Introduction to Computable General Equilibrium Model (CGE)

Dhazn Gillig
&
Bruce A. McCarl

Department of Agricultural Economics
Texas A&M University

Course Outline

- Overview of CGE
- **An Introduction to the Structure of CGE**
- An Introduction to GAMS
- Casting CGE models into GAMS
- Data for CGE Models & Calibration
- Incorporating a trade & a basic CGE application
- Evaluating impacts of policy changes and casting nested functions & a trade in GAMS
- Mixed Complementary Problems (MCP)

This Week’s Road Map

- Fundamental relationships in a simple CGE model
- Incorporating taxes
- Including demand for products and factors
- Interpretation of results
- Comparative analysis

Fundamental Structure

- **CGE models typically involve determination of economy wide levels of**

  1. commodity prices based on consumer demand and production possibilities/costs
  2. factor prices based on supply and production possibilities/product prices
  3. factor usage
  4. production levels
  5. income to households and resultant demand
  6. Government balance

Fundamental Structure

- **Basic relationships for a simple CGE**

1. Supply–Demand identities for factors & products
2. Zero profit condition for producing Industries
3. Factor demand by producers
4. Product demand by households
5. Income balance constraint for households
6. Government balance
7. Trade balance

Fundamental Structure

- Basic characteristics of CGE model solutions:

  A set of non-zero prices, consumption levels, production levels, and factor usages constitutes an economic equilibrium solution (also called a Walrasian equilibrium) and a solution to a CGE of the situation if

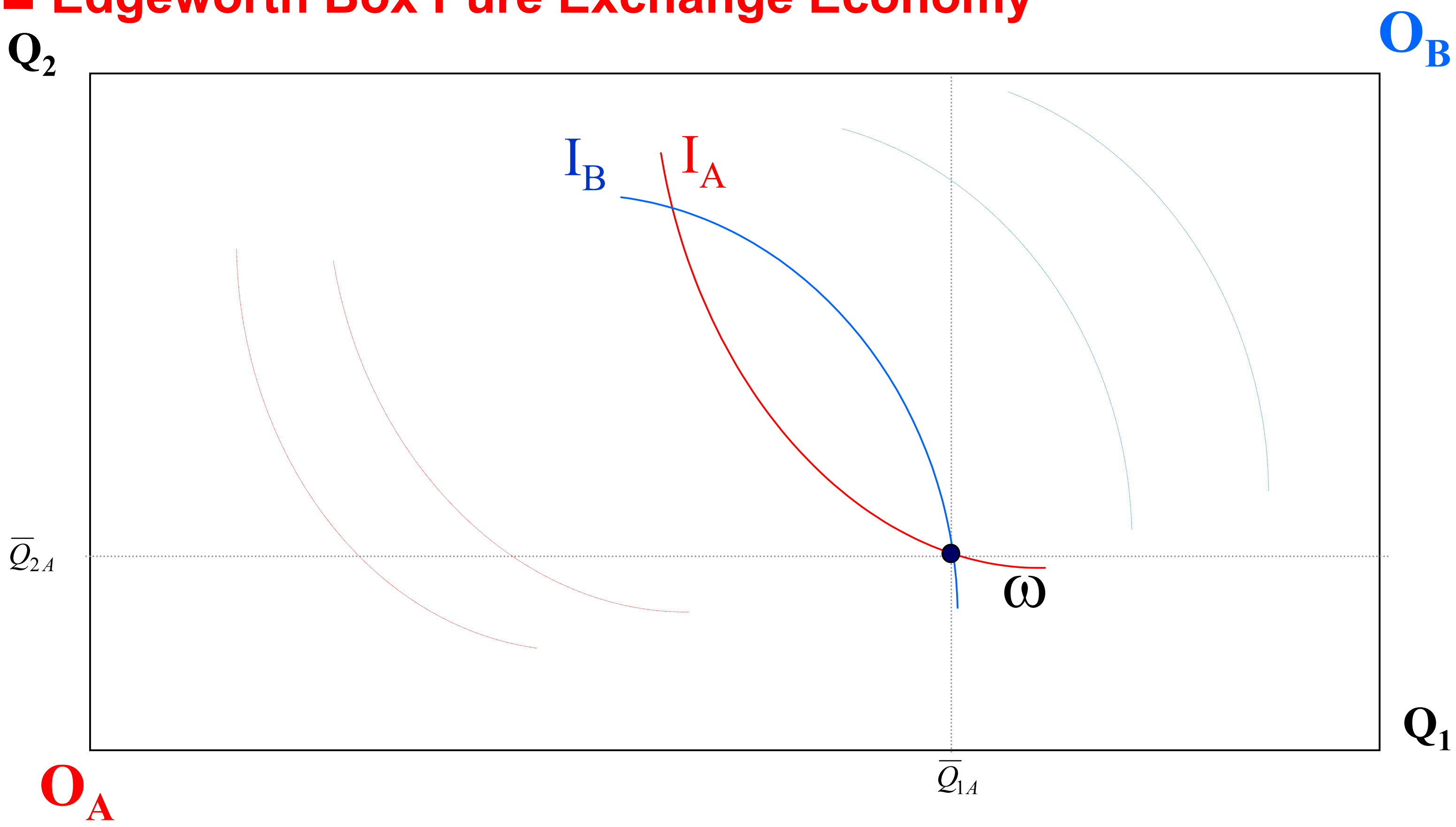
  1. Total market demand equals total market supply for each and every factor and output.

  2. Prices are set so that equilibrium profits of firms are zero with all rents accruing to factors.

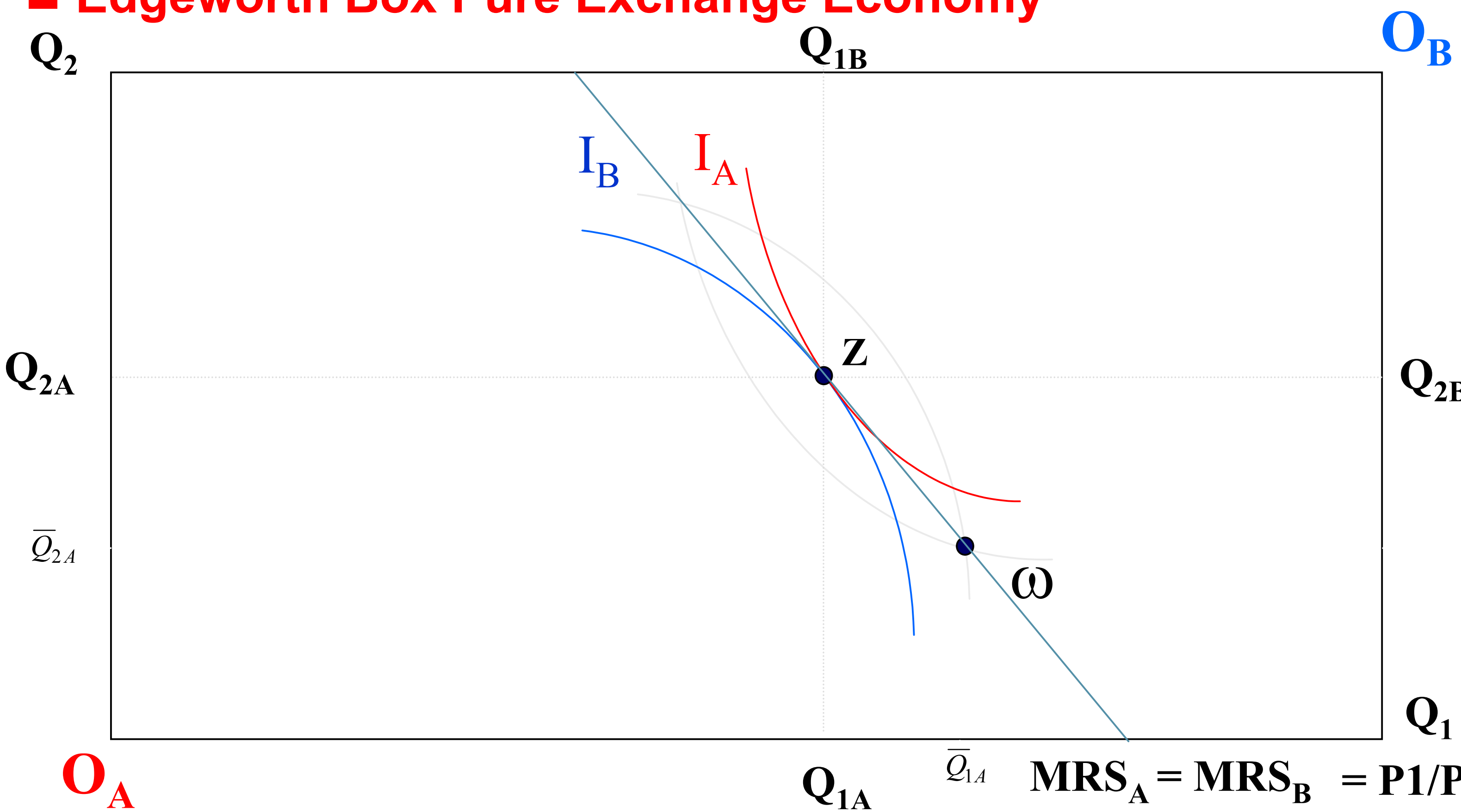
  3. Household income equals household expenditures.

  4. Government transfer payments to consumers equals taxes revenues.

■ Edgeworth Box Pure Exchange Economy



Edgeworth Box Pure Exchange Economy



$$\mathbf{MRS_A = MRS_B = P1/P2}$$

$$Max\ \mathbf{U}^{\mathbf{A}}$$
$$s.t\ P_1Q_1^{\mathbf{A}} + P_2Q_2^{\mathbf{A}} \le P_1\overline{Q}_{1\mathbf{A}} + P_2\overline{Q}_{2\mathbf{A}}$$

$$\frac{\partial L}{\partial Q_1^{\mathbf{A}}} \Rightarrow \frac{\partial \mathbf{U}^{\mathbf{A}}}{\partial Q_1^{\mathbf{A}}} - \lambda P_1 = 0 \ \textbf{ and } \ \frac{\partial L}{\partial Q_2^{\mathbf{A}}} \Rightarrow \frac{\partial \mathbf{U}^{\mathbf{A}}}{\partial Q_2^{\mathbf{A}}} - \lambda P_2 = 0$$
$$\frac{\partial \mathbf{U}^{\mathbf{A}}}{\partial Q_1^{\mathbf{A}}} \Big/ \frac{\partial \mathbf{U}^{\mathbf{A}}}{\partial Q_2^{\mathbf{A}}} = \frac{P_1}{P_2}$$

$$Max\ \mathbf{U}^{\mathbf{B}}$$
$$s.t\ P_1Q_1^{B} + P_2Q_2^{B} \le P_1\overline{Q}_{1B} + P_2\overline{Q}_{2B}$$

$$\frac{\partial L}{\partial Q_1^{B}} \Rightarrow \frac{\partial \mathbf{U}^{\mathbf{B}}}{\partial Q_1^{B}} - \lambda P_1 = 0 \ \textbf{ and } \ \frac{\partial L}{\partial Q_2^{B}} \Rightarrow \frac{\partial \mathbf{U}^{\mathbf{B}}}{\partial Q_2^{B}} - \lambda P_2 = 0$$
$$\frac{\partial \mathbf{U}^{\mathbf{B}}}{\partial Q_1^{B}} \Big/ \frac{\partial \mathbf{U}^{\mathbf{B}}}{\partial Q_2^{B}} = \frac{P_1}{P_2}$$

■ Edgeworth Box Pure Exchange Economy

Implications:

$$Q_{1A} + Q_{1B} = \overline{Q}_{1A} + \overline{Q}_{1B} = Q_1$$
$$Q_{2A} + Q_{2B} - \overline{Q}_{2A} + \overline{Q}_{2B} = Q_2$$

Feasible allocation

$$(Q_{1A} - \overline{Q}_{1A}) + (Q_{1B} - \overline{Q}_{1B}) = 0$$
$$(Q_{2A} - \overline{Q}_{2A}) + (Q_{2B} - \overline{Q}_{2B}) = 0$$

$$Z_1 = (Q_{1A} - \overline{Q}_{1A}) + (Q_{1B} - \overline{Q}_{1B}) = 0$$
$$Z_2 = (Q_{2A} - \overline{Q}_{2A}) + (Q_{2B} - \overline{Q}_{2B}) = 0$$

Excess demand

$$P_1 Q_{1A} + P_2 Q_{2A} = P_1 \overline{Q}_{1A} + P_2 \overline{Q}_{2A}$$
$$P_1 Q_{1B} + P_2 Q_{2B} = P_1 \overline{Q}_{1B} + P_2 \overline{Q}_{2B}$$

$$P_1 (Q_{1A} - \overline{Q}_{1A}) + P_2 (Q_{2A} - \overline{Q}_{2A}) = 0$$
$$P_1 (Q_{1B} - \overline{Q}_{1B}) + P_2 (Q_{2B} - \overline{Q}_{2B}) = 0$$

Budget constraints

$$P_1 (Q_{1A} - \overline{Q}_{1A}) + P_2 (Q_{2A} - \overline{Q}_{2A})$$
$$+ P_1 (Q_{1B} - \overline{Q}_{1B}) + P_2 (Q_{2B} - \overline{Q}_{2B}) = 0$$

$$P_1 Z_1 + P_2 Z_2 = 0$$ **Walras' Law**

Walras' Law

For any price vector P, PZ(P) = 0; i.e., the value of the excess demand, is identically zero, (Varian, page 317)

Fundamental Structure

- Basic characteristics of CGE model solutions:

  A set of non-zero prices, consumption levels, production levels, and factor usages constitutes an economic equilibrium solution (also called a Walrasian equilibrium) and a solution to a CGE of the situation if

  1. Total market demand equals total market supply for each and every factor and output.
  2. Prices are set so that equilibrium profits of firms are zero with all rents accruing to factors.
  3. Household incomes equal household expenditures.
  4. Government transfer payments to consumers + consumptions equal taxes revenues.

Fundamental Structure

- **Basic relationships for a simple CGE**

1. Supply–Demand identities for factors & products
2. Zero profit condition for producing Industries
3. Factor demand by producers
4. Product demand by households
5. Income balance constraint for households
6. Government balance

Fundamental Structure

- Basic notations:

Sectors denoted by j

Households denoted by h

Factors denoted by L and K and are owned by households

Q_j is the production of goods by the j^{th} sector

P_j is the price of goods produced by the j^{th} sector

$a_{j1,j}$ is the amount of goods used from sector $j1$ when producing one unit of goods in the j^{th} sector

L_j is the usage of labor by the j^{th} sector

W_L is the price of labor (the wage rate)

K_j is the usage of capital in the j^{th} sector

W_K is the price of capital

Fundamental Structure

1. **Supply-Demand identities for factors & products**

   a. Factor market:

   Total demand is less than or equal to total supply in every factor market or the excess demand in the factor market is less than or equal to zero.

$$\sum_j L_j - \sum_h \overline{L}_h \leq 0$$

$$\sum_j K_j - \sum_h \overline{K}_h \leq 0$$

Total supply is the sum across the household endowments

Fundamental Structure

1. **Supply-Demand identities (con't)**

   b. Product or output market:

   Total demand in every output market including consumer and intermediate production usage is less than or equal to total supply in that market or the excess demand in each output market is less than or equal to zero.

$$\sum_{h} X_{jh} \quad + \quad \sum_{j1} a_{j,j1} Q_{j1} \quad - \quad Q_j \ \leq \ 0 \qquad \forall\, j$$

Consumer consumption (ff)

Intermediate usage

Total production (ff)

Fundamental Structure

2. Zero profits in each sector

$$\underbrace{P_j Q_j}_{\text{Revenues}} = \underbrace{\sum_{j1} P_{j1}\, a_{j1,j} Q_j + W_L L_j + W_K K_j}_{\text{Costs}} \qquad \forall\ j$$

Note that: CRS + Perfect Competition

Unit price = unit cost

Fundamental Structure

3. Household income identity

This relationship implies that household exhausts its income.

$$Income_h \geq W_L \overline{L}_h + W_K \overline{K}_h$$

Note that:

Household consumptions (X_j) are a function of price and income.

Maximize $U(X_j)$

s.t. $\sum_j P_j X_j \leq W_L \overline{L} + W_K \overline{K} = Income$

Complementarity

Recall:

Walras' Law: *For any price vector **P**, **PZ(P)** = 0; i.e., the value of the excess demand*, *is identically zero,* (Varian, page 317)

Namely, if total **demand** is less than total **supply** for the factor/commodity markets then the **price** in that market must be zero; otherwise, prices will be nonzero only if supply equals demand

This implies PRICE OR EXCESS DEMAND = 0.

This leads to the following complementary relationships.

Complementarity (con't)

In a CGE model, a set of prices *P* and quantities *Q* are defined as variables such that D = S (**Walras' Law**)

$$(Qs-Qd)\mathbf{P} = 0$$

$$(Pd-P)\mathbf{Qd} = 0$$

$$(Ps-P)\mathbf{Qs} = 0$$

Implications:

Each equation must be binding or an associated complementary variable must be zero.

IF P > 0 then Qs = Qd

IF Qd > 0 then Pd = P,

IF Qs > 0 then Ps = P, and

This is similar to KT conditions of the following optimization model.

Complementarity (con't)

$$Max \;\; 6Qd\text{-}0.15Qd^2 - Qs - 0.1Qs^2$$
$$s.t \qquad Qd - Qs \le 0$$
$$Qd, Qs \ge 0$$

Pd = 6 - 0.3*Qd
Ps = 1 + 0.2*Qs

$$\frac{\partial L}{\partial Qd} = 0 \rightarrow \mathbf{6 - 0.3}Qd - P \le \mathbf{0} \qquad \left(\frac{\partial L}{\partial Qd}\right)Qd = 0$$

$$\frac{\partial L}{\partial Qs} = 0 \rightarrow \mathbf{1 + 0.2}Qs - P \ge \mathbf{0} \qquad \left(\frac{\partial L}{\partial Qs}\right)Qs = 0$$

$$\frac{\partial L}{\partial P} = 0 \rightarrow Qs - Qd \ge \mathbf{0} \qquad \left(\frac{\partial L}{\partial P}\right)P = 0$$

$$(Pd - P)Qd = 0$$

$$(Ps - P)Qs = \mathbf{0}$$

$$(Qs - Qd)P = \mathbf{0}$$

where P is the dual variable associated with the first constraint.

Complementarity (con't)

1.

$$0 \le W_L \perp \sum_j L_j - \sum_h \bar{L}_h \le 0$$

$$0 \le W_K \perp \sum_j K_j - \sum_h \bar{K}_h \le 0$$

representing complementary relationship

Factor prices must be zero if factors are not all used up.

Non zero prices exist if factors all are consumed.

2. $$0 \le P_j \perp \sum_h X_{jh} + \sum_{j1} a_{j,j1} Q_{j1} - Q_j \le 0 \quad \forall j$$

Product prices must be zero if products are not all consumed.

Non zero prices exist if products all are consumed.

Complementarity (con't)

3. $0 \leq Q_j \quad \perp \quad P_j Q_j \leq \sum_{j1} P_{j1} \mathbf{a_{j1,j}} Q_j + W_L L_j + W_K K_j$

Firm profits must equal zero and a non-zero production level is achieved.

Firm profits can be less than costs without the firm producing.

4. $0 \leq Income_h \quad \perp \quad Income_h \geq W_L \overline{L}_h + W_K \overline{K}_h$

Household incomes must be non-zero if expenditures exhaust incomes.

Complementarity (con't)

5. Factor demand by producers identity

(functional forms: CES, Cobb Douglas, Leontief?)

$$0 \le L_j \perp \quad L_j = \frac{1}{\phi_j} Q_j \left[\delta_j + (1-\delta_j) \left(\frac{\delta_j W_K}{(1-\delta_j) W_L} \right)^{(1-\sigma_j)} \right]^{\sigma_j/(1-\sigma_j)}$$

$$0 \le K_j \perp \quad K_j = \frac{1}{\phi_j} Q_j \left[\delta_j \left(\frac{(1-\delta_j) W_L}{\delta_j W_K} \right)^{(1-\sigma_j)} + (1-\delta_j) \right]^{\sigma_j/(1-\sigma_j)}$$

Complementarity (con't)

6. Product demand by households identity

(functional forms: CES, Cobb Douglas, LES ?)

$$0 \leq X_{jh} \quad \perp \quad X_{jh} = \frac{\alpha_j \left(Income_h \right)}{P_j^{\sigma} \sum_j \left(\alpha_j \left(P_j \right)^{1-\sigma} \right)}$$

Incorporating Taxes

- Basic characteristics of CGE model solutions:

A set of non-zero prices, consumption levels, production levels, and factor usages constitutes an economic equilibrium solution (also called a Walrasian equilibrium) and a solution to a CGE of the situation if

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2. Prices are set so that equilibrium profits of firms are zero with all rents accruing to factors.
3. Household income equals household expenditures.
4. **Government transfer payments to consumers equal taxes revenues.**

Incorporating Taxes (con't)

- Fundamental structure

1. Supply-Demand identities
2. Zero profits in each sector
3. Household income-expenditure balance

4. **Government tax revenue balance**

   This relationship implies that the government satisfies its budget constraint when total revenue is positive.

Incorporating Taxes (con't)

Assumptions:

1. Total tax revenues are
   a. redistributed to households in the form of transfer payments (TR_h) at the rate (s_h) => $TR_h = s_h R$

   OR b. expended on government purchase of goods (GP_j)

2. Government purchases are proportional to tax revenues at the rate (s_j) => $GP_j = s_j R$

$$\sum_h s_h + \sum_j s_j = 1$$

3. t_h percent is imposed on household income

   t_{fj} percent is imposed on factor f in sector j

   F_h is a total deduction imposed on household

Incorporating Taxes (con't)

- Modification

1. The market balance

   : include government purchases transformed to be in a quantity unit

$$\sum_{h} X_{jh} + \sum_{j1} \mathbf{a_{j,j1}} Q_{j1} - Q_j \boxed{+ s_j R / P_j} \le 0 \qquad \forall\, \mathbf{j}$$

2. The zero profit condition

   : include a tax effect as a cost of doing business

$$P_j Q_j \le \sum_{j1} P_{j1}\, a_{j1,j} Q_j + \boxed{1+t_{lj}}\, W_L L_j + \boxed{1+t_{kj}}\, W_K K_j \qquad \forall\, j$$

Incorporating Taxes (con't)

3. The household income equation

: include tax effects to reflect tax incidence and tax revenue redistribution

$$\boxed{(1-t_h)}\left(W_L \overline{L}_h + W_K \overline{K}_h - F_h\right) + \boxed{s_h \mathbf{R}} \le Income_h$$

4. Add the government tax revenue balance

: apply the household income tax **(t_h)** to gross revenue less deductions, and the factor tax **(t_{fj})**

$$R \le \sum_h t_h\left(W_L \overline{L}_h + W_K \overline{K}_h - F_h\right)$$
$$+ t_{lj} W_L L_j + t_{kj} W_K K_j$$

Incorporating Taxes (con't)

5. Complementary

$$0 \le R \quad \perp \quad R \;\le\; \sum_h t_h\left(W_L \overline{L}_h + W_K \overline{K}_h - F_h\right) + t_{lj} W_L L_j + t_{kj} W_K K_j$$

the government satisfies its budget constraint when total revenue is positive.

Including Demand for Products and Factors

- Specification of product and factor demand functions should be consistent with theory but analytically tractable.

  (1). Theoretical approach

  - choosing functional forms that satisfy

    : classical restrictions i.e. nonnegative, continuous, and homogenous degree zero in price

    : Walras's law => the value of market excess demands equals zero at all prices

  (2). Analytically tractable

  - choosing functional forms that are easy to evaluate e.g. Leontief, CD, CES, LES, etc.

Including Demand for Products and Factors

- Factor demand derived from CES function

  : Production function

$$Q_j = \phi_j(\delta_j L_j^{(\sigma_j - 1)/\sigma_j} + (1-\delta_j)K_j^{(\sigma_j - 1)/\sigma_j})^{\sigma_j/(\sigma_j - 1)}$$

: Factor demand

$$L_j = \frac{1}{\phi_j} Q_j \left[\delta_j + (1-\delta_j)\left(\frac{\delta_j W_K}{(1-\delta_j)W_L}\right)^{(1-\sigma_j)} \right]^{\sigma_j/(1-\sigma_j)}$$

$$K_j = \frac{1}{\phi_j} Q_j \left[\delta_j \left(\frac{(1-\delta_j)W_L}{\delta_j W_K}\right)^{(1-\sigma_j)} + (1-\delta_j) \right]^{\sigma_j/(1-\sigma_j)}$$

Note that: $\sigma \to 0$ then CES tends to Leontief

$\sigma \to 1$ then CES tends to Cobb - Douglas

Including Demand for Products and factors

- Household product demand from CES Utility function

Maximize Utility

$$U = \left[\sum_j (\alpha)^{1/\sigma} (X_j)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

s.t

$$\sum_j P_j X_j \leq W_L \overline{L} + W_K \overline{K} \equiv Income$$

Yields Demand Curve

$$X_j = \frac{\alpha_j (Income)}{P_j^{\sigma} \sum_j \left(\alpha_j (P_j)^{1-\sigma} \right)}$$

Numerical Example

Symbol	Brief Description
σ_h	Elasticity of substitution in household CES
α_{jh}	Consumption share in household CES
$\bar{L}_h, \bar{K}_h$	Household endowments of factors
$a_{j1,j}$	Use of goods in sector1 when producing in sector j
ϕ_j	Scale parameter in CES production function
δ_j	Distribution parameter in CES production
σ_j	Elasticity of production factor substitution
s_h	Household share of tax disbursements
s_j	Government goods purchase dependence on revenues
t_h	Household tax level
t_{fj}	Tax on factor *f* in sector *j*
F_h	Household tax exemptions

Parameter Specification

- **Example of simple 2x2x2 CGE (Shoven and Whalley 1984)**

Production Parameters

Sector (j)	ϕ_j	δ_j	σ_j
Food	1.5	0.6	2.0
Non-Food	2.0	0.7	0.5

t_{fj}

Labor	Capital
0.0	0.0
0.0	0.0

Consumer Parameters

Household (h)	σ_h	s_h	t_h	F_h
Farmer	0.75	0.6	0.00	0.0
Non-Farmer	1.5	0.4	0.00	0.0

	α_{hj}		Endowments	
	Food	Non-Food	**Labor**	**Capital**
Farmer	0.3	0.7	60	0
Non-Farmer	0.5	0.5	0	25

Government

Food	0.0
Non-Food	0.0

Equilibrium Results

1. **Total demand for each output exactly matches the amount produced**

```
----     323 PARAMETER HHdemand  Total quantity demand for output

                   Food      NonFood

NonFarmer        11.515       16.675
Farmer           13.428       37.704
Total            24.942       54.378


----     323 PARAMETER ProdQ  Total quantity produced

Food      24.942,     NonFood 54.378
```

$$X_{jh} = \frac{\alpha_j \left(Income_h\right)}{P_j^{\sigma} \sum_j \left(\alpha_j \left(P_j\right)^{1-\sigma}\right)}$$

$$Q_j = \phi_j(\delta_j L_j^{(\sigma_j - 1)/\sigma_j} + (1-\delta_j) K_j^{(\sigma_j - 1)/\sigma_j})^{\sigma_j/(\sigma_j - 1)}$$

Recall: $\sum_h X_{jh} + \cancel{\sum_{j1} a_{j,j1} Q_{j1}} - Q_j \leq 0 \quad \forall j$

Equilibrium Results (con't)

2. Producer revenues equal consumer expenditures

```
----     323 PARAMETER ConExpense  Consumer expenditures

                  Food      NonFood

NonFarmer       16.110       18.227
Farmer          18.787       41.213
Total           34.897       59.439
```

$\rightarrow \sum_h P_j X_{jh}$

```
----     323 PARAMETER ProdRev  Producer revenues

Food     34.897,     NonFood 59.439,     Total    94.337
```

$\rightarrow P_j Q_j$

Equilibrium Results (con't)

3. Labor and capital are exhausted

```
----     323 PARAMETER ResrcQuan  Factor endowment levels

                  Food      NonFood        Total

Labor           26.366       33.634       60.000
Capital          6.212       18.788       25.000
```

$$L_j = \frac{1}{\phi_j} Q_j \left[\delta_j + (1-\delta_j) \left(\frac{\delta_j W_K}{(1-\delta_j) W_L} \right)^{(1-\sigma_j)} \right]^{\sigma_j/(1-\sigma_j)}$$

$$K_j = \frac{1}{\phi_j} Q_j \left[\delta_j \left(\frac{(1-\delta_j) W_L}{\delta_j W_K} \right)^{(1-\sigma_j)} + (1-\delta_j) \right]^{\sigma_j/(1-\sigma_j)}$$

Equilibrium Results (con't)

4. Unit cost = selling price => zero profits

```
----     323 PARAMETER UnitCost  Unit cost

Food      1.399,     NonFood 1.093
```

$wl + rk$

```
----     323 PARAMETER UnitPrice  Unit selling price

Food      1.399,     NonFood 1.093
```

From model solutions

Equilibrium Results (con't)

5. Consumer factor incomes equal producer factor costs

```
----     323 PARAMETER ResrcIncom  Consumer factor incomes

                  Labor     Capital

NonFarmer                    34.337
Farmer           60.000
Total            60.000      34.337
```

$$W_L \overline{L}_h + W_K \overline{K}_h$$

WL = 1.00

WK = 1.37

```
----     323 PARAMETER ResrcCost  Producer factor costs

                 Food     NonFood       Total

Labor          26.366      33.634      60.000
Capital         8.532      25.805      34.337
```

$$W_L L_j + W_K K_j$$

Equilibrium Results (con't)

6. Household expenditures exhaust their incomes

```
----      323 PARAMETER ConExpense  Consumer expenditures

                     Food       NonFood

NonFarmer           16.110       18.227
Farmer              18.787       41.213
Total               34.897       59.439
```

$$\sum_j P_j X_j$$

```
----      323 PARAMETER ResrcIncom  Consumer factor incomes

                    Labor       Capital

NonFarmer                        34.337
Farmer              60.000
Total               60.000       34.337
```

$$W_L \overline{L}_h + W_K \overline{K}_h$$

Introducing Shocks

(1). Imposing a 50% tax on a capital used in a food sector

Production Parameters

Sector (*j*)	ϕ_j	δ_j	σ_j	t *j*
Food	1.5	0.6	2.0	0.0
Non-Food	2.0	0.7	0.5	0.0

t_{fj}

Labor	Capital
0.0	0.5
0.0	0.0

$$L_j = \frac{1}{\phi_j} Q_j \left[\delta_j + (1-\delta_j) \left(\frac{\delta_j (1+t_{kj}) W_K}{(1-\delta_j) W_L} \right)^{(1-\sigma_j)} \right]^{\sigma_j/(1-\sigma_j)}$$

$$K_j = \frac{1}{\phi_j} Q_j \left[\delta_j \left(\frac{(1-\delta_j) W_L}{\delta_j (1+t_{kj}) W_K} \right)^{(1-\sigma_j)} + (1-\delta_j) \right]^{\sigma_j/(1-\sigma_j)}$$

Introducing Shocks

1. The market balance

$$\sum_h X_{jh} - Q_j \quad \boxed{+ s_j R / P_j} \quad \leq 0$$

2. The zero profit condition

$$P_j Q_j \leq \quad (1 \cancel{+} t_{lj}) W_L L_j + (1 + t_{kj}) W_K K_j$$

3. The household income equation

$$(1 - \cancel{t_i}) \left(W_L \overline{L}_h + W_K \overline{K}_h - F_h \right) + s_h \mathbf{R} \leq Income_h$$

4. Add the government tax revenue constraint

$$R = \sum_h \cancel{t_h} \left(W_L \overline{L}_h + W_K \overline{K}_h - F_h \right) + \cancel{t_{lj}} W_L L_j + t_{kj} W_K K_j$$

Introducing Shocks

(2). At equilibrium, transfer payments = tax revenues

```
----     639 TaxRevenue.L  =   2.28  Total government tax revenues

----     639 PARAMETER cTransfer  Transfer payments

                       Tax

NonFarmer             0.91
Farmer                1.37
Total                 2.28
```

$$TR_h = s_h R$$

$$R = \sum_h \cancel{t_h}\left(W_L \bar{L}_h + W_K \bar{K}_h - F_h\right) + \cancel{t_{lj}} W_L L_j + t_{kj} W_K K_j$$

Comparative Analysis

Note that: Tax revenue under non-CGE

```
----     639 PARAMETER cUnitCost  Unit cost
                   NoTax          Tax
Labor               1.00         1.00
Capital             1.37         1.13
```

excluding a tax
including tax = 1.69

```
----     639 PARAMETER cRsrcQuan  Factor endowment levels
                        NoTax          Tax
Capital.Food             6.21         4.04
Capital.NonFood         18.79        20.96
Capital.Total           25.00        25.00
```

1.37 * 6.212 * **0.5** = **$4.265** VS. CGE tax revenue **=$2.28**

Factor Cost **Factor Quantity** **Tax level**

Comparative Analysis (con't)

1. Unit cost = selling price => zero profits

```
----     639 PARAMETER cUnitCost  Unit cost

               NoTax           Tax

Food            1.40          1.47
NonFood         1.09          1.01

----     639 PARAMETER cUnitPrice  Unit selling price

               NoTax           Tax

Food            1.40          1.47
NonFood         1.09          1.01
```

Relative price P1/P2

= 1.40/1.09 = 1.28 (no tax)

= 1.47/1.01 = 1.45 (with tax)

Comparative Analysis

2. **Total demand for each output exactly matches the amount produced**

```
----     639 PARAMETER cHHdemand  Total quantity demand for output

                          NoTax         Tax
NonFarmer.Food            11.51        8.99
NonFarmer.NonFood         16.67       15.83
Farmer   .Food            13.43       13.40
Farmer   .NonFood         37.70       41.48
Total    .Food            24.94       22.39
Total    .NonFood         54.38       57.31

----     639 PARAMETER cProdQ  Total quantity produced

                NoTax         Tax
Food            24.94       22.39
NonFood         54.38       57.31
```

Comparative Analysis (con't)

3. Producer revenues equal consumer expenditures

```
----      639 PARAMETER cConExpense  Consumer expenditures

                              NoTax        Tax

NonFarmer.Food                16.11        13.18
NonFarmer.NonFood             18.23        15.92
Farmer   .Food                18.79        19.65
Farmer   .NonFood             41.21        41.72
Total    .Food                34.90        32.83
Total    .NonFood             59.44        57.64


----      639 PARAMETER cProdRev  Producer revenues

                NoTax         Tax

Food            34.90       32.83
NonFood         59.44       57.64
Total           94.34       90.47
```

Comparative Analysis (con't)

4. Labor and capital are exhausted

```
----      639 PARAMETER cRsrcQuan  Factor endowment levels
                         NoTax         Tax
Labor   .Food            26.37       26.00
Labor   .NonFood         33.63       34.00
Labor   .Total           60.00       60.00
Capital.Food              6.21        4.04
Capital.NonFood          18.79       20.96
Capital.Total            25.00       25.00
```

Comparative Analysis (con't)

5. Consumer factor incomes equal producer factor costs

```
----     639 PARAMETER cRsrcCost  Producer factor costs
                          NoTax        Tax
Labor   .Food             26.37      26.00
Labor   .NonFood          33.63      34.00
Labor   .Total            60.00      60.00
Capital.Food               8.53       6.83
Capital.NonFood           25.81      23.64
Capital.Total             34.34      30.47
----     639 PARAMETER cRsrcIncom  Consumer factor incomes
                            NoTax        Tax
NonFarmer.Capital           34.34      29.10
Farmer   .Labor             60.00      60.00
Farmer   .Capital                       1.37
Total    .Labor             60.00      60.00
Total    .Capital           34.34      30.47
```

Comparative Analysis (con't)

6. Household expenditures exhaust their incomes

```
----     639 PARAMETER cConExpense  Consumer expenditures

                        NoTax        Tax

NonFarmer.Food          16.11      13.18
NonFarmer.NonFood       18.23      15.92
Farmer   .Food          18.79      19.65
Farmer   .NonFood       41.21      41.72
Total    .Food          34.90      32.83
Total    .NonFood       59.44      57.64

----     639 PARAMETER cRsrcIncom  Consumer factor incomes

                        NoTax        Tax

NonFarmer.Capital       34.34      29.10
Farmer   .Labor         60.00      60.00
Farmer   .Capital                   1.37
Total    .Labor         60.00      60.00
Total    .Capital       34.34      30.47
```

Wrap Up

- Overview of CGE
- Fundamental relationship of simple CGE model
- Interpretation of results
- Incorporating shocks (Taxes)
- Comparative analysis

Next:

- Using GAMS for CGE Modeling
- What is GAMS?
- GAMS IDE
- Dissecting GAMS Formulation

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