **Error! Reference source not found.**__, III.A.5__				
6. Given a general problem statement, write a computer program that solves a deterministic CD-DP problems using both rounding and linear interpolation. (1 hour time limit)	Lecture 11 PS3.4__, II.B.5__				

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	Prerequisites (target class # by which MT should be attempted)	**1**	**2**	**3**	**4**
7. Given a general problem statement, write a computer program that solves a stochastic DD-DP problem. (2 hour limit)	Lecture 9 PS3.5__, II.B.5__				
8. Economically interpret the solution to a stochastic DP problem (1 hour limit)	Lecture 9 PS3.5__, III.B.3__				
9. Write computer code that generates the optimal paths that follow from the solution to a stochastic DD-DP problem and explain and analyze the paths and the solution to the problem.	Lecture 9 PS3.5__, III.B.7__				
10. Solve analytically a simple DP problem with a continuous choice variable.	Lecture 9 ps3.1__				
C. Advanced skills					
1. Write a computer program that solves a stochastic infinite-horizon DD-DP and simulates the optimal path over a finite horizon.	Lecture 9 PS3.5 III.B.9 __				
2. Economically interpret the solution to a stochastic infinite-horizon DD-DP. You should be able to use as appropriate graphical or tabular presentation of your results.	Lecture 9 III.C.1 __				
3. Demonstrate the ability to use policy iteration and one other acceleration technique for the solution of ∞-horizon problems	Lecture 12 PS3.5__, III.C.1 __				
4. Write a computer program that solves a deterministic CC-DP problem using a hill-climbing algorithm.	Lecture 11 III.B.7 __				
5. Write a computer program that solves a problem with multiple state variables.	Lecture 11 III.B.7 __				
6. Demonstrate the ability to use nonlinear interpolation and spline methods for problems with continuous state variables.	Lecture 11 III.C.1 __				
7. Describe in general terms how Rust's nested fixed point algorithm would be used to estimate the parameters of a dynamic decision problem. *Or* Apply Approximate Dynamic Programming methods to a stochastic dynamic optimization problem.	Lecture 13 III.C.3 __				

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