Introduction to Computable General Equilibrium Model (CGE)

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Course Outline

- Overview of CGE
- An Introduction to the Structure of CGE
- An Introduction to GAMS
- Casting CGE models into GAMS
- **Data for CGE Models & Calibration**
- Incorporating a trade & a basic CGE application
- Evaluating impacts of policy changes and casting nested functions & a trade in GAMS
- Mixed Complementary Problems (MCP)

This Week’s Road Map

- More than 2 goods or factors --Hierarchical (nested) functions
- Social Accounting Matrices
- Input-output table
- Building benchmark equilibrium data sets
- Parameter calibration

Why use Hierarchical (nested) Functions?

Why use hierarchical (nested) functions?

: Allows different elasticity of substitution among factors and/or among intermediate inputs in the production

: Allows different elasticity of substitution among goods in the consumption

: Expands the number of elasticity parameters used in a calibration

Hierarchical (nested) functions - production

New Functions:

: Building a system allowing substitution throughout the production structure in the model

a. Factors producing a new item called value-added (VA)

b. Intermediate inputs producing a new item of intermediate inputs (INT)

Hierarchical (nested) functions - production

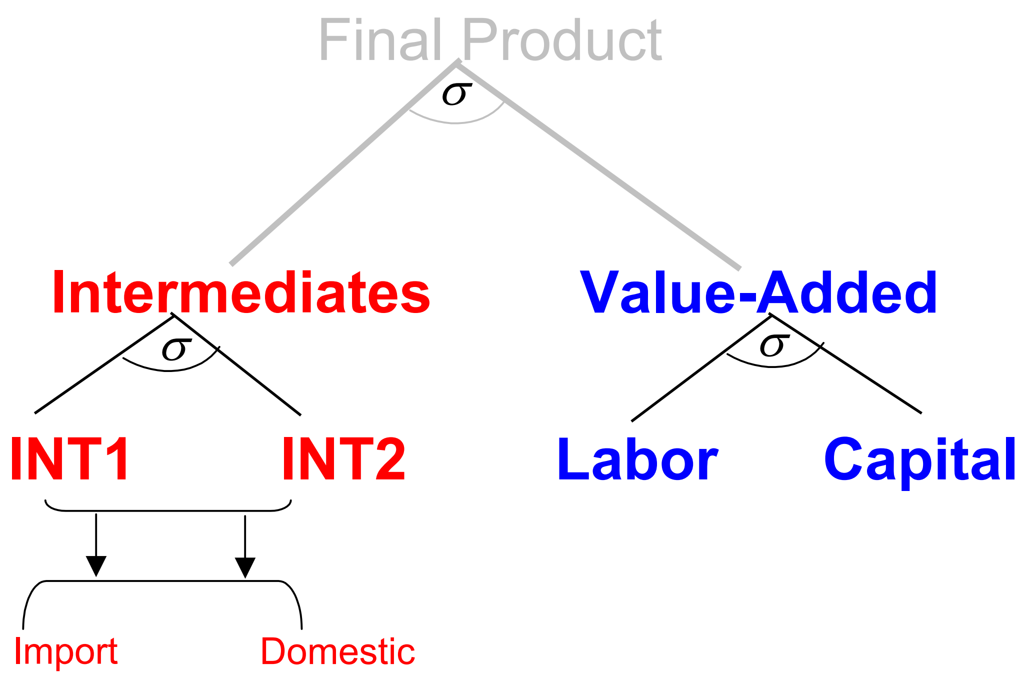
- **Levels of production possibilities**
- **The bottom level**

a. Substitution exists among factors depends on σ

b. Substitution exists among INT depends on σ

if $\sigma \Rightarrow 0$ then there is no substitution

if $0 < \sigma < 1$ then there is some degree of substitution



Hierarchical (nested) functions - production

Value-Added (VA) production function at the bottom level using labor (**L**) and capital (**K**) for a CES function is:

$$\mathbf{QVA_j = f_j\ (L_j, K_j;\ \alpha_j, \delta_j, \sigma_j)}$$

where α, δ, σ are efficiency, share, and elasticity of substitution between **L** and **K** factors in sector **j** parameters.

Factor demand derived from a CES function:

$$\mathbf{L_j\ =\ g_j\ (QVA_j, PVA_j, L_j, K_j, w;\ \delta_j, \sigma_j)}$$

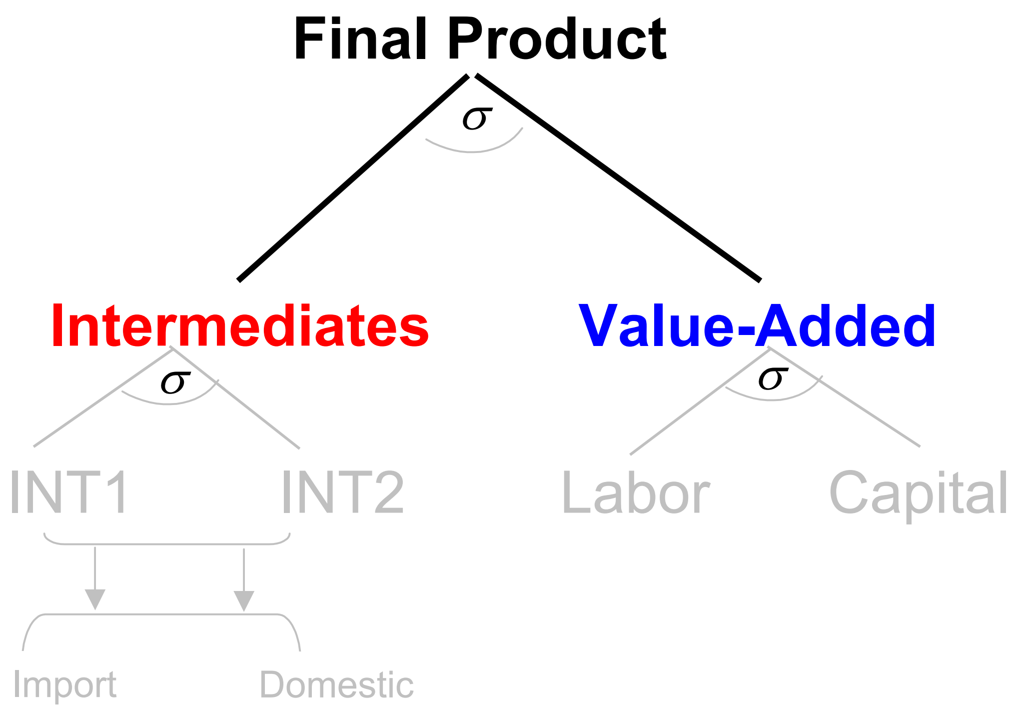
$$\mathbf{K_j\ =\ h_j\ (QVA_j, PVA_j, L_j, K_j, r;\ \delta_j, \sigma_j)}$$

where w and r are a wage rate and a capital rent.

See Appendix - A for details of these functions.

Hierarchical (nested) functions - production

- **Levels of production possibilities**
- **The top level**
  a. Outputs are derived from INT and VA
  b. Substitution between INT and VA depends on σ



Hierarchical (nested) functions - production

INT and VA production at the **top** level assuming **CES** technology:

$$Q_j = f_j\ (QINT_j, QVA_j;\ \alpha'_j, \delta'_j, \sigma'_j)$$

where α', δ', σ' are efficiency, share, and elasticity of substitution between intermediate inputs and value-added in sector **j** parameters.

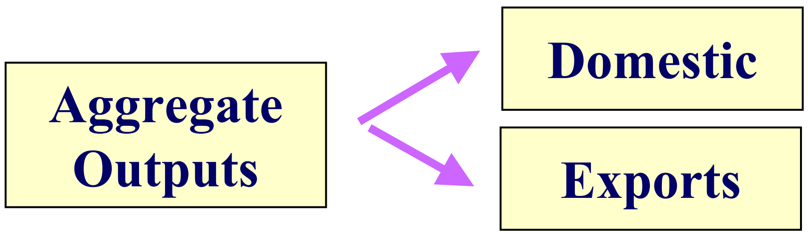
See Appendix - B for details of the functions.

Hierarchical (nested) functions - production

Opened Economy:

The aggregated domestic output is sold domestic or exported based on the imperfect transformability assumption.

Recall:



Constant Elasticity of Transformation (CET) Function:

$$QQ_j = f'_j\,(QD_j, QX_j;\ \alpha^t_j, \delta^t_j, \sigma^t_j)$$

where QQ_j, QD_j, QX_j, are aggregate outputs, domestic outputs, and exports in sector **j**, and α^t_j, δ^t_j, and σ^t_j are efficiency, share, and elasticity of substitution between domestic outputs and exported goods in sector **j** parameters. **See Appendix - C for details of these functions**.

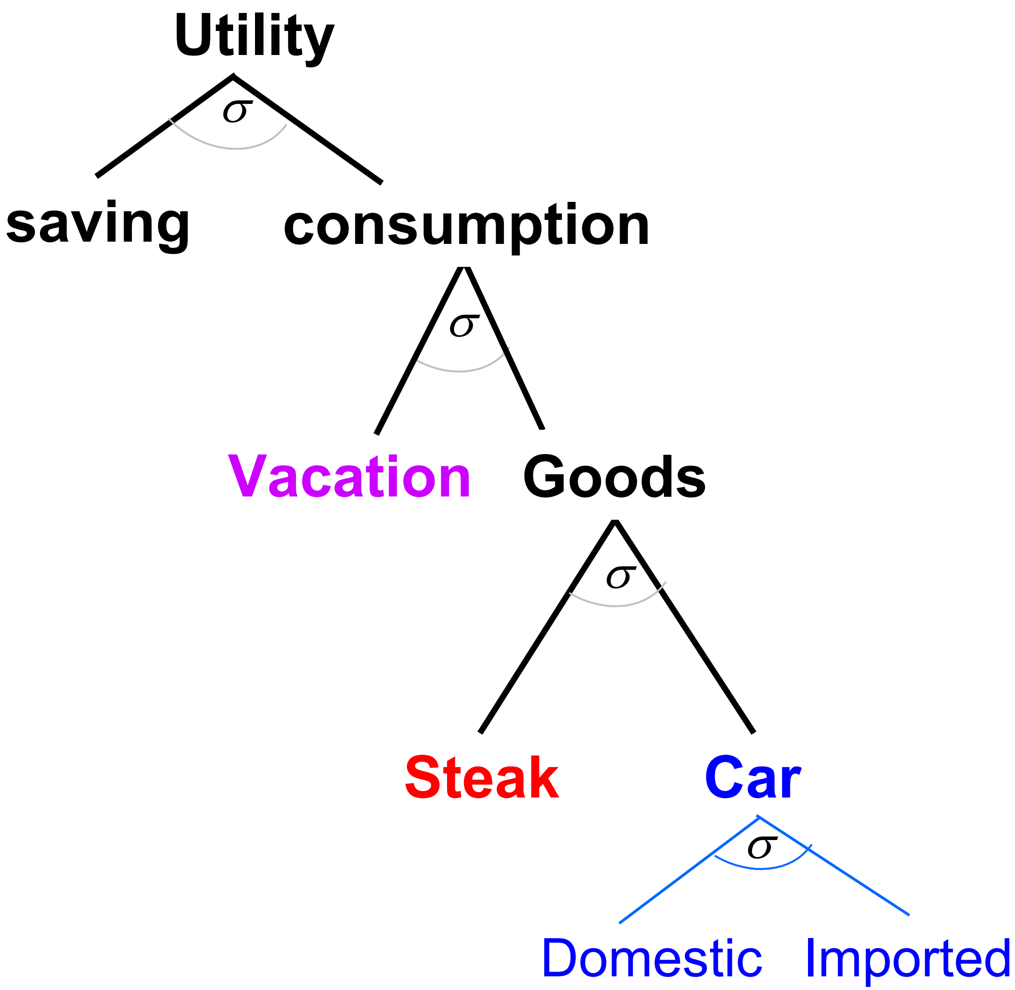
Hierarchical (nested) functions - production

Characteristics

- Allows factor (capital, labor, land, etc.) substitution in the value-added
- Allows input substitution in the intermediate input
- Allows the value-added and intermediate input substitution

Hierarchical (nested) functions - consumption

- **Nested utility function**



Substitution between saving and consumption goods

Substitution between leisure and goods

Substitution between steak and car

Substitution between domestic and imported car

Hierarchical (nested) functions - consumption

Household Consumption (X) assuming derived from the CES utility maximization subject to a budget constraint is:

$$X_{jh} = Y_j\,(P_j, M_h;\ \alpha_{jh}, \sigma_h)$$

where P_j is prices of goods **j**, M_h is household h income, and α_{jh} and σ_h are consumption share and elasticity of substitution in household **h** in sector **j** parameters.

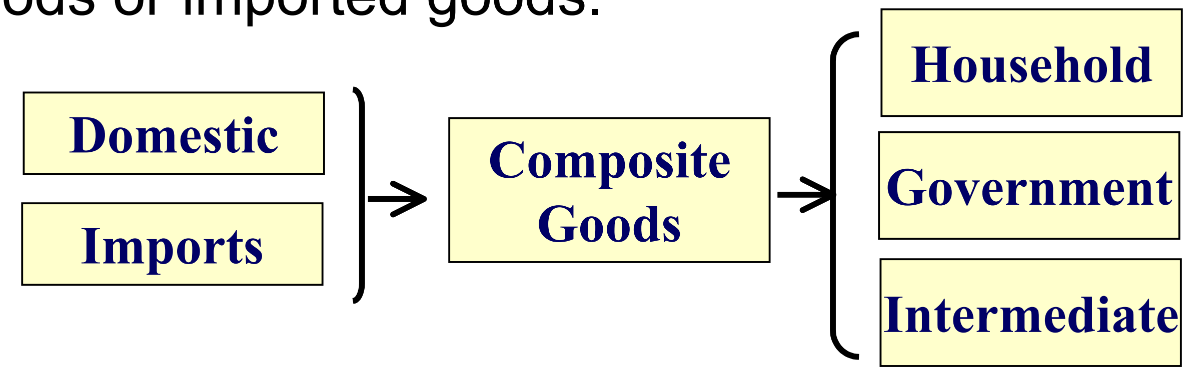
See Appendix - D for details of these functions.

Hierarchical (nested) functions - consumption

Open Economy:

The domestic demands are for a composite goods made up of domestic goods or imported goods.

Recall:



Armington Function (Imperfect substitutability):

$$QC_j = f''_j\ (QD_j, QM_j;\ \alpha^c_j, \delta^c_j, \sigma^c_j)$$

where QC_j, QD_j, QM_j, are composite, domestic, and imported goods in sector **j**, and α^c_j, δ^c_j, and σ^c_j are efficiency, share, and elasticity of substitution between domestic goods and imported goods in sector **j** parameters. See Appendix - E for details of these functions.

What is a SAM?

A Social Accounting Matrix (SAM) represents

- : an economy wide accounting of expenditures and incomes of agents like an input output table but differs in that households are included and all accounts are fully balanced.
- : a column = payments, a row = receipt and a column sum = a row sum

Mil. of $US	Food Act	NonFood Act	Food	NonFood	Labor	Capital	Household	Government	Tax	ROW	Total
Food Activity	0	0	1430	0				0			1430
NonFood Activity	0	0	0	850				0			850
Food	0	100	0	0			1020	10		300	1430
NonFood	200	0	0	0			910	40		0	1150
Labor	900	200									1100
Capital	300	500									800
Household					1100	800			30		1930
Government			0				0		50		50
Tax	30	50							0		80
ROW			0	300							300
Total	1430	850	1430	1150	1100	800	1930	50	80	300	

SAM - implications

Mil. of $US	Food Act	NonFood Act	Food	NonFood	Labor	Capital	Household	Government	Tax	ROW	Total
Food Activity	0	0	1430	0				0			1430
NonFood Activity	0	0	0	850				0			850
Food	0	100	0	0			1020	10		300	1430
NonFood	200	0	0	0			910	40		0	1150
Labor	900	200									1100
Capital	300	500									800
Household					1100	800			30		1930
Government			0				0		50		50
Tax	30	50							0		80
ROW			0	300							300
Total	1430	850	1430	1150	1100	800	1930	50	80	300	

Example: Food Sector

Payments to Food Activity = 1430 =

Receipt from

- **INT = 100**
- **HH = 1020**
- **GOV = 10**
- **Exports = 300**

double-entry bookkeeping

Example: Food Activity

Payments to

- **INT cost = 200**
- **Labor = 900**
- **Capital = 300**
- **TAX = 30**

= **Receipt from Food Sector = 1430**

SAM - implications

Mil. of $US	Food Act	NonFood Act	Food	NonFood	Labor	Capital	Household	Government	Tax	ROW	Total
Food Activity	0	0	1430	0				0			1430
NonFood Activity	0	0	0	850				0			850
Food	0	100	0	0			1020	10		300	1430
NonFood	200	0	0	0			910	40		0	1150
Labor	900	200									1100
Capital	300	500									800
Household					1100	800			30		1930
Government			0				0		50		50
Tax	30	50							0		80
ROW			0	300							300
Total	1430	850	1430	1150	1100	800	1930	50	80	300	

Example: NonFood Sector

Payments to NonFood Activity+ROW =1150 =

Receipt from
- INT = 200
- HH = 910
- GOV = 40
- Exports = 0

double-entry bookkeeping ⟷

Example: NonFood Activity

Payments to
- INT cost = 100
- Labor = 200
- Capital = 500
- TAX = 50

= Receipt from NonFood Sector = 850

Inconsistent Data

Because calibration relies on the benchmark data, what to do if

: Data/Accounting inconsistency

=> demand ≠ supply

=> expenditures exceed incomes

=> consumer expenditure classification does not match production classification

=> lack of data

DATA PROCESSING & ADJUSTMENT! => No uniform adjustment

=> adjustment varies from case to case

=> interpolation and use of other economic data

=> use previous year data with some adjustment

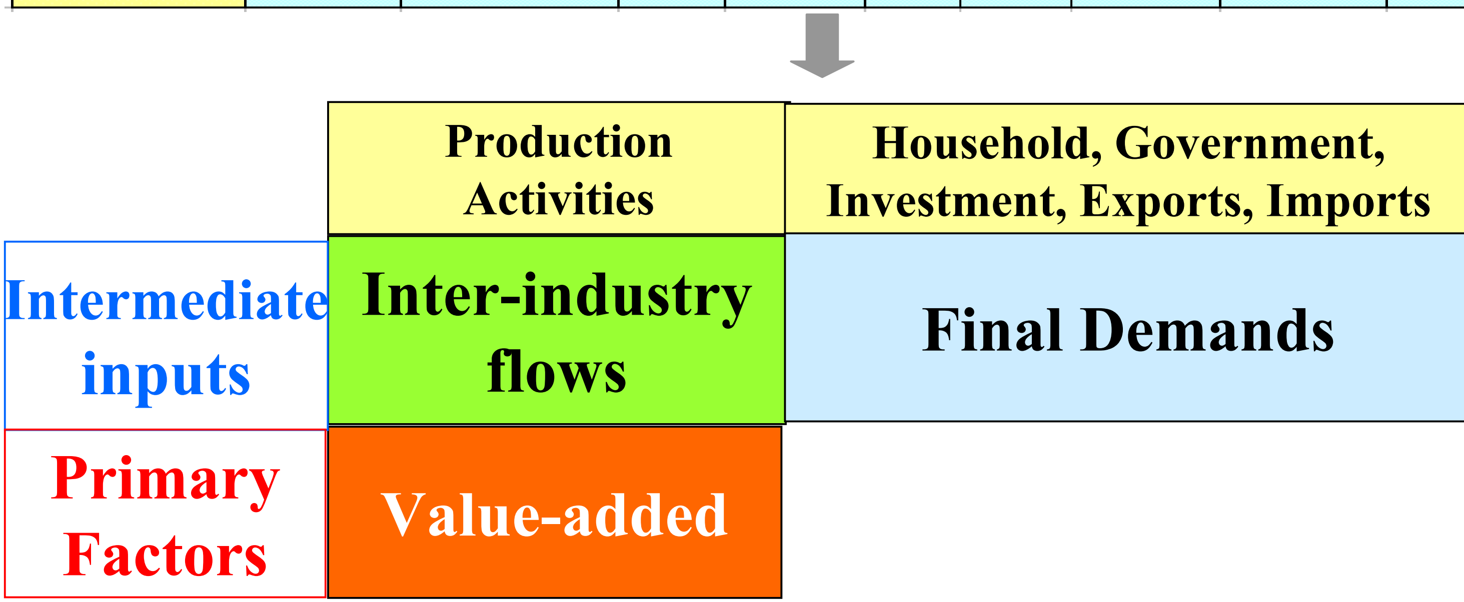
=> RAS (row-and-column-sum) procedure

=> modeler's judgment

Suggested Reading: St-Hilaire, F., and J. Whalley. "A microconsistent equilibrium data set for Canada for use in tax policy analysis." Review of Income and Wealth 29, 175-204.

From SAM to Input-output Table

Mil. $US	Food Act	NonFood Act	Food	NonFood	Labor	Capital	Household	Government	Tax	ROW	Total
Food Activity	0	0	1430	0				0			1430
NonFood Activity	0	0	0	850				0			850
Food	0	100	0	0			1020	10		300	1430
NonFood	200	0	0	0			910	40		0	1150
Labor	900	200									1100
Capital	300	500									800
Household					1100	800			30		1930
Government			0				0		50		50
Tax	30	50							0		80
ROW			0	300							300
Total	1430	850	1430	1150	1100	800	1930	50	80	300	



Input-output Table

	Production Activities	Household, Government Investment, Exports, Imports
Intermediate inputs	Inter-industry flows	Final Demands
Primary Factors	Value-added	

Mil. $US	Food	NonFood	Household	Government	Net Exports	Total Consumption
Food	0	100	1020	10	300	1430
NonFood	200	0	910	40	-300	850
Labor	900	200				
Capital	300	500				
Tax	30	50				
Total Production	1430	850				

	Household	Government
Transfer Payments	30	50

	Household
Labor	1100
Capital	800

Building the Basic Data – things to do

Things to be considered when building the basic data

1. Check the classifications among data sets

   e.g. HH expenditures categories vs. industry product categories

2. Decide on units for goods and factors so that prices and quantities are separately obtained

   e.g. choose units for goods and factors so that they have a price of unity in the benchmark equilibrium

   Note: in the CGE model only the **relative price** is the focus and the absolute price is not important.

Units are in million $US

	Food	NonFood	Household	Government	Net Exports	Total Consumption
Food	0	100	1020	10	300	1430
NonFood	200	0	910	40	-300	850
Labor	900	200				
Capital	300	500				
Tax	30	50				
Total Production	1430	850				

choosing units for goods and factors so that the benchmark equilibrium price is one, then we have

Units are in million **quantities**

	Food	NonFood	Household	Government	Net Exports	Total Consumption
Food	0	100	1020	10	300	1430
NonFood	200	0	910	40	-300	850
Labor	900	200				
Capital	300	500				
Tax	30	50				
Total Production	1430	850				

Building the Basic Data – things to do

3. Check if the data is consistent with the equilibrium conditions e.g.

   a. Demands = Supplies (consumption = production)

   b. Zero profits (revenues = costs)

   c. All agents (i.e. HH, Government, ROW) exhaust their budgets

   d. Resources are used up.

Suggested Reading:
St-Hilaire, F., and J. Whalley. “A microconsistent equilibrium data set for Canada for use in tax policy analysis.” Review of Income and Wealth 29, 175-204.

Checking data consistency – output market balance

Units are in million **quantities**

	Food	NonFood	Household	Government	Net Exports	Total Consumption
Food	0	100	1020	10	300	1430
NonFood	200	0	910	40	-300	850
Labor	900	200				
Capital	300	500				
Tax	30	50				
Total Production	1430	850				

Output markets balance:

	HH	+ INT	+ Exports	+ Govt	= Production	+ Imports
Food:	1020	+ 100	+ 300	+ 10	= 1430	+ 0
NonFood:	910	+ 200	+ 0	+ 40	= 850	+ 300

Checking data consistency – factor market balance

Units are in million **quantities**

	Food	NonFood	Household	Government	Net Exports	Total Consumption
Food	0	100	1020	10	300	1430
NonFood	200	0	910	40	-300	850
Labor	900	200				
Capital	300	500				
Tax	30	50				
Total Production	1430	850				

	Household
Labor	1100
Capital	800

Factor markets balance:

	Food	+	NonFood	=	Endowment
Labor:	900	+	200	=	1100
Capital:	300	+	500	=	800

This also implies resources are used up.

Checking data consistency – zero profits

Units are in million **quantities**

	Food	NonFood	Household	Government	Net Exports	Total Consumption
Food	0	100	1020	10	300	1430
NonFood	200	0	910	40	-300	850
Labor	900	200				
Capital	300	500				
Tax	30	50				
Total Production	1430	850				

Zero profits:

Costs: Factors + INT + Tax = Revenues (PxQ)

Food: 900+300 + 200 + 30 = 1430

NonFood: 200+500 + 100 + 50 = 850

Checking data consistency – household income balance

Units are in million **quantities**

	Food	NonFood	Household	Government	Net Exports	Total Consumption
Food	0	100	1020	10	300	1430
NonFood	200	0	910	40	-300	850
Labor	900	200				
Capital	300	500				
Tax	30	50				
Total Production	1430	850				

	Household	Government
Transfer Payments	30	50

	Household
Labor	1100
Capital	800

Household income balance:

Labor income + Capital income + Transfer Payments = Expenditures

1100 + 800 + 30 = 1930

Checking data consistency – government income balance

Units are in million **quantities**

	Food	NonFood	Household	Government	Net Exports	Total Consumption
Food	0	100	1020	10	300	1430
NonFood	200	0	910	40	-300	850
Labor	900	200				
Capital	300	500				
Tax	30	50				
Total Production	1430	850				

	Household	Government
Transfer Payments	30	50

Government income balance:

Capital Tax = Transfer Payments to HH + Government consumption

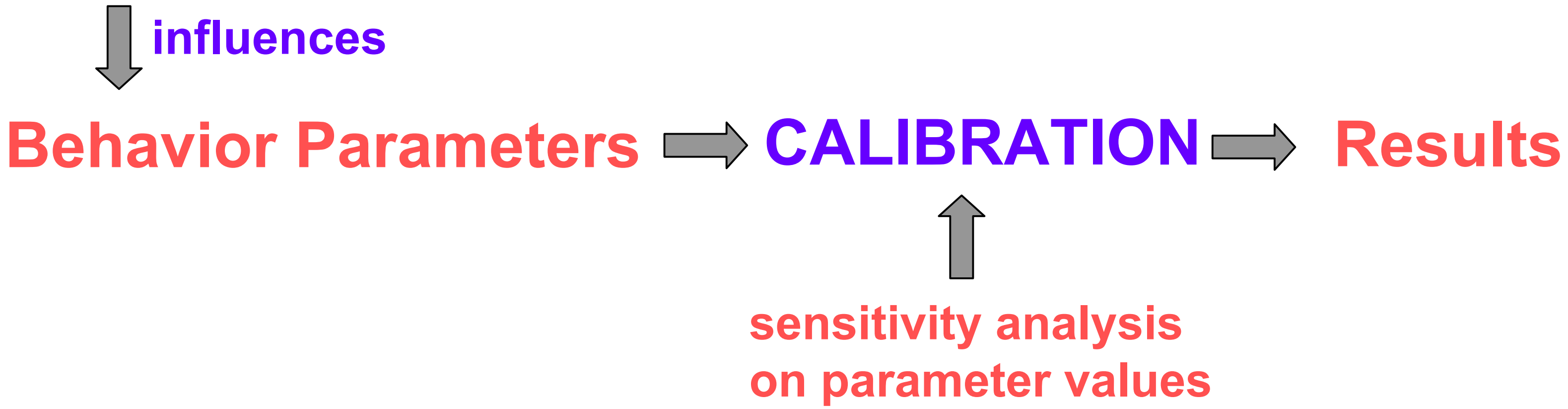
30 + 50 = 30 + 50

Building the Basic Data – things to do

4. Decide on functional forms e.g. Cobb-Douglas, CES, Leontief, LES, etc.

e.g. Cobb-Douglas => the benchmark data is sufficient to determine behavior parameter values

e.g. CES or LES => exogenous elasticity values are required



Cobb Douglas vs. CES

Cobb Douglas

Pros

- A special case of CES
- Easy to work with
- Unique calibration

Cons

- Income and own-price elast. = 1
- Cross-price elast. = 0

CES

Pros

- Commonly used in the CGE work
- Flexible for nested functions

Cons

- Not unique calibration
- Same elasticity of substitution between pair of goods or factors
- Messy math

Calibration – numerical example

Calibration of a Cobb Douglas production function w/o nested functions

$$Q_F = A_F K_F^{\alpha} L_F^{1-\alpha}$$

	Food	NonFood	Price
Labor	900	200	1
Capital	300	500	1
Total Production	1200	700	1
α	0.25	0.71	
1-α	0.75	0.29	
A	1.75	1.82	
Replication			
Total Production	1200	700	
Labor	900	200	
Capital	300	500	

please finish the rest as your exercise

$$\alpha_F = \frac{r_F K_F}{P_F Q_F} = \frac{1 \times 300}{1 \times 1200} = 0.25$$

$$A_F = \frac{Q_F}{K_F^{\alpha} L_F^{1-\alpha}} = \frac{1200}{300^{0.25} 900^{0.75}} = 1.75$$

$$Q_F = A_F K_F^{\alpha} L_F^{1-\alpha} = 1.75 \times 300^{0.25} \times 900^{0.75} = 1200$$

$$K_F = \frac{Q_F}{A_F}\left(\frac{\alpha w}{(1-\alpha) r}\right)^{1-\alpha} = \frac{1200}{1.75}\left(\frac{0.25 \times 1}{0.75 \times 1}\right)^{0.75} = 300$$

Note: see McCarl and Gillig for CES calibration

Calibration – numerical example

Calibration of a Cobb Douglas production function w/ nested functions

	Food	NonFood	Price
Food	0	100	1
NonFood	200	0	1
Value-Added	1200	700	1
Labor	900	200	1
Capital	300	500	1
Total Production	1400	800	1

Replication	Food	NonFood
Food	0	100
NonFood	200	0
Value-Added	1200	700
Labor	900	200
Capital	300	500
Total Production	1400	800

please check replication as your exercise

Top Level (Leontief)		
α Food	0	0.125
α NonFood	0.143	0
α Value-Added	0.857	0.875

Bottom Level (CD)		
α	0.250	0.714
1-α	0.750	0.286
A	1.755	1.819

$$\alpha_{F,NF} = \frac{QINT_{F,NF}}{Q_{NF}} = \frac{100}{800} = 0.125$$

$$\alpha_{NF,F} = \frac{QINT_{NF,F}}{Q_{F}} = \frac{200}{1400} = 0.143$$

$$\alpha_{F}^{va} = \frac{QVA_{F}}{Q_{F}} = \frac{1200}{1400} = 0.857$$

$$\alpha_{F} = \frac{rK_{F}}{PVA_{F}QVA_{F}} = \frac{1\times 300}{1\times 1200} = 0.25$$

please finish the rest as your exercise

Calibration – numerical example

Calibration of a CES utility function

	Food	NonFood	Labor	Capital	Transfer payments
Household	1020	910	1100	800	30
Price	1	1	1	1	
σ	0.7	0.7			
ω	1.121	0.892			
θ	1.00	1.00			
α	0.5285	0.4715			

$$\omega_{F,NF} = \frac{P_F X_F}{P_{NF} X_{NF}} = \frac{1 \times 1020}{1 \times 910} = 1.121$$

$$\omega_{NF,F} = \frac{P_{NF} X_{NF}}{P_F X_F} = \frac{1 \times 910}{1 \times 1020} = 0.892$$

$$\theta_{F,NF} = \frac{P_F}{P_{NF}} = 1$$

$$\alpha_{F,NF} = \frac{\omega_{F,NF}}{(\theta_{F,NF})^{1-\sigma} + \omega_{F,NF}} = 1.121/(1+1.121) = 0.5285$$

$$\alpha_{NF,F} = \frac{\omega_{NF,F}}{(\theta_{NF,F})^{1-\sigma} + \omega_{NF,F}} = 0.892/(1+0.892) = 0.4715$$

$$X_F = \frac{(\alpha_{F,NF})(Income)}{P_F^{\sigma}(\alpha_{F,NF} \times P_F^{1-\sigma} + \alpha_{NF,F} \times P_{NF}^{1-\sigma})}$$

$$= \frac{0.528 \times (1100 + 800 + 30)}{1^{0.7} \times (0.528 \times 1^{1-0.7} + 0.472 \times 1^{1-0.7})} = 1020$$

Replication	Food	NonFood
Total Consumption	1020	910

⇩

please finish the rest as your exercise

Wrap Up

- Hierarchical (nested) function & functional forms
- SAM & Input-output data
- Building benchmark equilibrium data sets
- Parameters calibration

Next:

- Shoven, J. B. and J. Whalley. “Applied General-Equilibrium Models of Taxation and International Trade: An Introduction and Survey.” *J. Economic Literature*, 22:1007-1051, 1984.

Appendix

NOTE:

Materials presented in Appendices A to E are based on TMD Discussion Paper No. 75 by Lofgren et al. (2001). There are several equations or functions that are not presented in the class notes due to the limitations in time and space. However, one who is interested to explore the CGE profoundly can get a copy of this paper at http:www.cgiar.org/ifpri/divs/tmd/dp.htm.

Appendix - A

VA Production function:

$$QVA_j = \alpha_j^{va}\left(\delta_j^{va} L_j^{(\sigma_j^{va}-1)/\sigma_j^{va}} + (1-\delta_j^{va})K_j^{(\sigma_j^{va}-1)/\sigma_j^{va}}\right)^{\sigma_j^{va}/(\sigma_j^{va}-1)}$$

where $\alpha_j^{va}, \delta_j^{va}, and\ \ \sigma_j^{va}$ are efficiency, share, and elasticity of substitution between **L** and **K** factors in sector **j** parameters.

Factor demand function:

$$r_j \overline{r}_j = PVA_j(1-tva_j)\,QVA_j\left[\delta_j^{va} L_j^{(\sigma_j^{va}-1)/\sigma_j^{va}} + (1-\delta_j^{va})K_j^{(\sigma_j^{va}-1)/\sigma_j^{va}}\right]^{-1}\delta_j^{va} K_j^{-1/\sigma_j^{va}}$$

$$w_j \overline{w}_j = PVA_j(1-tva_j)\,QVA_j\left[\delta_j^{va} L_j^{(\sigma_j^{va}-1)/\sigma_j^{va}} + (1-\delta_j^{va})K_j^{(\sigma_j^{va}-1)/\sigma_j^{va}}\right]^{-1}\delta_j^{va} L_j^{-1/\sigma_j^{va}}$$

where r, w, and PVA_j are capital, labor, and value-added prices, tva_j is value-added tax.

Appendix - B

Top level production function:

$$Q_j = \alpha_j \left(\delta_j QVA_j^{(\sigma_j - 1)/\sigma_j} + (1 - \delta_j) QINT_j^{(\sigma_j - 1)/\sigma_j} \right)^{\sigma_j/(\sigma_j - 1)}$$

where

Q_j	=	output in sector **j**
$QINT_j$	=	quantity of intermediate inputs in sector **j**
QVA_j	=	quantity of value-added in sector **j**

and α_j, δ_j, σ_j are efficiency, share, and elasticity of substitution between intermediate inputs and value-added in sector **j** parameters.

Appendix - C

Constant Elasticity of Transformation (CET) function:

$$QQ_j = \alpha_j^t \left(\delta_j^t QD_j^{(\sigma_j^t - 1)/\sigma_j^t} + (1 - \delta_j^t) QX_j^{(\sigma_j^t - 1)/\sigma_j^t} \right)^{\sigma_j^t/(\sigma_j^t - 1)}$$

where QQ_j, QD_j, QX_j, are aggregate outputs, domestic outputs, and exports in sector **j**, and α^t_j, δ^t_j, and σ^t_j are efficiency, share, and elasticity of substitution between domestic outputs (QD_j) and exported goods (QX_j) in sector **j** parameters.

Appendix - D

Household consumption function:

Utility maximization

$$U_h = \left[\sum_j \left(\alpha_{jh}\right)^{1/\sigma_h} \left(X_{jh}\right)^{(\sigma_h - 1)/\sigma_h} \right]^{\sigma_h/(\sigma_h - 1)}$$

s.t

$$\sum_j P_j X_{jh} \leq W_L \overline{L}_h + W_K \overline{K}_h \equiv Income_h$$

yields demand function:

$$X_{jh} = \frac{\alpha_{jh}\left(Income_h\right)}{P_j^{\sigma_h} \sum_j \left(\alpha_{jh}\left(P_j\right)^{1-\sigma_h}\right)}$$

where P_j is prices of goods **j**, and α_{jh} and σ_h are consumption share in household **h** in sector **j** parameters and elasticity of substitution in household **h** parameters.

Appendix - E

Armington function:

$$QC_j = \alpha_j^c \left(\delta_j^c QD_j^{(\sigma_j^c - 1)/\sigma_j^c} + (1 - \delta_j^c) QM_j^{(\sigma_j^c - 1)/\sigma_j^c} \right)^{\sigma_j^c/(\sigma_j^c - 1)}$$

where QC_j, QD_j, QM_j, are composite, domestic, and imported goods in sector **j**, $\alpha_j^c, \delta_j^c, and\ \sigma_j^c$ are efficiency, share, and elasticity of substitution between domestic goods (QD_j) and imported goods (QM_j) in sector **j** parameters.

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