

AGEC 637
Problem Set #1

Problem sets are not graded, but they are prerequisites for some MTs.

When you have completed a problem, you should submit an electronic copy of your answer (this will **not** be evaluated in any way) and request an answer key.

1. (Lecture #1) For each of the problems listed below, identify the following:
 - (i) The time horizon for your problem,
 - (ii) At least one control variable,
 - (iii) At least one state variable,
 - (iv) A benefit function,
 - (v) A state equation for each of your state variables
 - (vi) A function representing a salvage value. If not appropriate, explain why not.
 - (vii) An objective function for the problem.

While you will want to define variables and may need to specify general functional representations, specific functional forms are often not needed.

For each problem (below) create a table as below.

Sample question: The problem of a farmer managing a farm for a single season. Management practices will influence crop productivity and final yield.

<u>Time horizon</u>	$t = 0$ is the day that planting occurs and $t=T$ is the day of harvest. t measures the number of days that have passed since planting, which could be measured as a real number or integer.
<u>Control Variables, z_t</u>	Management choices made by the farmer, such as irrigation, fertilizer, tilling, etc.
<u>State Variables, x_t</u>	Quantity of crops standing in the field, x_t^c , and a measure of weeds present in the field, x_t^w .
<u>Benefit Function, $u(\cdot)$</u>	During the growing season there are only costs. Hence the “benefit function” actually takes on negative values over the season, we might write, $B_t = -c(z_t)$.
<u>State Equation</u>	The change in x_t^c is equal to the rate of growth in the crop. This would typically be a function of x_t^c , x_t^w , and z_t . For x_t^w , the change in the amount of weeds is $x_{t+1}^w - x_t^w = f(x_t^w) - z_t^w$ where z_t^w is weeding done on day t and $f(x_t^w)$ is a function describing how fast weeds grow.
<u>Salvage value</u>	The salvage value is a function of the state variable(s) after all choices have been made. In this case it would be price times the crop minus harvesting costs, $S(x_{T+1}) = (p^c - c^h) \cdot x_{T+1}^c$
<u>Objective Function</u>	In this case the objective would be the maximization of the discounted profits, which equals the discounted end-of-period

	<p>revenue, $S(\mathbf{x}_T)$, minus the discounted sum of the costs over the season:</p> $\max_{z_t} \frac{S(\mathbf{x}_{T+1})}{(1+r)^{T+1}} - \sum_{t=1}^T \frac{c(\mathbf{z}_t)}{(1+r)^t}$ <p>If I allowed choices to be continuous time</p> $\max_{z_t} e^{-rT} S(\mathbf{x}_T) - \int_0^T e^{-rt} c(\mathbf{z}_t) dt$
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- a. A firm with a very simple production process that produces goods every day during the month but only sells its inventory at the end of each month.
 - b. A firm selling a product. The number of goods it sells is a function of its reputation. Reputation can be improved slowly over time through advertising or diminishes slowly over time if advertising is insufficient.
 - c. A social planner's problem to address the problem of global climate change. Damages associated with climate change are a function of the temperature, which is an increasing function of the global stock of greenhouse gases in the atmosphere. Those gases increase with the burning of fossil fuels. Reducing emissions of greenhouse gases is expensive.
2. (Lecture #2) **Do this question before Lecture 4.**
 Let $f(x; \mu, \sigma)$ be the p.d.f. of a normal distribution centered at μ with variance σ^2 .
- a. Without doing any algebra (i.e., use the function in the general form, $f(x; \mu, \sigma)$), use Leibniz's rule to evaluate the following derivative:

$$\frac{d}{d\sigma} \int_{\mu-\sigma}^{\mu+\sigma} f(x; \mu, \sigma) dx.$$

If you don't remember Leibniz's rule, consult Wolfram's MathWorld (<http://mathworld.wolfram.com/>). Provide a concise derivation of the derivative applying Leibniz's rule.
 - b. Explain in words the meaning of this derivative and state its exact numerical value? Your answer should consist of a one or two sentence explanation. No algebra is required for this answer.
3. (Matlab Tutorial) For each of the differential equations below
- (i) Solve the problem by hand, showing all work.
 - (ii) Solve the problem using a symbolic algebra computer program.
 - (iii) Explain why it is or is not *autonomous*. Your answer should consist of a one or two sentence explanation for each along with any necessary equations.
- a. $\dot{x} = -e^x + ax$ with $x_0 = 0$
 - b. $2\dot{x} = \frac{1}{x \cdot t}$ with $x_1 = 2$
 - c. $\ddot{x} = n$ with $\dot{x}(0) = 1$ and $x(0) = 2$

4. (Lecture #2) For each of the following systems of first-order differential equations:
- (i) Derive the functions that are needed to draw the phase diagrams. Your answer should consist of clearly and well-organized mathematical derivations with your steps explained.
 - (ii) Draw phase diagrams identifying the steady states of the systems (showing all the underlying work).
 - (iii) Identify the steady states of the systems (numerical values). Your answer should consist of mathematical derivation of the values of the steady states and indicate these on your figure.
 - (iv) State the system of linear equations that would be used to characterize the stability of all equilibria.
 - (v) Calculate, either by hand or using a computer, the Eigen values of the linear approximation of the system in the neighborhood of every equilibrium.
 - (vi) Using the Eigen values and/or the linearization directly, characterize the stability of the equilibrium or equilibria.

a. $\dot{x}_1 = 5x_1 + 2x_2 - 7$ $\dot{x}_2 = 2x_1 - 2x_2$

b. $\dot{x}_1 = 3 + x_1 - 0.3x_1^2 - x_2$ $\dot{x}_2 = 1 - x_2$

Reminder: You will learn the material best if you have worked through the problem set independently and struggled on your own. While you can work with others, simply seeing someone else's answer is unlikely to be helpful and you should not request an answer key until you feel you have the correct answer.
