

AGEC 637 2025

Problem Set #3

Problem sets are not graded, but must be completed before taking some MTs.

Answers can be requested one question at a time.

Problems 2-4 below are all based on Burt & Allison (1963). These problems start out more simplified than Burt & Allison's paper, and then become progressively more complicated. The problems are designed to walk you through the range of dynamic programming problems and specifications and a variety of techniques that can be used to solve these problems.

Programming advice and assistance.

- It is imperative that you are able to carry out these simple tasks before you begin writing your code.
- *Do not start programming until you have thought carefully about how your program works.* I strongly recommend that you develop pseudo code before beginning to write your programs. I am happy to discuss your pseudo code or general structure of the problem with you before you start programming.
- You can use these programs as a reference, but always start writing your program with a blank program then you can copy and paste pieces of the other code that you really need. Trust me, this will save you lots of time in the long run.
- *Do not waste time on syntax problems. Use Chat GPT.*
- *Questions 2-5 are all related and build on each other.* If you pay attention to what you will have to do next, it will save you time as you write up your programs.

1. (Lecture # 7) Suppose that in period 2 (the terminal stage) you will be in state 0 or state 1. The value of being in state 1 is 1 and the value of being in state 0 is 0. Your choice in period 1 will influence which state you will reach in period 2. Specifically, if you spend $z \in [0,1]$, the probability of being in state 1 is a monotonically increasing and continuous function $F(z)$ with $F(0)=0$ and $F(1)=1$. Your objective is to maximize the present value of expected net benefits, using the discount rate r .
 - a. Write out the Bellman's equation for this problem.
 - b. Using a general form, solve for the interior optimal spending in period 1. Your answer should be a concise derivation of the optimum and your solution.
 - c. If $F(z) = z^{1/2}$, what would be the optimal level of spending? Your answer should be an explanation or derivation of the optimum.
 - d. If $F(z) = z^2$, what would be the optimal level of spending? *Hint: this is a trick question.*
2. (Lecture #7) A manual DP problem. Suppose that you are a farmer planning your planting for the next 4 years ($V_{T+1}(\cdot) = 0$). Your choice each year is whether to plant wheat or leave your land fallow (i.e., to let the land rest for a year).
 - The profits from planting and fallowing depend on the soil moisture as indicated in the table below.

Soil Moisture	Net return under Fallow	Net return with planting
0	-2	-4
0.9	-2	7
1.8	-2	9
2.7 or greater	-2	14

- If you plant wheat, the soil moisture will decline by 0.9 inches. If you leave the land fallow, soil moisture increases by 1.8 inches. The soil's maximum saturation is 2.7 inches so that even if you fallow at that level, the soil's moisture will not increase further.
 - You do not discount future profits, i.e., r with $r=0\%$ per year.
 - **If two policies yield the same value, choose to fallow.**
- a. Using a circle and arrow diagram to solve this problem. Identify the optimal policy (plant or fallow) and the value function in each state and each stage over your four year planning horizon. This can be handwritten and does not need to be excessively neat, in other words, don't waste time on making your answer look pretty. A template is provided on the last page of this assignment that you can use if you like. Your answer should be a circle and arrow diagram, highlighting optimal policies and writing in the circles the value function at each stage-state combination. You should be able to present your results as tables with your value function and policy function.
 - b. Very explicitly, write down, the Bellman's equation that is solved at $t=3, x=1.8$.
 - c. **ECONOMICALLY** discuss the problem and your results by answering the following questions:
 - Consider the decision in $t=3, x_t=1.8$. Calculate Δu and $\Delta V(x_{t+1})$, comparing the outcomes when $z=\text{Fallow}$ and $z=\text{Plant}$. Then, in the spirit of Dorfman, explain **WHY** the choice you found was optimal at this state-stage combination, deconstructing the trade-offs between present benefits and future consequences.

3. (Lecture #8) In this problem you must solve the problem described in question 2 using a computer program.
- What is the Bellman's equation that must be solved at each point in the state space at each point in time? This should be specific to the problem, but not specific to a particular point (i.e. you should use x_t , not 0.9). The Bellman's equation must always include the specific state equation and benefit function. Boundary conditions must be explicitly addressed.
 - Write a computer program to solve the optimization problem. (Remember to include comments). Your program should be written in a way that you can vary some parameters of the model (see part e). Your computer code should include a discount rate, even if that is set to zero, and it should include the simulation loop.
 - Identify using comments how the Bellman's equation in part a is solved in your program.
 - Write a short loop at the end of your program to simulate your optimal policy over the time horizon assuming an initial soil moisture level of 1.8 inches.
Your comments should clearly identify where this piece of code appears.
Present in a table or figure the results of the simulation for three different planning horizons, $T=5$, $T=10$, and $T=20$.
 - Varying parameters of the model, give economic answers to the following questions:
 - Compare the value function at time $t=1$ and at time $t=T-1$, and shadow value of the state variable at these two time points in time. Do they change for different values of T ? Why? Consider 3 values for T , $T=5$, $T=10$, and $T=20$? Discuss the differences, including two tables, one for $V(x)$ and one for $\lambda(x)$. Then write 2-4 sentences explaining **why** this variation occurs?
 - Using a 10 year horizon, discuss how the optimal policy function, $z^*(x,t)$, changes for discount rates of 0%, 10% and 90%. You should create a table presenting the policy function (not the policy path) for each value of r , and a one paragraph discussion of how and why the optimal policy function is affected by the discount rate.
 - Discuss why there are differences between value function at $t=1$ and $t=5$.

4. (Lecture #8) In this problem you must alter the problem considered in question 2, with the change that the time horizon is now infinite and you discount future net benefits at the rate of 10% per year.
- Write down the infinite horizon Bellman's equation. How does this Bellman's equation differ from that solved in question 3? Again, this should be specific to the problem, but not specific to a particular point.
Explain how this equation differs from that in question 3.
 - Write a computer program to solve the optimization problem. This should be a modified version of the program used in question 3. (Your code must correctly solve an infinite-horizon program, not simply a finite-horizon program for a long horizon). Comment your code, identifying the main differences in your program compared to question 3.
 - After solving the DP problem, your program should include code that simulates the optimal policy over 20 years assuming an initial soil moisture level of 1.8 inches. Make handwritten notes on your hard copy pointing out the main differences in your program compared to question 3.
 - Create tables or graphs that present your optimal value and policy functions and the simulated policy path.
 - Referring to your value function, policy function, and simulated path, discuss how and why the infinite horizon solution differs (or does not differ) from the finite horizon problem. (Hint: focus on the last 1 or 2 periods of the finite horizon problem for differences and the first periods for similarities).
Write a one or two paragraph discussion with appropriate references to the tables or figures in part d.
 - Carry out some sensitivity analysis and evaluate the numerical importance of the discount rate. How does an increase in the discount rate affect the number iterations to convergence?
Write short paragraph with appropriate numerical evidence in table or graphical form.
 - Carry out some sensitivity analysis and economically evaluate the importance of changes in the discount rate. How does the discount rate affect the optimal policy vector and the corresponding simulated path? Discuss why these changes take place and what are the consequences for the soil's moisture level?
Write a short paragraph with appropriate numerical evidence in table or graphical form.

5. (Lecture #12) This problem further modifies the problem by making the state equation stochastic. Specifically, the soil moisture level changes stochastically following the state transition probabilities in Table 1 from the Burt & Allison paper:

MTM - Fallow $P(F)$						MTM - Wheat $P(W)$					
	0.0	0.9	1.8	2.7	≥ 3.6		0.0	0.9	1.8	2.7	≥ 3.6
0.0	0	1/20	5/20	7/20	7/20	0.0	9/23	7/23	7/23	0	0
0.9	0	0	1/20	5/20	14/20	0.9	9/23	7/23	7/23	0	0
1.8	0	0	0	1/20	19/20	1.8	9/23	7/23	7/23	0	0
2.7	0	0	0	0	1	2.7	9/23	7/23	7/23	0	0
≥ 3.6	0	0	0	0	1	≥ 3.6	9/23	7/23	7/23	0	0

Retain the infinite horizon specification as in question 4 with a 10% discount rate to start out. The profit from planting if the soil moisture is ≥ 3.6 is 20. All other values are as in 2.

You need to program a solution to this problem in two ways: first using value function iteration and then (part f) using policy iteration. You may submit one or two programs. If a single program is submitted, be sure to include programs where the option of using policy iteration is incorporated.

Your program must:

- 1) solve the DP,
 - 2) store the value and policy functions (in a spreadsheet if using VB),
 - 3) carry out stochastic simulation, storing the results in a spreadsheet, and
 - 4) calculate the limiting probability distribution.
- a. Write the Bellman's equation for this problem that must be solved at each point in each stage of the algorithm. How does this Bellman's equation differ from that solved in question 4?
 - b. Present your value function, optimal policy function, and optimal Markov transition matrix in formatted tables.
 - c. Present a printed version of the limiting probability distribution and a brief discussion of what it tells us.
 - d. Present and discuss 4 simulated stochastic paths of the state and choice variables, each starting at $x_0=1.8$ and lasting for 20 years. Present the simulated results in graphical or tabular form and briefly discuss of what you see.
 - e. Points awarded as follows:
 Correctness and clarity of the main program to solve the problem using the successive approximation approach.
 Handwritten notes explaining the differences in this program compared to 4.
 Code that carries out a simulation of the optimal
 - f. Modify your main program to solve the problem using policy iteration.
 - g. Use a timer to compare the run time of the successive approximation and policy iteration approaches. Which method is faster? Does the discount rate affect which algorithm is faster?

Name: _____

Problem Set 3-6

Stage
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1

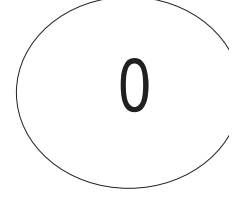
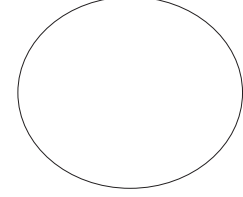
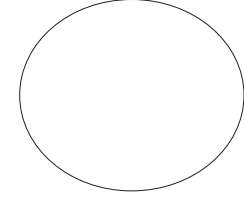
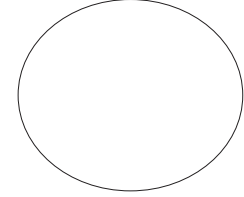
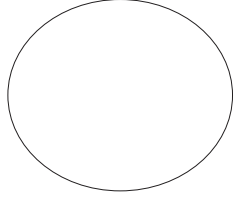
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3

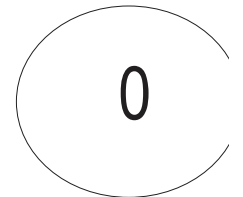
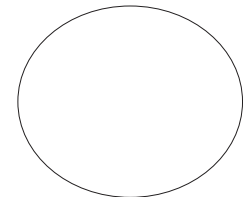
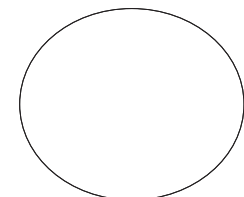
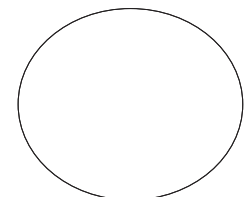
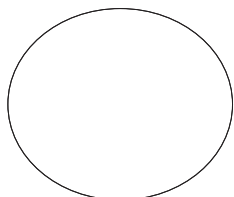
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T+1

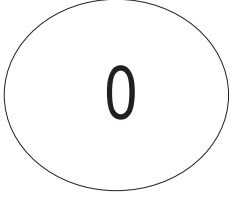
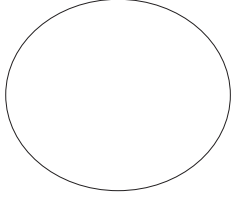
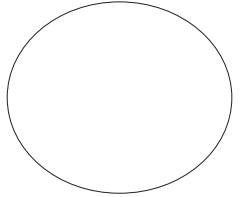
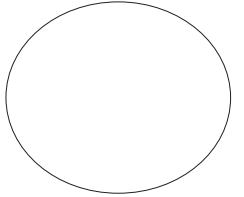
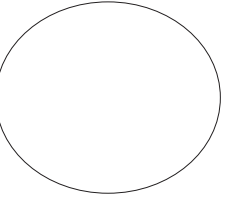
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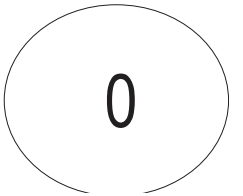
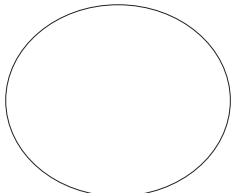
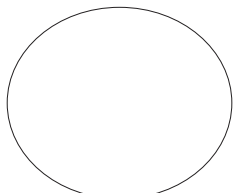
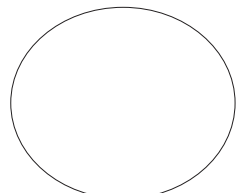
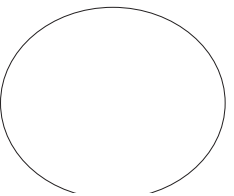
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2



1



State
↑

$$V^*(x,t)$$

$x \setminus t$	$t = \underline{\quad}$	$t = \underline{\quad}$	$t = \underline{\quad}$	$t = \underline{\quad}$
$x = \underline{\quad}$				
$x = \underline{\quad}$				
$x = \underline{\quad}$				
$x = \underline{\quad}$				

$$z^*(x,t)$$

$x \setminus t$	$t = \underline{\quad}$	$t = \underline{\quad}$	$t = \underline{\quad}$	$t = \underline{\quad}$
$x = \underline{\quad}$				
$x = \underline{\quad}$				
$x = \underline{\quad}$				
$x = \underline{\quad}$				