

Introduction to Computable General Equilibrium Model (CGE)

Dhazn Gillig
&
Bruce A. McCarl

Department of Agricultural Economics
Texas A&M University

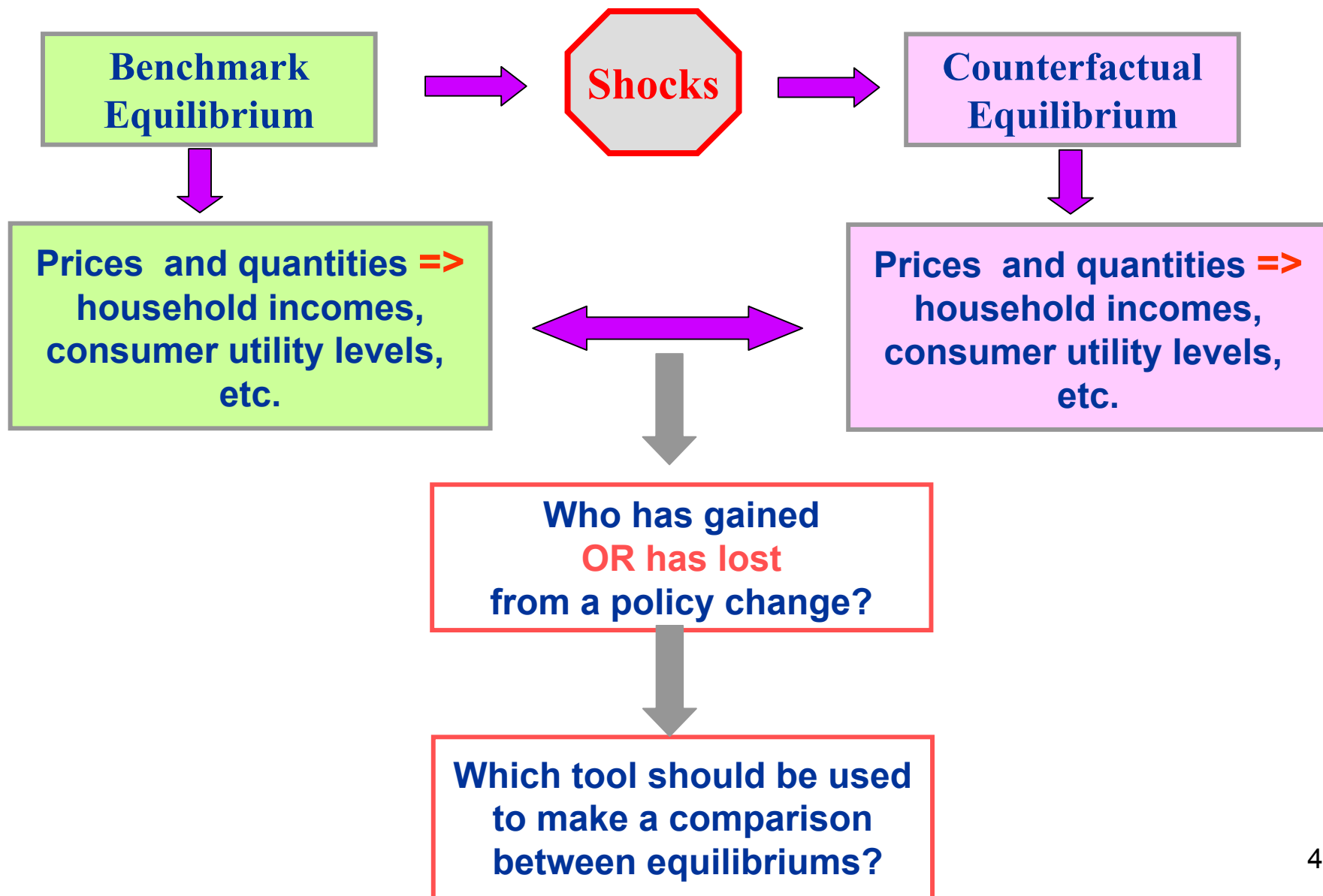
Course Outline

- Overview of CGE
- An Introduction to the Structure of CGE
- An Introduction to GAMS
- Casting CGE models into GAMS
- Data for CGE Models & Calibration
- Incorporating a trade & a basic CGE application
- **Evaluating impacts of policy changes and casting nested functions & a trade in GAMS**
- Mixed Complementary Problems (MCP)

This Week's Road Map

- **Discussions on technical matters on modeling that have not been discussed in the class.**
 - **Deriving welfare impacts of policy changes**
 - **Building nested functions in GAMS**
 - **Extending a closed economy to a small open economy in GAMS**
- **CGE application of Manne & Richels**

Evaluating impacts of policy changes



Evaluating impacts of policy changes

There are several approaches to evaluate impacts of policy changes; however, most of the CGE literature on the effects of policy changes focus on **welfare measures:**

1. Compensating Variation (CV)
2. Equivalent Variation (EV)

Welfare measures – CV

Compensating Variation:

How much money is necessary to compensate someone for price changes?

$$CV = E(U^1, P^1) - E(U^0, P^1) \Rightarrow CV = [(U^1 - U^0) / U^1] * Y^1$$

STEPS to calculate CV:

1. Calculate a new utility level

$$U = \sum_j \left[(\alpha_j)^{1/\sigma} (X_j)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

2. Calculate a new income level

$$Y^1 = W_L^1 \bar{L}_h + W_K^1 \bar{K}_h + Transfers$$

3. Calculate a utility difference

Welfare measures –EV

Equivalent Variation:

How much money is a particular change equivalent to?

$$EV = E(U^1, P^0) - E(U^0, P^0) \Rightarrow EV = [(U^1 - U^0) / U^0] * Y^0$$

STEPS to calculate EV:

1. Calculate a new utility level

$$U = \sum_j \left[(\alpha_j)^{1/\sigma} (X_j)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

2. Calculate a utility difference

Price & quantity measures

An alternative to evaluating impacts of policy changes is **price and quantity measures**. These measures involve developing price and quantity indexes which can be used to describe how large adjustments are between a base scenario and an alternative scenario.

The simplest price and quantity measures are:

1. Laspeyres price index
2. Laspeyres quantity index
3. Paasche price index
4. Paasche quantity index

Price & quantity measures

The Laspeyres price index: $L^P = \sum_j P_j^1 X_j^0 / \sum_j P_j^0 X_j^0$

shows the ratio between the aggregate value of all commodities at prices in the new equilibrium but quantities in the old equilibrium and the aggregate value of all commodities at the old equilibrium prices and quantities.

The Laspeyres quantity index: $L^Q = \sum_j P_j^0 X_j^1 / \sum_j P_j^0 X_j^0$

shows the ratio between the aggregate value of all commodities at quantity in the new equilibrium but prices in the old equilibrium and the aggregate value of all commodities at the old equilibrium prices and quantities.

Note that: This price index is similar to EV where we compare the aggregate value of all goods with the old equilibrium value of all goods.

P's are prices and X's are quantity. Subscripts '1' and '0' refer to a new and old equilibriums, respectively.

Price & quantity measures

The Paasche price index:
$$P^P = \frac{\sum_j P_j^1 X_j^1}{\sum_j P_j^0 X_j^1}$$

shows the ratio between the aggregate value of all commodities considered at prices and quantities in the new equilibrium and the aggregate value of all commodities at the new equilibrium quantities but prices in the old equilibrium.

The Paasche quantity index:
$$P^Q = \frac{\sum_j P_j^1 X_j^1}{\sum_j P_j^1 X_j^0}$$

shows the ratio between the aggregate value of all commodities considered at prices and quantities in the new equilibrium and the aggregate value of all commodities at prices in the new equilibrium but quantities in the old equilibrium.

Note that: This price index is similar to CV where we compare the aggregate value of all goods with the new equilibrium value of all goods.

Building nested functions in GAMS

Suppose we want to put nested functions in GAMS.

Recall: With the use of nested functions, a system allows substitution in the model

Assumptions:

1. Leontief technology using INT and VA **at a top level**
2. Cobb Douglas technology using L and K **at a bottom level**
3. Non-nested CES utility function

Building nested functions in GAMS – Modifications on variables

VARIABLE

TaxRevenue Total government tax revenues ;

POSITIVE VARIABLE

FactorPrice(AllSets)	Factor price
FactorQuan(AllSets,AllSets1)	Factor use by a producing sector
ComPrice(AllSets)	Commodity price
DemCommod(AllSets,AllSets1)	Commodity demand by households
Production(AllSets)	Production quantity levels
HHIncome(AllSets)	Household income
QIntA(AllSets,AllSets1)	Intermedite inputs quantity
PIntA(AllSets,AllSets1)	Intermedite inputs price
QValAdd(AllSets)	Value-added quantity
PValAdd(AllSets)	Value-added price

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Building nested functions in GAMS – Modifications on equations

EQUATIONS

FactorMkt (AllSets)	Factor market balances
FactorDem (AllSets, AllSets1)	Factor demand by a sector
CommodMkt (AllSets)	Commodity market balance
CommodDem (AllSets, AllSets1)	Commodity Demand by Households
Profit (AllSets)	Zero profit condition
Income (AllSets)	Household budget constraint
GovBal	Government budget constraint

QintAEq (AllSets, AllSets1)	Intermedite inputs quantity equation
PIntAEq (AllSets, AllSets1)	Intermedite price equation
CDQVAEq (AllSets)	Value-added quantity equation
PVAEq (AllSets)	Value-added inputs price equation

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Building nested functions in GAMS – Modifications on VA

Steps:

1. Define quantities of value-added (QVA)
2. Define prices of VA
3. Define factor demands used in producing VA

$$QVA_j = A_j K_j^\alpha L_j^{1-\alpha} \quad \longrightarrow \quad QVA_j = A_j \prod_f F_{jf}^{\alpha_{jf}}$$

CDQVAEq(Sector)..

QValAdd(Sector)

=E= BigA(Sector)

***(PROD(Factor,FactorQuan(Factor,Sector)**AlphaCD(Factor,Sector))) ;**

Building nested functions in GAMS – Modifications on VA

Prices of VA are derived from the relationship between revenues and costs of producing final goods where **revenue (PxQ) is exhausted by payments for VA and INT inputs.**

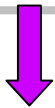
PVAEq(Sector)..

ComPrice(Sector)* Production(Sector)

=G=

PValAdd(Sector)*QValAdd(Sector)

**+ SUM(Activity,PIntA(Activity,Sector) * QIntA(Activity,Sector)
\$YesQIntA(Activity,Sector)) ;**



PARAMETER YesQIntA(AllSets,AllSets1) **Yes there are intermediate inputs;**

YesQIntA(Activity,Sector) = NO ;

YesQIntA("Food","NonFood") = YES ;

YesQIntA("NonFood","Food") = YES ;

Building nested functions in GAMS – Modifications on VA

$$\text{Factor demand : } F_{fj} = \frac{QVA_j}{A_j} \left(\frac{\alpha_{fj} W_{f'}(1+t_{f'})}{\alpha_{fj} W_f(1+t_f)} \right)^{\alpha_{fj}}$$

FactorDem(Factor, Sector)..

FactorQuan(Factor, Sector)

=E=

(QValAdd(Sector) / BigA(Sector))

* SUM(MapFact(Factor, Factor1),

((AlphaCD(Factor, Sector)

(FactorPrice(Factor1)(1 + TaxFactor(Factor1))))

/ (AlphaCD(Factor1, Sector)

* (FactorPrice(Factor)*(1 + TaxFactor(Factor))))

)**(AlphaCD(Factor1, Sector))

) ;

Building nested functions in GAMS – Modifications on INT

Steps:

1. Define quantities of intermediate input (QINT)
2. Define prices of INT

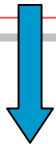
$QINT_{j1j} = a_{j1,j} Q_j$ where $a_{j1,j}$ is I-O coefficients, using $j1$ to produce j

QIntAEq(Sector,Activity)..

QIntA(Sector,Activity)

=E=

AlphaLeon(Sector,Activity) * Production(Activity) ;



AlphaLeon(Sector,Activity)

= SAM(Sector,Activity) /SAM("Total",Activity) ;

Building nested functions in GAMS – Modifications on INT

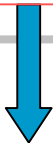
Intermediate input prices depend on **commodity prices** and **intermediate input coefficient** representing the quantity of input per unit of INT input (not output).

PIntAEq(Activity,Sector)..

PIntA(Activity,Sector)

=E=

AlphaLeonc(Activity,Sector) * ComPrice(Sector) ;



AlphaLeonc(Activity,Sector)

= SAM(Activity,Sector) /SUM(Sector1,SAM(Activity,Sector1)) ;

Building nested functions in GAMS – Complementarity

MODEL CGEModel

/ FactorMkt.FactorPrice
FactorDem.FactorQuan
Commoddem.DemCommod
CommodMkt.ComPrice
Profit.Production
Income.HHincome
Govbal.TaxRevenue

QintAEq.QintA
PIntAEq.PIntA
CDQVAEq.QValAdd
PVAEq.PValAdd

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Incorporating a small open economy in GAMS

Now, we are going to see how to incorporate a small open economy in GAMS.

Assumptions:

1. World prices and an exchange rate are exogenous.
2. Elasticity of substitution in household CES utility function = 0.7
3. Free of taxes at the benchmark equilibrium
4. Let an exchange rate be numeraire.
5. **Counterfactual equilibrium**

If an export tax is imposed, then 100% of tax revenues is used to **purchase the government goods/services.**

Incorporating a small open economy in GAMS – Modifications on variables

VARIABLE

TaxRevenue Total government tax revenues ;

POSITIVE VARIABLE

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FactorQuan(AllSets,AllSets1) Factor use by a producing sector
ComPrice(AllSets) Commodity price
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PIntA(AllSets,AllSets1) Intermedite inputs price
QValAdd(AllSets) Value-added quantity
PValAdd(AllSets) Value-added price

PExp(AllSets) US Export price paid by ROW but received
PImp(AllSets) US Import price paid by US consumers but
QExp(AllSets) US Export quantity
QImp(AllSets) US Import quantity
PWExp(AllSets) ROW or world export price
PWImp(AllSets) ROW or world import price
Exchange Exchange rate

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Incorporating a small open economy in GAMS – Modifications on equations

EQUATIONS

FactorMkt (AllSets)	Factor market balances
FactorDem (AllSets, AllSets1)	Factor demand by a sector
CommodMkt (AllSets)	Commodity market balance
CommodDem (AllSets, AllSets1)	Commodity Demand by Households
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Income (AllSets)	Household budget constraint
GovBal	Government budget constraint
QintAEq (AllSets, AllSets1)	Intermedite inputs quantity equation
PIntAEq (AllSets, AllSets1)	Intermedite price equation
CDQVAEq (AllSets)	Value-added quantity equation
PVAEq (AllSets)	Value-added inputs price equation
PExpBal (AllSets)	US Export price
PImpBal (AllSets)	US Import price
QExpBal (AllSets)	US Export demand equation
QImpBal (AllSets)	US Import supply equation
PUSExpPBal (AllSets)	US domestic export price relationship
PUSImpPBal (AllSets)	US domestic import price relationship
TradeBal	Trade balance

1. The commodity market balance

$$Q_j \geq \sum_h X_{hj} + \sum_{j1} a_{j,j1} QIntA_{jj1} + s_j R / P_j + QExp_j - QImp_j$$

CommodMkt(Sector)..

Production(Sector)

=G=

sum(Households, DemCommod(Households, Sector))

+ sum(OtherSector\$QintA0(Sector, OtherSector),

QIntA(Sector, OtherSector))

+ GovTaxShare(Sector)*(TaxRevenue/ComPrice(Sector))

+ QExp(Sector)\$ExTrade(Sector)

- QImp(Sector)\$ImTrade(Sector) ;

2. The government tax revenue balance

$$\begin{aligned}
 R \leq & \sum_h \left(t_h \sum_f \bar{F}_{fh} W_f \right) + \sum_{fj} t_{fj} W_f F_{fj} \\
 & + \sum_{j \in ex} t_{Exp_j} Q_{Exp_j} P_j + \sum_{j \in im} t_{Imp_j} Q_{Imp_j} P_j
 \end{aligned}$$

GovBal..

TaxRevenue

=L=

SUM(Households,

Incometax(Households)

*** SUM(Factor,Endowment(Factor,HouseHolds)* FactorPrice(Factor)))**

+ SUM((Factor,Sector),TaxFactor(Factor,Sector)*FactorPrice(Factor)

***FactorQuan(Factor,Sector))**

+ SUM(Sector,(TaxExp(Sector)

***QExp(Sector)*ComPrice(Sector))\$ExTrade(Sector))**

+ SUM(Sector,(TaxImp(Sector)

***QImp(Sector)*ComPrice(Sector))\$ImTrade(Sector))**

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3. The domestic trade price equations

$$(PWExp_j \times Exchange) - TransCost_j \geq PExp_j \quad \forall j \in ex$$

PExpBal(sector)\$ExTrade(Sector)..

(PWExp(Sector)*Exchange) - Transcost(Sector)

=G= PExp(Sector) ;

$$(PWImp_j \times Exchange) + TransCost_j \geq PImp_j \quad \forall j \in im$$

PImpBal(sector)\$ImTrade(Sector)..

(PWImp(Sector)*Exchange) + Transcost(Sector)

=G= PImp(Sector) ;

4. The domestic trade quantity equations

$$QExp_j = f(PWExp_j, \epsilon) = a_j * PWExp_j^{\epsilon_j} \quad \forall j \in ex$$

QExpBal(sector)\$ExTrade(Sector)..

QExp(Sector) =E=

ExpDem("cons",Sector) *(PWExp(Sector)**ExpDem("slope",Sector)) ;

$$QImp_j = f(PWImp_j, \mu) = b_j * PWImp_j^{\mu_j} \quad \forall j \in im$$

QImpBal(sector)\$ImTrade(Sector)..

QImp(Sector) =E=

ImpDem("cons",Sector) *(PWImp(Sector)**ImpDem("slope",Sector)) ;

5. The domestic & trade price relationship

$$PExp_j = (1 - tExp_j) P_j \quad \forall j \in ex$$

PUSExpPBal(Sector)\$ExTrade(Sector)..

$$PExp(Sector) = E =$$

$$(1 - TaxExp(Sector)) * ComPrice(Sector) ;$$

$$PImp_j = (1 + tImp_j) P_j \quad \forall j \in im$$

PUSImpPBal(Sector)\$ImTrade(Sector)..

$$PImp(Sector) = E =$$

$$(1 + TaxImp(Sector)) * ComPrice(Sector) ;$$

6. The zero trade balance

$$\sum_{j \in im} PWImp_j QImp_j \leq \sum_{j \in ex} PWExp_j QExp_j$$

TradeBal..

SUM(Sector\$ImTrade(Sector),PWImp(Sector)*QImp(Sector))

=L=

SUM(Sector\$ExTrade(Sector),PWExp(Sector)*QExp(sectors))

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Incorporating a small open economy in GAMS – Complementarity

MODEL CGEModel

/ FactorMkt.FactorPrice
FactorDem.FactorQuan
Commoddem.DemCommod
CommodMkt.ComPrice
Profit.Production
Income.HHincome
Govbal.TaxRevenue

QintAEq.QintA
PVAEq.PValAdd
PIntAEq.PIntA
CDQVAEq.QValAdd

Qexpbal.Qexp
Qimpbal.Qimp
PUSexpPbal.Pexp
PUSimpPbal.Pimp
Pexpbal.Pwexp
Pimpbal.Pwimp
Tradebal.Exchange

/;

Incorporating a small open economy in GAMS – Results

---- 1112 PARAMETER Compare Comparative analysis

		NoTax	ExpTax20%
ExpPrice	.Food	1.000	1.072
ImPrice	.NonFood	1.000	1.725
USExPrice	.Food	1.000	1.340
USIMPrice	.NonFood	1.000	1.725

$$PExp_j = (1 - tExp_j) P_j$$

distortion in prices

Incorporating a small open economy in GAMS – Results

---- 1112 PARAMETER Compare Comparative analysis

		NoTax	ExpTax20%
HH demand	.Food	1000.000	656.847
HH demand	.NonFood	900.000	591.162
Gov demand	.Food		30.000
Gov demand	.NonFood		23.301
Int demand	.Food	100.000	57.965
Int demand	.NonFood	200.000	149.259
Domestic Demand	.Food	1100.000	744.812
Domestic Demand	.NonFood	1100.000	763.722
Export Demand	.Food	300.000	300.000
Demand	.Total	2500.000	1808.534
Domestic Supply	.Food	1400.000	1044.812
Domestic Supply	.NonFood	800.000	463.722
Import Supply	.NonFood	300.000	300.000
Supply	.Total	2500.000	1808.534

 $s_j R / P_j$

How do you explain these results?

Key Elements – MERGE

MERGE:

- **Model for Evaluating Regional and Global Effects**
- **Stanford University & Electric Power Research Institute**
- **Multi-sectors CGE model**
- **9 regions**
- **The energy sector**
- **International trade => carbon emission rights**
- **Non-energy & energy inputs**
- **Changes in the cost of energy => production**
- **Sinks and non-CO2 gas**

More on MERGE see <http://www.stanford.edu/group/MERGE/>

Wrap Up

- Evaluating results from CGE models
- Incorporating nested functions and a trade relationship in GAMS

Next:

- MCP
- MacCracken, C. N., J. A. Edmonds, S. H. Kim, and R. D. Sands. “The Economics of the Kyoto Protocol,” in *The Costs of the Kyoto Protocol: A Multi-Model Evaluation*, John Weyant (ed.), special issue of *The Energy Journal*, 1999.
- Incorporating environmental aspects (e.g. Ghg emissions) in the CGE model and in GAMS

Reference:

Shoven, J. B. and J. Whalley. “Applying general equilibrium.” ***Surveys of Economic Literature***, Chapter 5, 1998.

Manne A. S. and R. G. Richels. “An alternative approach to establishing trade-offs among greenhouse gases.” ***Nature*** 410, 675-677 (2001).