Agricultural Economics 622, Spring 2005
Midterm Exam on Math programming Topic

1. (25 points) Darius's dairy is trying to figure out how to move milk to customers. Darius operates two dairies (Brenham and Stephenville) and serves customers in 3 cities (Houston, Dallas, and Austin). In Brenham they have 200,000 gallons available per day that costs 60 cents to produce while in Stephenville they have 500,000 gallons available per day that costs 55 cents per gallon. Their existing contracts are for 225,000 in Austin, 175,000 in Houston and 250,000 in Dallas. In addition a potential client in College Station has offered to buy up to 50,000 gallons at $1 per gallon. Costs per gallon for milk movement from Brenham are 10 cents to Houston and Austin, 7 cents to College Station and 13 cents to Dallas. Costs per gallon for milk movement from Stephenville are 18 cents to Houston, 12 cents to Austin, 13 cents to College Station and 8 cents to Dallas.

   Set up an LP model of this situation.

2. (15 points) In the context of problem number 1 above state a small case where each of the assumptions of LP might be violated (using no more than 2-3 sentences for each assumption).

3. (15 points) Given the model

Max	c1 * x1		+ c2*x2			
	2 * x1		+ 4 * x2	≤		55
	4 * x1		+ x2	≤		45
	x1	,	x2	≥		0

   Suppose that over time c1 and c2 have been uncertain where the mean of c1 is 10 and the mean of c2 is 11 while the standard error of c1 is 4, the standard error of c2 is 5, and their correlation is 0.5. Set a model that considers the risk and returns and tell how you numerically solve it for different risk return possibilities (note the covariance between c1 and c2 is the correlation times the product of the standard deviations).

4. (15 points) Give an example of where you would use the LP in problem 1 in each of
   a. a predictive setting and
   b. a prescriptive setting.

   Limit the answer for each to no more than 4 sentences.

5. (10 points) Suppose you have the model

Max	3 * x1		+ 2 *x2	-10* Y		
			+ 4 * x2	-50*Y ≤		0
	4 * x1		+ x2	≤		45
	x1	,	x2	≥		0
				Y is 0 or 1		

Explain the coefficients and function of the variable Y in this problem along with the constraint relating x2 and Y.

6. (20 points) Given the following problem where the
   - first two variables tell number of hogs to skin or scald in head
   - the next 4 tell how to cut up (disassemble)the hogs in head
   - the next 5 variables give products sold in number of skins or lbs for other products
   - the last variable gives waste disposed of in lbs

   - the first equation is profits in $
   - the second equation is hogs available in number of head
   - the next 5 equations balance products in number of skins and lbs for others
   - the next to last equation gives waste to be disposed of in lbs
   - the last equation limits labor in hours

   and the Excel solution below it
   a. indicate where you find the amount of each variable in the solution and provide a one sentence interpretation one would place on one nonzero and one zero variable
   b. indicate where you find the value of the shadow price for each constraint and provide a one sentence interpretation one would place on one of those shadow prices
   c. indicate where you find the value of the reduced costs for each variable not produced and provide a one sentence interpretation one would place on one of those reduced costs which is non zero.
   d. Tell what the objective function value is and give a one sentence interpretation one would place on that

	Skin	Scald	Hog pattern 1	Hog pattern 2	Hog pattern 3	Hog pattern 4	Skins	Hams	Bacon	Sausage	By Product	Waste		
Profit	-50	-48					8	1.9	1.85	1	0.4	-0.01		
Hogs for slaughter	-1	-1	1	1	1	1							≤	0
Skins	-1						1						≤	0
Ham			-40	-40				1					≤	0
Bacon			-20		-30				1				≤	0
Sausage			-50	-70	-90	-130				1			≤	0
By product		-30	-50	-50	-40	-30					1		≤	0
Waste			-40	-40	-40	-40						1	=	0
Labor	6	4	2.5	3.2	2.2								≤	5000

Plus all variables greater than or equal to zero

The solution follows

EXCEL answer sheet

Target Cell (Max)

Cell	Name	Original Value	Final Value
O18		-50	132000

Adjustable Cells

Cell	Name	Original Value	Final Value
B2	Skin	1	0
C2	Scald	0	1250
D2	Hog pattern 1	0	0
E2	Hog pattern 2	0	0
F2	Hog pattern 3	0	0
G2	Hog pattern 4	0	1250
H2	Skins	0	0
I2	Hams	0	0
J2	Bacon	0	0
K2	Sausage	0	162500
L2	By Product	0	75000
M2	Waste	0	50000

EXCEL sensitivity sheet

Adjustable Cells

Cell	Name	Final Value	Reduced Cost
B2	Skin	0	-58.8
C2	Scald	1250	0
D2	Hog pattern 1	0	-25.00
E2	Hog pattern 2	0	-60.48
F2	Hog pattern 3	0	-38.58
G2	Hog pattern 4	1250	0
H2	Skins	0	0
I2	Hams	0	0
J2	Bacon	0	0
K2	Sausage	162500	0
L2	By Product	75000	0
M2	Waste	50000	0

Constraints

Cell	Name	Final Value	Shadow Price
O20	Hogs for slaughter	0	141.6
O21	Skins	0	8
O22	Ham	0	1.9
O23	Bacon	0	1.85
O24	Sausage	0	1
O25	By product	0	0.4
O26	Waste	0	-0.01
O27	Labor	5000	26.4

Note this is 15 by 15 and does not imply the problem is that size

Test Answers and Grading Key

1. LP model (25 points)

   Darius's dairy is trying to figure out how to move milk to customers. Darius operates two dairies (Brenham and Stephenville) and serves customers in 3 cities (Houston, Dallas, and Austin). In Brenham they have 200,000 gallons available per day that costs 60 cents to produce while in Stephenville they have 500,000 gallons available per day that costs 55 cents per gallon. Their existing contracts are for 225,000 in Austin, 175,000 in Houston and 250,000 in Dallas. In addition a potential client in College Station has offered to buy up to 50,000 gallons at $1 per gallon. Costs per gallon for milk movement from Brenham are 10 cents to Houston and Austin, 7 cents to College Station and 13 cents to Dallas. Costs per gallon for milk movement from Stephenville are 18 cents to Houston, 12 cents to Austin, 13 cents to College Station and 8 cents to Dallas.

   - Point allocation
     - Transport variables to existing cities (6 points),
     - Objective function transport costs (3 points)
     - Objective function production costs (2 points),
     - Supply constraints (2 points)
     - Demand constraints (3 points)
     - Transport variables to college station (1 points),
     - College station demand constraint (1 points)
     - College station max demand constraint (1 points)
     - College station demand variable (2 points)
     - non-negativity (3 points)

Variant 1

Here we have explicit variables for production cost and demand

	Brenham milk	Stephenville milk	Brenham to Austin	Brenham to Houston	Brenham to Dallas	Brenham to College Station	Stephenville to Austin	Stephenville to Houston	Stephenville to Dallas	Stephenville to College Station	Demand College Station			
Objective	60	55	10	10	13	7	12	18	8	13	-100			
Brenham prod supply	1												≤	200000
Stephenville prod supply		1											≤	500000
Brenham supply	-1		+1	+1	+1	+1							≤	0
Stephenville supply		-1					+1	+1	+1	+1			≤	0
Austin Demand			+1				+1						≥	225000
Houston Demand				+1				+1					≥	175000
Dallas demand					+1				+1				≥	250000
College Station						+1				+1	-1		≥	0
Max college station											+1		≤	50000
All variables are non negative														

Variant 2

Here we add production cost to transport cost and have a variable for demand

	Brenham to Austin	Brenham to Houston	Brenham to Dallas	Brenham to College Station	Stephenville to Austin	Stephenville to Houston	Stephenville to Dallas	Stephenville to College Station	Demand College Station			
Objective	70	70	73	67	67	73	63	68	-100			
Brenham prod supply	+1	+1	+1	+1							≤	200000
Stephenville prod supply					+1	+1	+1	+1			≤	500000
Brenham supply											≤	0
Stephenville supply											≤	0
Austin Demand	+1				+1						≥	225000
Houston Demand		+1				+1					≥	175000
Dallas demand			+1				+1				≥	250000
College Station				+1				+1	-1		≥	0
Max college station									+1		≤	50000

Variant 3

Here we add production cost to transport cost and subtract cs price demand

	Brenham to Austin	Brenham to Houston	Brenham to Dallas	Brenham to College Station	Stephenville to Austin	Stephenville to Houston	Stephenville to Dallas	Stephenville to College Station			
Objective	70	70	73	-33	67	73	63	-32			
Brenham prod supply	+1	+1	+1	+1						≤	200000
Stephenville prod supply					+1	+1	+1	+1		≤	500000
Brenham supply										≤	0
Stephenville supply										≤	0
Austin Demand	+1				+1					≥	225000
Houston Demand		+1				+1				≥	175000
Dallas demand			+1				+1			≥	250000
College Station				+1				+1		≥	0
Max college station				+1				+1		≤	50000

2. Violation of LP assumptions (15 points)
    - 7 assumptions (2 points per each)
    - 1 point for showing up

1) Objective Function Appropriateness
    - Obj. Fn in the above case might not be the sole criteria for choosing the decision variables. The firm might take the supply risk into account
2) Decision Variable Appropriateness
    - We might have omitted major choice variables like buy milk from elsewhere
3) Constraint Appropriateness
    - There might be any omitted restrictions such as transport capacity
4) Proportionality
    - transport cost per unit shipped might be increasing or decreasing as the firm produces more
5) Additivity
    - There might be interactions between shipping routes lower or adding to costs
6) Divisibility
    - The firm might not be permitted to sell 4x4s or 2x4s in amounts smaller than one bundle
7) Certainty
    - The supply might vary.

(15 points)

3. Suppose that over time c1 and c2 have been uncertain where the mean of c1 is 10 and the mean of c2 is 11 while the standard error of c1 is 4, the standard error of c2 is 5, and their correlation is 0.5. Set a model that considers the risk and returns and tell how you numerically solve it for different risk return possibilities (note the covariance between c1 and c2 is the correlation times the product of the standard deviations).

$$\begin{array}{llllll} \text{Max} & 10 * x1 & + 11*x2 - RAP* x'Sx & & & \\ & 2 * x1 & + 4 * x2 & \leq & 55 \\ & 4 * x1 & + \quad x2 & \leq & 45 \\ & x1 \quad , & \quad x2 & \geq & 0 \end{array}$$

$$S=\begin{pmatrix} 16 & 10 \\ 10 & 25 \end{pmatrix} \quad x'Sx= 16\,x1^2 + 25\,x2^2 + 2 * 10 * x1 * x2$$

Vary RAP from 0 to larger values to get different risk return possibilities

Obj expected values	2 points
RAP and x'Sx	5 points
Empirical Var covar matrix	2 points
Use of RAP	6 points

4. Predictive and Prescriptive (15 points)

- 7 points for (a) and 8 points for (b)

(a) Predictive
- Consequence of adding customers or milk supply or changing transport costs

(b) Prescriptive
- What is the optimal transport pattern and should we take on college station.

3. Integer programming (10 points)

Max	3 * x1	+ 2 *x2	-10* Y		
		+ 4 * x2	-50*Y	≤	0
	4 * x1	+ x2		≤	45
	x1 ,	x2		≥	0
			Y is 0 or 1		

Explain the coefficients and function of the variable Y in this problem along with the constraint relating x2 and Y.

Y must be nonzero for x2 to be nonzero and is a zero one variable	(5 points)
-10 y is fixed cost	(3 points)
-50 y is maximum capacity for number of 4 x2 's	(2 points)

4. Interpretation of the Excel Solution

- 5 points for each question (2 points for values and 3 points for interpretation)

(a) Decision variables

Final Value column under Adjustable Cells in EXCEL answer sheet
- Scald 1250 hogs,
- Cut with pattern 4
- Sell 162500 lbs sausage, 75000 lbs of by product
- the last variable gives lbs disposed of which is 50000
- do not skin any hogs or cut any with patterns 1-3 or sell any skins, hams or bacon

(b) Shadow price
- Shadow price column under Constraints in **EXCEL sensitivity sheet**
  - the second equation is hogs available and one more worth 141.6
  - one more skin worth 8
  - one more lb ham worth 1.9
  - one more lb bacon worth 1.85
  - one more lb sausage worth 1
  - one more lb by product worth 0.4
  - the next to last equation gives waste to be disposed of and one lb costs 0.01
  - the last equation limits labor and one hr worth 26.4

(c) Reduced cost
- Reduced cost column under Adjustable Cells in **EXCEL sensitivity sheet**
  - Skinning would cost $58.50 per hog
  - Using cutting pattern 1 would cost $25 per hog

- Using cutting pattern 2 would cost $60.48 per hog
- Using cutting pattern 3 would cost $38.58 per hog
- All other variables have zero reduced cost and are produced to point where marginal profit contribution is zero.

(d) Objective function value s in EXCEL answer sheet

- Final value under Target Cell
  Objective Function Value = 132000
  The value is the maximum profit given the resources and parameters