Introduction to Computable General Equilibrium Model (CGE)

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Course Outline

- Overview of CGE
- An Introduction to the Structure of CGE
- An Introduction to GAMS
- Casting CGE models into GAMS
- Data for CGE Models & Calibration
- Incorporating a trade & a basic CGE application
- Evaluating impacts of policy changes and casting nested functions & a trade in GAMS
- **Mixed Complementary Problems (MCP)**

This Week's Road Map

- **The class of models that our CGEs fall into in GAMS is the MCP model type which we solve with PATH. Here we explore several issues related to PATH for solving MCP models.**
  - Gaining an understanding of how PATH works
  - Gaining an understanding of what happens when a system of equations is inconsistent?
- **Plotting a useful feature in GAMS**
  - Output improvement and management using GNUPLOT and GNUPLTXY

Investigating how PATH works

Suppose we wish to solve a single variable, single nonlinear equation for a root x

$f(x)=0$

A first order Taylor series around x*

$f(x)=f(x^*)+f'(x^*)\,(x-x^*)$

Now if we want $f(x)=0$ then

$0=f(x^*)+f'(x^*)\,(x-x^*)$

or $\quad x=x^*-f(x^*)/f'(x^*)$

This is the **Newton-Raphson method**

Investigating how PATH works

Now given x* we iterate using $x = x^* - f(x^*)/f'(x^*)$
Until we obtain convergence

Example f(x)=x**2 - 5*x+6=0 starting from x=10

	x	f	f'
1	10.00000	56.00000	15.00000
2	6.26667	13.93778	7.53333
3	4.41652	3.42305	3.83304
4	3.52348	0.79752	2.04696
5	3.13387	0.15180	1.26775
6	3.01414	0.01434	1.02827
7	3.00019	0.00019	1.00039
8	3.00000	3.775990E-8	1.00000
9	3.00000	1.77636E-15	1.00000
10	3.00000		1.00000

Example f(x)=x**2 - 5*x+6=0 starting from x=0

	x	f	f'
1		6.00000	-5.00000
2	1.20000	1.44000	-2.60000
3	1.75385	0.30675	-1.49231
4	1.95940	0.04225	-1.08121
5	1.99848	0.00153	-1.00305
6	2.00000	2.317831E-6	-1.00000
7	2.00000	5.37259E-12	-1.00000
8	2.00000	8.88178E-16	-1.00000
9	2.00000		-1.00000

Investigating how PATH works

Observations

Iterative convergence to a root

Starting point helps determine speed of convergence and root found

Investigating how PATH works

Now suppose we have multiple variables and equations

$f_i(x)=0$ for all i (must be square)

The Multivariate Newton-Raphson

$x = x^* - J^{-1}(x^*)\, f(x^*)$

Where x is now a vector as if $f(x^*)$ and $J(x)$ is the Jacobian of first derivatives $\partial f_i(x)/\partial x_j$. This is the multivariate multiequation Newton-Raphson method and is the basic method used by PATH with an extension to maintain complementarity

Investigating how PATH works

Applying $x = x^* - J^{-1}(x^*) f(x^*)$ to the solution of the equations

1/2*sin(y("x1")*y("x2"))-y("x2")/(4*pi)-y("x1")/2=0

(1-1/(4*pi))*(exp(2*y("x1"))-e)+e*y("x2")/pi -2*e*y("x1")=0

	x1	x2	eq1	eq2
1	5.40000	13.00000	-3.29232	45099.39135
2	4.90016	16.86011	-3.38916	16589.52225
3	4.40041	20.96459	-4.32414	6103.06136
4	3.90100	19.01579	-3.93280	2243.65327
5	3.40029	27.96940	-3.54808	830.09479
6	2.89266	35.65083	-4.02325	312.20442
7	2.36204	38.93706	-4.65990	122.01104
8	1.73993	43.08567	-4.50788	55.19010
9	-0.21167	101.79790	-8.20907	87.32933
10	-0.42347	-0.17065	0.26141	0.04718
11	-0.29431	0.46853	0.04114	0.01438
12	-0.26265	0.61347	0.00229	0.00105
13	-0.26061	0.62251	9.140456E-6	4.531039E-6
14	-0.26060	0.62254	1.48592E-10	7.46654E-11

Investigating how PATH works

So PATH uses an iterative scheme starting from a starting point and then tries to converge to the solution.

It uses a variant of the Newton-Raphson multivariable, multi-equation solution method.

A good starting point helps in model solution.

Solutions may not be unique

PATH and Inconsistent Systems

PATH cannot always achieve solutions to the systems of equations and may hide the fact that it could not.

Let's look at some linear programming examples.

Suppose we have the problem

Max	2*	X1	+3*	X2		
s.t.		X1	+	X2	$\leq$	2
		X1	+	X2	$\leq$	3
		X1	,	X2	$\geq$	0

PATH and Inconsistent Systems

Max	2*	X1	+3*	X2		
s.t.		X1	+	X2	$\leq$	2
		X1	+	X2	$\leq$	3
		X1	,	X2	$\geq$	0

Leads to KKT conditions

U1 +	U2			$\geq$	2	$\perp$ X1
U1 +	U2			$\geq$	3	$\perp$ X2
		X1+	X2	$\leq$	2	$\perp$ U1
		X1+	X2	$\leq$	3	$\perp$ U2
U1 ,	U2 ,	X1 ,	X2	$\geq$	0	

PATH and Inconsistent Systems

```
POSITIVE VARIABLES  U1,  U2,  X1,  X2     ;
EQUATIONS           EQ1, EQ2, EQ3, EQ4    ;
EQ1..               U1+U2 =G= 2 ;
EQ2..               U1+U2 =G= 3 ;
* X1+X2 =L=  2 ;
EQ3..              -X1-X2 =G= -2 ;
* X1+X2 =L=  3 ;
EQ4..              -X1-X2 =G= -3 ;
OPTION MCP    = PATH;
MODEL  LPKKT    /EQ1.X1,EQ2.X2,EQ3.U1,EQ4.U2/;
SOLVE  LPKKT USING MCP;
DISPLAY LPKKT.SOLVESTAT;
```

Path Solution is OK

```
**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL

                      LOWER        LEVEL        UPPER        MARGINAL
---- VAR U1             .          3.0000       +INF            .
---- VAR U2             .            .          +INF          1.0000
---- VAR X1             .            .          +INF          1.0000
---- VAR X2             .          2.0000       +INF            .
```

PATH and Inconsistent Systems

Max	2*	X1	+3*	X2		
s.t.		X1	+	X2	$\geq$	2
		X1	+	X2	$\geq$	3
		X1	,	X2	$\geq$	0

(an Unbounded LP)

Leads to KKT conditions

-U1 -	U2			$\geq$	2	$\perp$ X1
-U1 -	U2			$\geq$	3	$\perp$ X2
		X1+	X2	$\leq$	2	$\perp$ U1
		X1+	X2	$\leq$	3	$\perp$ U2
U1 ,	U2 ,	X1 ,	X2	$\geq$	0	

PATH and Inconsistent Systems

```
POSITIVE VARIABLES  U1,  U2,  X1,  X2     ;
EQUATIONS           EQ1, EQ2, EQ3, EQ4    ;
EQ1..              -U1-U2 =G= 2 ;
EQ2..              -U1-U2 =G= 3 ;
EQ3..               X1+X2 =G= 2 ;
EQ4..               X1+X2 =G= 3 ;
OPTION MCP    = PATH;
MODEL  LPKKT    /EQ1.X1,EQ2.X2,EQ3.U1,EQ4.U2/;
SOLVE  LPKKT USING MCP;
DISPLAY LPKKT.SOLVESTAT;
```

Path Solution is Infeasible

```
**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      4 INFEASIBLE

                   LOWER        LEVEL        UPPER       MARGINAL
---- EQU EQ1        2.0000        .          +INF           .      INFES
---- EQU EQ2        3.0000        .          +INF           .      INFES
---- EQU EQ3        2.0000        .          +INF           .      INFES
---- EQU EQ4        3.0000        .          +INF           .      INFES

                   LOWER        LEVEL        UPPER       MARGINAL

---- VAR U1          .            .          +INF        -2.0000
---- VAR U2          .            .          +INF        -3.0000
---- VAR X1          .            .          +INF        -2.0000
---- VAR X2          .            .          +INF        -3.0000
```

PATH and Inconsistent Systems

$$
\begin{array}{llllllll}
\text{Max} & 2^* & X1 & +3^* & X2 & & \\
\text{s.t.} & & X1 & + & X2 & = & 2 \\
 & & X1 & + & X2 & \geq & 3 \\
 & & X1 & , & X2 & \geq & 0
\end{array}
$$

(an Infeasible LP)

Leads to KKT conditions with U1<>0

$$
\begin{array}{lllllll}
U1\ + & U2 & & & \geq & 2 & \perp X1 \\
U1\ + & U2 & & & \geq & 3 & \perp X2 \\
 & & X1+ & X2 & = & 2 & \perp U1 \\
 & & X1+ & X2 & \geq & 3 & \perp U2 \\
 & U2\ , & X1\ , & X2 & \geq & 0 &
\end{array}
$$

PATH and Inconsistent Systems

```
POSITIVE VARIABLES       U2,  X1,  X2     ;
VARIABLES             U1      ;
EQUATIONS             EQ1, EQ2, EQ3, EQ4    ;
EQ1..                 U1+U2 =G= 2 ;
EQ2..                 U1+U2 =G= 3 ;
EQ3..                 X1+X2 =E= 2 ;
EQ4..                 X1+X2 =G= 3 ;
OPTION MCP    = PATH;
MODEL  LPKKT    /EQ1.X1,EQ2.X2,EQ3.U1,EQ4.U2/;
SOLVE  LPKKT USING MCP;
DISPLAY LPKKT.SOLVESTAT, LPKKT.MODELSTAT;
```

Path Solution is Infeasible

```
**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      4 INFEASIBLE

                     LOWER        LEVEL        UPPER       MARGINAL
---- EQU EQ1          2.0000         .         +INF            .      INFES
---- EQU EQ2          3.0000         .         +INF            .      INFES
---- EQU EQ3          2.0000         .          2.0000         .      INFES
---- EQU EQ4          3.0000         .         +INF            .      INFES

                     LOWER        LEVEL        UPPER       MARGINAL
---- VAR U2            .            .         +INF         -3.0000
---- VAR X1            .            .         +INF         -2.0000
---- VAR X2            .            .         +INF         -3.0000
---- VAR U1          -INF           .         +INF         -2.0000
```

PATH and Inconsistent Systems

Make sure your model solution status is
optimal and do not just use numbers

Can display modelname.solvestat
modelname.modelstat

Other features in GAMS – GNUPLOT

GNUPLOT is a command-driven data plotting program that can be used during a GAMS run.

GNUPLTXY is a procedure developed by Uwe Schneider and Bruce McCarl that can be used to plot GAMS generated data during a run.

By inserting a couple of commands in a GAMS program on a windows machine you can generate and display graphs during any PC GAMS runs.

MORE ON GNUPLOT see

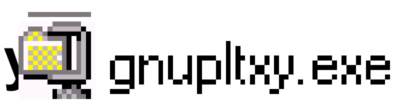
http://ageco.tamu.edu/faculty/mccarl/gnuplot/gnuplot.html

Other features in GAMS – GNUPLTXY

GNUPLTXY causes graphs to be drawn and presented during a GAMS run. To graph data with **GNUPLTXY** follow these three basic steps.

1. Download the **GNUPLTXY** software which includes the gnupltxy.gms file and windows gnuplot executable from

   http://ageco.tamu.edu/faculty/mccarl/gnuplot/gnuplot.html

   When y 

   you get **Gnupltxy.exe** which is a self extracting archive. In turn double click on this file and extract the files to your GAMSIDE directory (now c:\program files\gams20.5).

Other features in GAMS – GNUPLTXY

2. Fill a three dimensional array where you pick the name

i. the first dimension represents names of lines in a plot

ii. the second dimension represents the points on the lines

iii. the third dimension represents the data values (ie. X and Y)

```
SETS
LINES        Lines in graph /A,B/
POINTS       Points on line /1*10/
ORDINATES    ORDINATES      /X-AXIS,Y-AXIS/   ;

TABLE GRAPHDATA(LINES,POINTS,ORDINATES)
         X-AXIS    Y-AXIS
A.1         1         1
A.2         2         4
A.3         3         9
A.4         5        25
A.5        10       100
B.1         1         2
B.2         3         6
B.3         7        15
B.4        12        36
 ;
```

Other features in GAMS – GNUPLTXY

3. **Call GNUPLTXY by adding the statement**

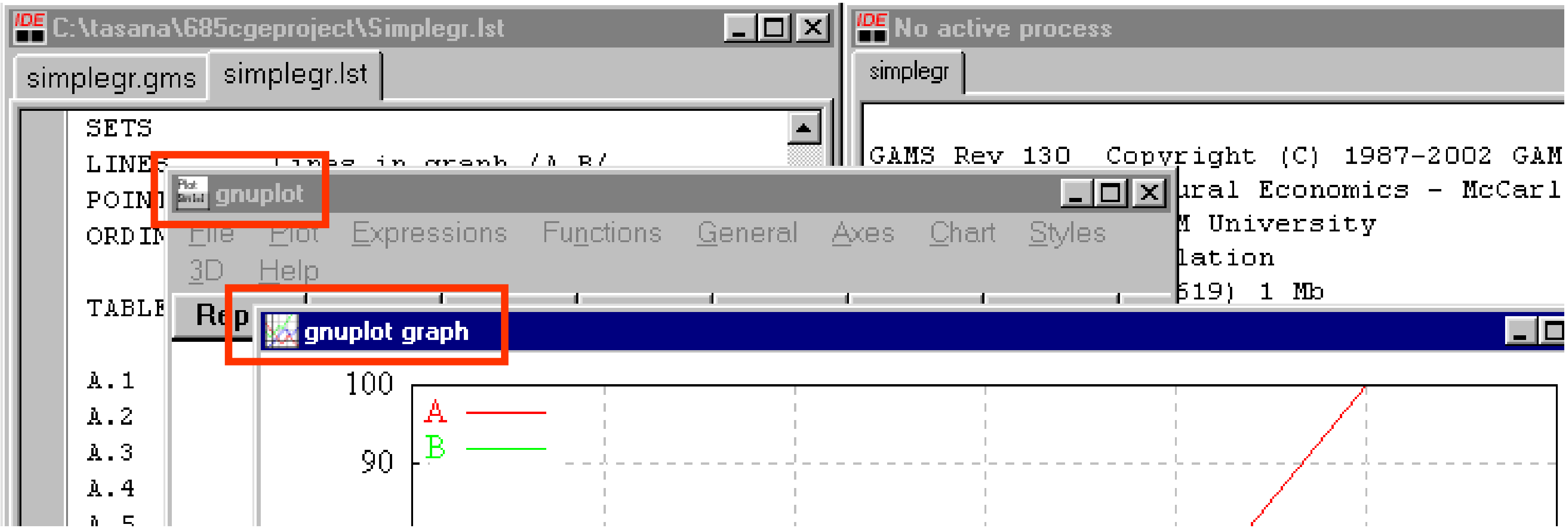
$LIBINCLUDE GNUPLTXY GRAPHDATA Y-AXIS X-AXIS

```
SETS
LINES        Lines in graph /A,B/
POINTS       Points on line /1*10/
ORDINATES    ORDINATES       /X-AXIS,Y-AXIS/   ;

TABLE GRAPHDATA(LINES,POINTS,ORDINATES)
          X-AXIS    Y-AXIS
A.1          1         1
A.2          2         4
A.3          3         9
A.4          5        25
A.5         10       100
B.1          1         2
B.2          3         6
B.3          7        15
B.4         12        36
 ;
$LIBINCLUDE GNUPLTXY GRAPHDATA Y-AXIS X-AXIS
```

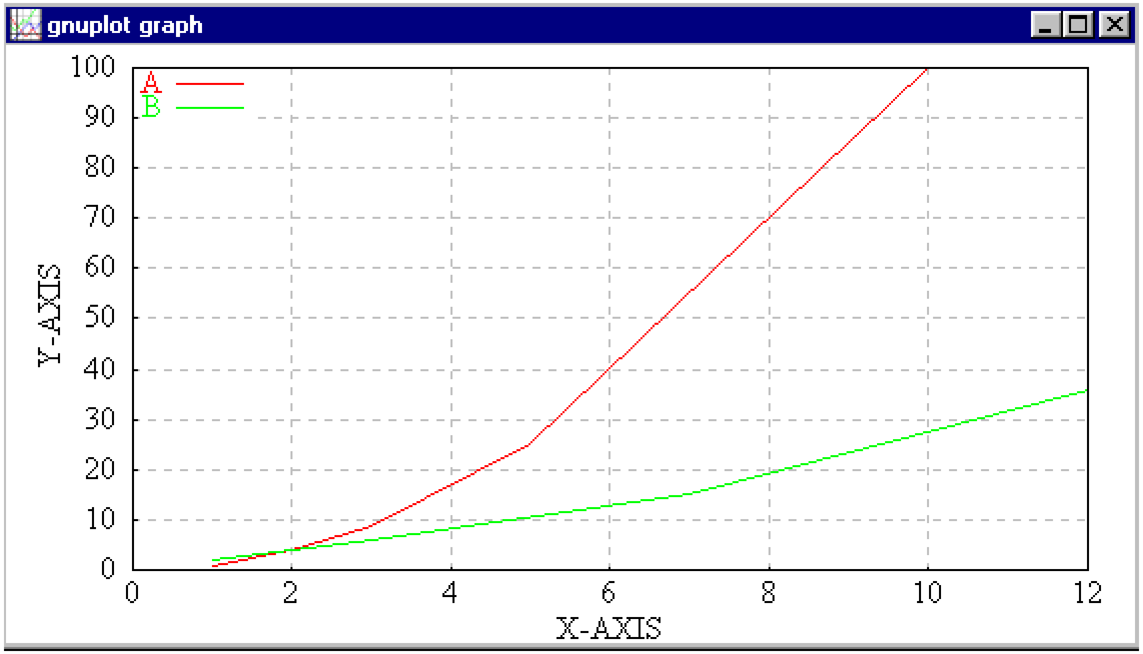
Other features in GAMS – GNUPLTXY

After running the GAMS program, two new windows that automatically open.

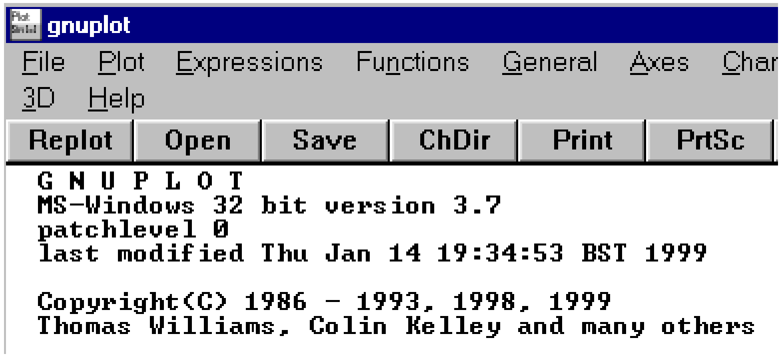


Other features in GAMS – GNUPLTXY

1. The window labeled gnuplot graph is the graph of the data



2. The window labeled gnuplot is the execution of the source gnuplot program



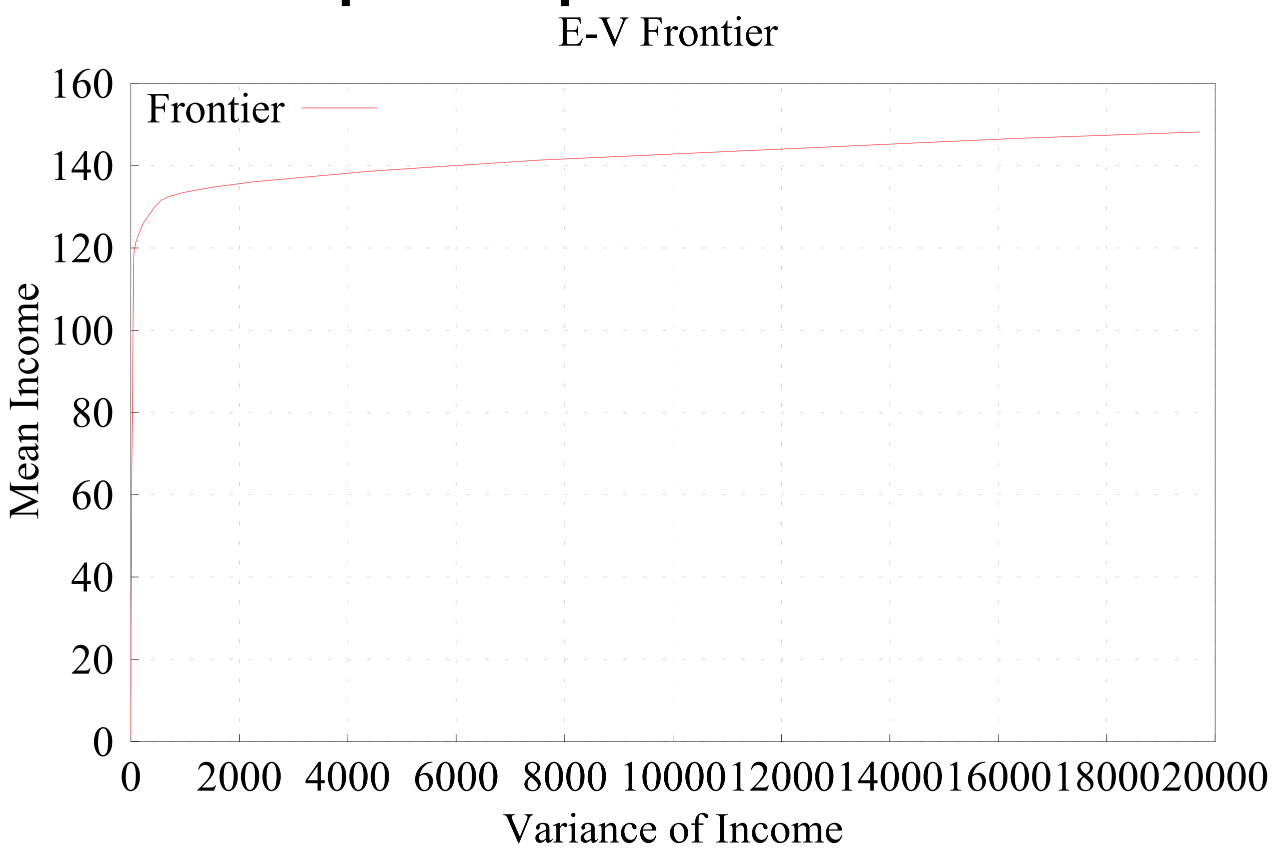
Other features in GAMS – GNUPLTXY

Gnupltxy can also be used to graph data generated during a model run for example in a portfolio case

```
 LOOP (RAPS,RAP=RISKAVER(RAPS);
           SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;
           VAR = SUM(STOCK, SUM(STOCKS,
               INVEST.L(STOCK)*COVAR(STOCK,STOCKS)*INVEST.L(STOCKS))) ;
           OUTPUT("RAP",RAPS)=RAP;
           OUTPUT(STOCKS,RAPS)=INVEST.L(STOCKS);
           OUTPUT("OBJ",RAPS)=OBJ.L;
           OUTPUT("MEAN",RAPS)=SUM(STOCKS, MEAN(STOCKS) * INVEST.L(STOCKS));
           OUTPUT("VAR",RAPS) = VAR;
           OUTPUT("STD",RAPS)=SQRT(VAR);
           OUTPUT("SHADPRICE",RAPS)=INVESTAV.M;
           OUTPUT("IDLE",RAPS)=FUNDS-INVESTAV.L
             );
 DISPLAY OUTPUT;
parameter graphit (*,raps,*);
graphit("Frontier",raps,"Mean")=OUTPUT("MEAN",RAPS);
graphit("frontier",raps,"Var")=OUTPUT("std",RAPS)**2;
*$include gnu_opt.gms
* titles
$setglobal gp_title  "E-V Frontier "
$setglobal gp_xlabel "Variance of Income"
$setglobal gp_ylabel "Mean Income"
$libinclude gnupltxy graphit mean var
```

Other features in GAMS – GNUPLTXY

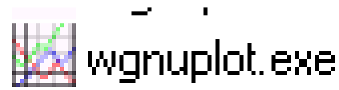
Gnupltxy can also be used to graph data generated during a model run for example in a portfolio case



Where the portfolio is a function of the decision variables choices and we are summarizing 28 solutions

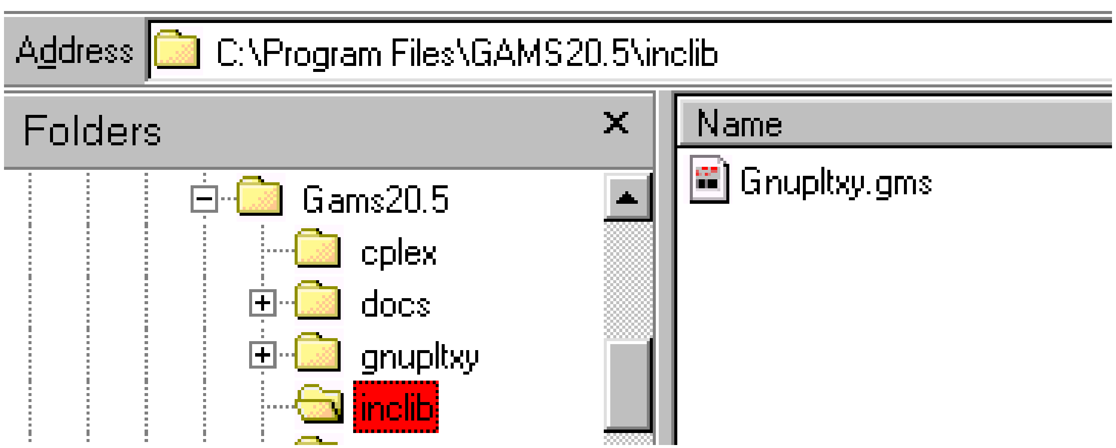
Other features in GAMS – GNUPLTXY

When using **GNUPLTXY** several things need to be present.

1. The gnuplot executable 

needs to be in the GAMS system directory (c:\program files\gams20.5\wgnuplot.exe)

2. The **gnupltxy.gms** file must be in the **inclib** directory under the GAMS system directory (c:\program files\gams20.5\inclib\gnupltxy.gms)



Wrap Up

- Investigating how MCP works
- Insight on how PATH approaches a problem solution?
- Information on what happens with inconsistent conditions
- Output improvement and management using **GNUTPLOT**