Agricultural Economics 622, Spring 2004
Midterm Exam on Math programming Topic

1. (25 points) Temple Outland forest group is trying to set up a plan for utilizing its land and wood processing facilities. Temple harvests trees and makes wood chips for sending to pulp and paper mills along with 2x4's and 4x4's. Labor can be used to either cut trees or make wood products.

   Temple has 300 sections of mature forest land and 38,000 hours of labor available.

   When trees are harvested the cost is $1000 per section with 50 units of labor used. The tree yield per section is 5 truckloads.

   When trees are brought into the saw mill they can be cut by one of three processes.

   Process 1- Whole log chipping- costs $120 per truck load, uses 10 hours of labor and yields 75 cwt of wood chips.

   Process 2- 2x4 targeting - costs $140 per truck load, uses 18 hours of labor and yields 20 cwt of wood chips, 10 bundles of 4x4's and 61 bundles of 2x4's.

   Process 3- 4x4 targeting - costs $100 per truck load, uses 15 hours of labor and yields 15 cwt of wood chips, 40 bundles of 4x4's and 5 bundles of 2x4's.

   Wood chips sell for $4 per cwt, 4x4's for $110 per bundle and 2x4's for $55 per bundle.

   Set up an LP model of this situation.

2. (15 points) In the context of problem number 1 above state a small case where each of the assumptions of LP might be violated (using no more than 2-3 sentences for each assumption).

3. (15 points) Describe the E-V model algebraically and state how one
   a. might use the risk aversion parameter to generate solutions with different amounts of risk
   b. might interpret the shadow prices

4. (15 points) Give an example of where you would use the LP in problem 1 in each of

   a. a predictive setting and
   b. a prescriptive setting.

   Limit the answer for each to no more than 4 sentences.

5. (10 points) Why might you use integer programming?

   Limit the answer to no more than 4 sentences.

6. (20 points) Given the following problem and the Excel solution below it what is the
   a. amount of each variable in the solution and a one sentence interpretation one would place on one nonzero and one zero variable
   b. value of the shadow price for each constraint and a one sentence interpretation one would place on one of those shadow prices which is zero and one which is non zero
   c. value of the reduced costs for each variable not produced and a one sentence interpretation one would place on one of those reduced costs which is non zero.
   d. objective function value and a one sentence interpretation one would place on that

	Make steel	Make Ingots	Make pig iron	Sell steel	Sell Ingots	Sell pig iron		
Profit Objective	-120	-200	-100	3000	100	100		
Ore			1				<=	20
Coal	10	8	5				<=	300
Labor	11	5	1				<=	250
Rolling Mill	22						<=	250
Smelter			20				<=	300
Steel	-0.7			1			<=	0
Ingots	1	-1			1		<=	0
Pig Iron		1	-1			1	<=	0

Plus all variables greater than or equal to zero

The solution follows

EXCEL answer sheet

Microsoft Excel 9.0 Answer Report
Worksheet: [lptest2003.xls]Sheet1
Report Created: 2/10/2004 3:41:39 PM

Target Cell (Max)

Cell	Name	Original Value	Final Value
$J$25	Profit Objective Row sum	19090.90869	19090.90869

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$35	vars Make steel	11.36363636	11.36363636
$C$35	vars Make Ingots	11.36363636	11.36363636
$D$35	vars Make pig iron	15.00000005	15.00000005
$E$35	vars Sell steel	7.95454532	7.95454532
$F$35	vars Sell Ingots	0	0
$G$35	vars Sell pig iron	3.636363686	3.636363686

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$J$26	<= Row sum	15.00000005	$J$26<=$I$26	Not Binding	4.99999995
$J$27	<= Row sum	279.5454548	$J$27<=$I$27	Not Binding	20.4545452
$J$28	<= Row sum	196.8181819	$J$28<=$I$28	Not Binding	53.18181813
$J$29	<= Row sum	250	$J$29<=$I$29	Binding	0
$J$30	<= Row sum	300.000001	$J$30<=$I$30	Binding	0
$J$31	<= Row sum	-1.34869E-07	$J$31<=$I$31	Binding	0
$J$32	<= Row sum	0	$J$32<=$I$32	Binding	0
$J$33	<= Row sum	0	$J$33<=$I$33	Binding	0
$B$35	vars Make steel	11.36363636	$B$35>=0	Not Binding	11.36363636
$C$35	vars Make Ingots	11.36363636	$C$35>=0	Not Binding	11.36363636
$D$35	vars Make pig iron	15.00000005	$D$35>=0	Not Binding	15.00000005
$E$35	vars Sell steel	7.95454532	$E$35>=0	Not Binding	7.95454532
$F$35	vars Sell Ingots	0	$F$35>=0	Binding	0
$G$35	vars Sell pig iron	3.636363686	$G$35>=0	Not Binding	3.636363686

EXCEL sensitivity sheet

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [lptest2003.xls]Sheet1
Report Created: 2/10/2004 3:45:26 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Gradient
$B$35	vars Make steel	11.36363636	0
$C$35	vars Make Ingots	11.36363636	0
$D$35	vars Make pig iron	15.00000005	0
$E$35	vars Sell steel	7.95454532	0
$F$35	vars Sell Ingots	0	-200
$G$35	vars Sell pig iron	3.636363686	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
$K$26	Ore Row sum	15.00000005	0
$K$27	Coal Row sum	279.5454548	0
$K$28	Labor Row sum	196.8181819	0
$K$29	Rolling Mill Row sum	250	76.36363474
$K$30	Smelter Row sum	300.000001	0
$K$31	Steel Row sum	-1.34869E-07	3000
$K$32	Ingots Row sum	0	300
$K$33	Pig Iron Row sum	0	100

Test Answers and Grading Key

1. LP model (25 points)
    - Point allocation
        - Proper Decision variables (3 points),
        - Objective function (3 points)
        - Forest harvest linked to processing (3),
        - Joint products (3)
        - Labor constraint (3)
        - Product balances linking processing and sale (3)
        - Product sales properly specified (3)
        - Forest land constraint (3)
        - non-negativity (1)

variant 1

	Forest (sec)	Processing (in truckloads)			Sell Product					
Decision Variables	**Harvesting**	**Process 1**	**Process 2**	**Process 3**	**W. Chips**	**4*4**	**2*4**	**Sum**		**RHS**
Objective Function	-1000	-120	-140	-100	4	110	55	0	Max	
Constraints										
Forest resource	1							0	≤	300
Labor resource	50	10	18	15				0	≤	38000
Logs balance	-5	1	1	1				0	≤	0
Chips balance		-75	-20	-15	1			0	≤	0
4*4 balance			-10	-40		1		0	≤	0
2*4 balance			-61	-5			1	0	≤	0

note all variables non negative

variant 2 with forest harvest combined with processing

	Cut Forest and Process (in truck loads)			Sell Product					
Decision Variables	**Process 1**	**Process 2**	**Process 3**	**W. Chips**	**4*4**	**2*4**	**Sum**		**RHS**
Objective Function	-320	-340	-300	4	110	55	0	Max	
Constraints									
Forest resource	0.2	0.2	0.2				0	≤	300
Labor resource	20	28	25				0	≤	38000
Chips balance	-75	-20	-15	1			0	≤	0
4*4 balance		-10	-40		1		0	≤	0
2*4 balance		-61	-5			1	0	≤	0

note all variables non negative

variant 3 with forest harvest combined with processing and sale also folded in

	Cut + Process +Sell Products (in truck loads)					
Decision Variables	**Process 1**	**Process 2**	**Process 3**	**Sum**		**RHS**
Objective Function	-20	4195	4,435	0	Max	
Constraints						
Forest resource	0.2	0.2	0.2	0	≤	300
Labor resource	20	28	25	0	≤	38000

note all variables non negative

2. Violation of LP assumptions (15 points)
    - 7 assumptions (2 points per each)
    - 1 point for showing up

1) Objective Function Appropriateness
    - Obj. Fn in the above case might not be the sole criteria for choosing the decision variables. The firm might take the business risk into account
2) Decision Variable Appropriateness
    - We might have omitted major choice variables like processing logs other than in the 3 cases for which we have data
3) Constraint Appropriateness
    - There might be any omitted restrictions such as labor by facility (ie harvest labor is probably not sawmill labor)
4) Proportionality
    - Harvesting cost per section or processing cost per truck load might be increasing or decreasing as the firm produces more
    - Labor use per section or truck load might be decreasing as produces more
5) Additivity
    - There might be interactions between sale items leading to higher prices as collective volume goes up
6) Divisibility
    - The firm might not be permitted to sell 4x4s or 2x4s in amounts smaller than one bundle
7) Certainty
    - The yields per log of product might vary

3. E-V Model (15 points)

Model specification (5 points)

Max $E - \phi\sigma^2{}_Z = \bar{c}X - RAP * X'SZ = \sum_j \bar{c}_j X_j - RAP * \sum_j \sum_k s_{jk} X_j X_k$ (3 points)

s.t. $\sum_j X_j \leq funds$ (1 points)

$X_j \geq 0$ for all j (1 points)

- 5 points for each (a) and (b)

(a) We can solve the E-V Model for various RAP values starting at zero and then becoming larger heading toward infinity, and then we would obtain a set of possible solutions, which include maximum return (rap=0), minimum risk (rap=infinity), and intermediate efficient mean-variance combination (0<rap< infinity), across various risk preferences of investor (RAP) . This yields different portfolios with different mean and variance.

(b) In E-V Model, shadow price is equal to $\partial E/\partial X - \Phi^*(\partial\sigma/\partial X)$, where X is total fund available.

Shadow price is the difference between "change in expected return which is mean return" and "the product of marginal change in variance and risk preference (RAP)" when we increase one unit of fund. It is the risk discounted rate of return.

4. Predictive and Prescriptive (15 points)

- 7 points for (a) and 8 points for (b)

(a) Predictive

- If we buy additional forest section or hire additional labor, what are the projected profit and optimal decision variable consequences
- If one of the processing patterns is unavailable, then how does it affect the profit and optimal decision variables

(b) Prescriptive

- What is the optimal harvesting, processing, and sale pattern the firm should follow to reach maximum profit

5. Integer programming (10 points)

We use it to handle

- Decision variables are integer or binary (indivisible)
- Lumpy investment and fixed costs
- Logical Conditions – mutual exclusivity of items etc.

6. Interpretation of the Excel Solution

- 5 points for each question (2 points for values and 3 points for interpretation)

(a) Decision variables

- Final Value column under Adjustable Cells
  Make steel = 11.36, Make ingots = 11.36, Mike pig iron = 15
  Sell steel = 7.95, Cell ingots = 0, Sell pig iron = 3.63
- Those numbers give the amount of each product which the firm should make or sell. The firm should make 11.36 of ingots but sell none of ingots to maximize profit.

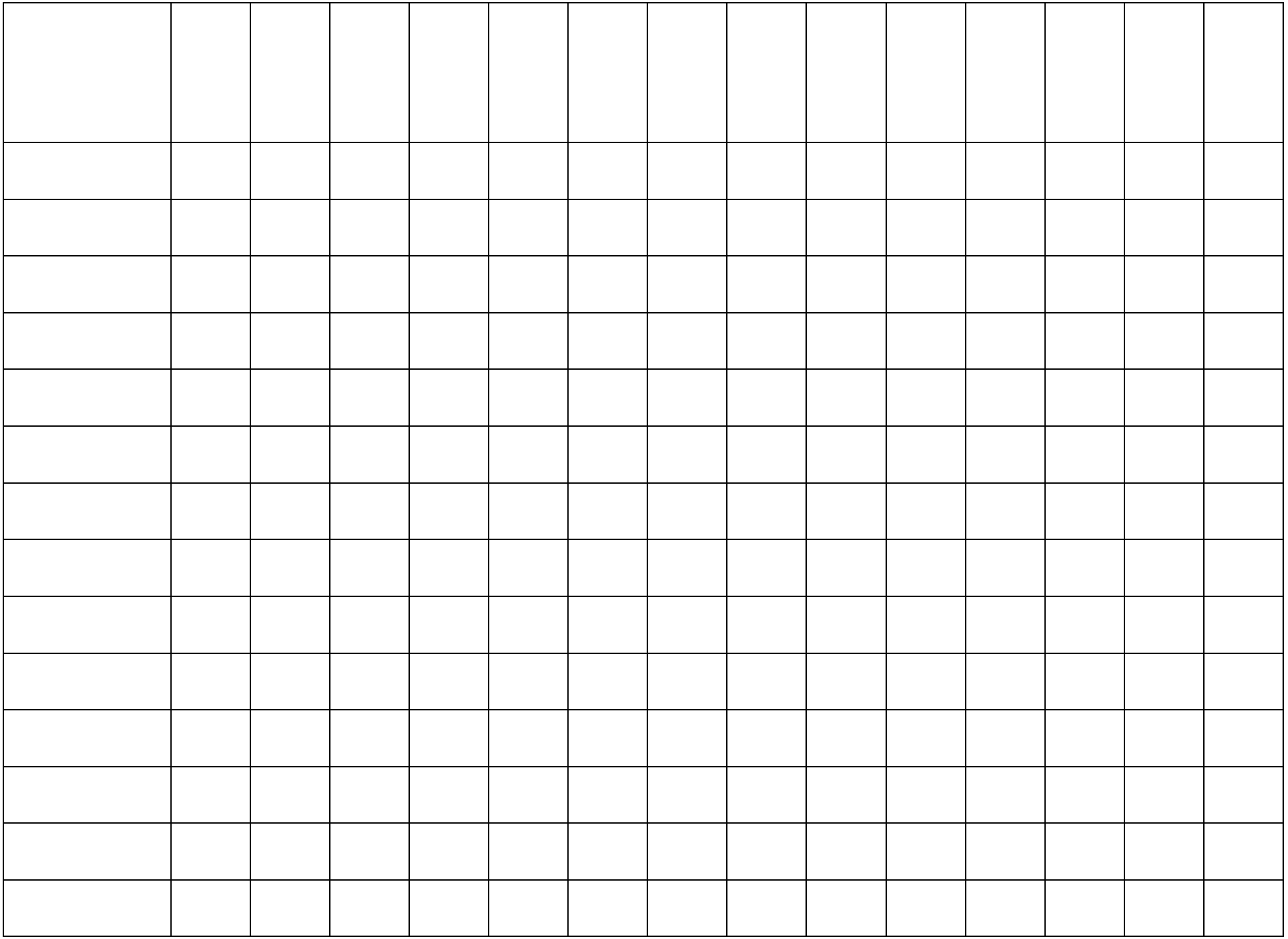
(b) Shadow price

- Lagrange Multiplier column under Constraints
  Ore = 0, Coal = 0, Labor = 0, Rolling Mill = 76.36, Smelter = 0
  Steel = 3000, Ingots = 300, Pig Iron = 100
- Those numbers give the change in the objective function value when one unit of those constraints is changed. If the firm increases one unit of rolling mill capacity, then profit will be increased by 3000. However one unit of incensement in Ore is worth nothing because Ore is non-binding resource

(c) Reduced cost

- Reduced Gradient column under Adjustable Cells
  Sell Ingots = -200
- Reduced cost means the change in objective function value when one unit of a decision variable that is not in the solution is forced into solution. If the firm alters operations to sell one unit of Ingots, then the obj. fn value would be decreased by 200

(d) Objective function value
- Final value under Target Cell
  Objective Function Value = 19,090.9
- The value is the maximum profit that the achieves given the resources and parameters



Note this is 15 by 15 and does not imply the problem is that size