Agricultural Economics 622
Midterm Exam on LP Topic
Feb, 2003

1. (25 points) Dollar Hog Company is planning its weekly hog cutting operation. Dollar buys hogs and can either skin them or scald them and then cut them up via one of four patterns. Technical data follow.

Table 1. Technical Data by Processing Type

	Hog Processing Type	
	Skin	Scald
Cost of processing/Hog	10	8
AddedByproduct[1/] /hog	0 lbs.	30 lbs.
Skin yield/hog	1	0
Labor to skin or scald/hog	6	4

[1/] byproduct yield in addition to those shown in Table 2.

Table 2. Yield of product in pounds by cutting pattern

	Cutting Pattern			
Product	1	2	3	4
Ham	40	40		
Bacon	20		30	
Sausage	50	70	90	130
By-product	50	50	40	30
Waste	40	40	40	40
Labor hours/hog	2.5	3.2	2.2	2.1

Products sell for the following prices:

Skins: $8 Bacon: $1.85/lb. Byproduct: $0.40/lb.
Ham: $1.90/lb. Sausage:$1.00/lb.

Hogs cost $40. 5000 man hours of labor are available. Waste costs $0.01/lb. to dispose of. Formulate a linear programming problem to determine the best operation.

2. (25 points) In the context of problem number 5 below state a small case where each of the assumptions of LP might be violated (using no more than 2-3 sentences for each assumption).

3. (20 points) Suppose that you have solved an LP problem and found the optimal solution for a client in Mexico but you set the problem up in dollars. Now suppose the client wants the answer in pesos. Use the LP matrix approach to determine whether changing the objective function to Pesos will
    a. Cause the current solution (basis) to cease to be optimal.
    b. Cause the current solution (basis) to cease to be feasible.
    c. Alter the value of the decision variables.
    d. Alter the value of the objective function.
    e. Alter the value of the shadow prices.
    f. Alter the value of the reduced costs.

4. (10 points) State tests you would use to check the units of the coefficients in the rows and columns of a linear programming formulation.

5. (20 points) Given the following problem which is the feed problem from class making 100kg of feed at minimum cost and the Excel solution below it what is the
    a. amount of each variable in the solution and a one sentence interpretation one would place on one nonzero and one zero variable
    b. shadow prices on the resources and a one sentence interpretation one would place on one of those shadow prices which is zero and one which is non zero
    c. reduced costs for variables not produced and a one sentence interpretation one would place on one of those reduced costs which is non zero.
    d. objective function value and the one sentence interpretation one would place on that value

	Corn	Hay	Soybean	Urea	Dical	Salt	Vitamin A	Slurry		
Cost	0.113	0.077	0.3	0.332	0.498	0.11	0.286	0.2		
Protein Max	0.075	0.127	0.438	2.62				0.032	≤	0.13
Fat Max	0.0357	0.022	0.013					0.009	≤	0.05
Salt Max						1			≤	0.02
Calcium Max	0.002	0.0125	0.0036		0.2313			0.002	≤	0.01
Phospohrous Max	0.0035	0.0023	0.0075	0.68	0.1865			0.0024	≤	0.013
Calories Min	1.48	0.49	1.29					1.39	≥	1.34
Protein Min	0.075	0.127	0.438	2.62				0.032	≥	0.07
Vitamin A Min	600	50880	80				2204600		≥	2200
Salt Min						1			≥	0.015
Calcium Min	0.002	0.0125	0.036		0.2313			0.002	≥	0.0025
Phosphorous Min	0.0035	0.0023	0.0075	0.68	0.1865			0.0024	≥	0.0035
Volume	1	1	1	1	1	1	1	1	=	100

Plus all variables greater than or equal to zero

The solution follows on this and the next page

EXCEL sensitivity sheet

Adjustable Cells

Cell	Name	Final Value	Reduced Gradient
$B$3	Corn	86.61526027	0
$C$3	Hay	11.85594856	0
$D$3	Soybean	0	0.19220763
$E$3	Urea	0.02879104	0
$F$3	Dical	0	0.364491785
$G$3	Salt	1.5	0
$H$3	Vitamin A	0	0.227504397
$I$3	Slurry	0	0.090671288

Constraints

Cell	Name	Final Value	Lagrange Multiplier
$K$21	Protein Max	8.077282512	0
$K$22	Fat Max	3.35299566	0
$K$23	Salt Max	1.5	0
$K$24	Calcium Max	0.321429878	0
$K$25	Phospohrous Max	0.35	0
$K$32	Volume	99.99999987	0.05849562
$K$26	Calories Min	134	0.035876104
$K$27	Protein Min	8.077282512	0
$K$28	Vitamin A Min	655199.8188	0
$K$29	Salt Min	1.5	0.05150438
$K$30	Calcium Min	0.321429878	0
$K$31	Phosphorous Min	0.35	0.402212306

EXCEL answer sheet

Target Cell (Min)

Cell	Name	Original Value	Final Value
$K$20	Cost	10.87499108	10.87499108

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Corn	86.61526027	86.61526027
$C$3	Hay	11.85594856	11.85594856
$D$3	Soybean	0	0
$E$3	Urea	0.02879104	0.02879104
$F$3	Dical	0	0
$G$3	Salt	1.5	1.5
$H$3	Vitamin A	0	0
$I$3	Slurry	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$K$21	Protein Max	8.077282512	$K$21<=$K$6	Not Binding	4.922717488
$K$22	Fat Max	3.35299566	$K$22<=$K$7	Not Binding	1.64700434
$K$23	Salt Max	1.5	$K$23<=$K$8	Not Binding	0.5
$K$24	Calcium Max	0.321429878	$K$24<=$K$9	Not Binding	0.678570122
$K$25	Phospohrous Max	0.35	$K$25<=$K$10	Not Binding	0.95
$K$32	Volume	99.99999987	$K$32=$K$17	Binding	0
$K$26	Calories Min	134	$K$26>=$K$11	Binding	0
$K$27	Protein Min	8.077282512	$K$27>=$K$12	Not Binding	1.077282512
$K$28	Vitamin A Min	655199.8188	$K$28>=$K$13	Not Binding	435199.8188
$K$29	Salt Min	1.5	$K$29>=$K$14	Binding	0
$K$30	Calcium Min	0.321429878	$K$30>=$K$15	Not Binding	0.071429878
$K$31	Phosphorous Min	0.35	$K$31>=$K$16	Binding	0
$B$3	Corn	86.61526027	$B$3>=0	Not Binding	86.61526027
$C$3	Hay	11.85594856	$C$3>=0	Not Binding	11.85594856
$D$3	Soybean	0	$D$3>=0	Binding	0
$E$3	Urea	0.02879104	$E$3>=0	Not Binding	0.02879104
$F$3	Dical	0	$F$3>=0	Binding	0
$G$3	Salt	1.5	$G$3>=0	Not Binding	1.5
$H$3	Vitamin A	0	$H$3>=0	Binding	0
$I$3	Slurry	0	$I$3>=0	Binding	0

Note this is 17 by 15 and does not imply the problem is that size

First Alternative Answer to #1

	Skin	Scald	Hog pattern 1	Hog pattern 2	Hog pattern 3	Hog pattern 4	Skins	Hams	Bacon	Sausage	By Product	Waste		
Profit	-50	-48					8	1.90	1.85	1.00	0.40	-0.01		
Hogs for slaughter	-1	-1	1	1	1	1							≤	0
Skins	-1						1						≤	0
Ham			-40	-40				1					≤	0
Bacon			-20		-30				1				≤	0
Sausage			-50	-70	-90	-130				1			≤	0
By-product		-30	-50	-50	-40	-30					1		≤	0
Waste			-40	-40	-40	-40						1	= or ≥	0
Labor	6	4	2.5	3.2	2.2	2.1							≤	5000

Plus all variables greater than or equal to zero.
Points 25

Obj +3, variables +5, parts bal +3, skin scald +3, waste +3, product sale +3, labor +3, bonus +2

Second Alternative Answer #1

	Skin and cut Hog pattern 1	Skin and cu Hog pattern 2	Skin and cu Hog pattern 3	Skin and cu Hog pattern 4	Scald and cut Hog pattern 1	Scald and cu Hog pattern 2	Scald and cu Hog pattern 3	Scald and cu Hog pattern 4	Skins	Hams	Bacon	Sausage	By Product	Waste		
Profit	-50	-50	-50	-50	-48	-48	-48	-48	8	1.90	1.85	1.00	0.40	-0.01		
Hogs for slaughter	1	1	1	1	1	1	1	1							≤	0
Skins	-1	-1	-1	-1					1						≤	0
Ham	-40	-40			-40	-40				1					≤	0
Bacon	-20		-30		-20		-30				1				≤	0
Sausage	-50	-70	-90	-130	-50	-70	-90	-130				1			≤	0
By-product	-50	-50	-40	-30	-50-30	-50-30	-40-30	-30-30					1		≤	0
Waste	-40	-40	-40	-40	-40	-40	-40	-40						1	= or ≥	0
Labor	6+2.5	6+3.2	6+2.2	6+2.1	4+2.5	4+3.2	4+2.2	4+2.1							≤	5000

Plus all variables greater than or equal to zero.
Points 25

Answer #2

7 assumptions
4 points for anything
1 point for stating each of the 7 assumptions
2 points for each of the 7 interpretations

Objective Function Appropriateness
In case above have right objective as cost min and numbers therein are right

Decision Variable Appropriateness
In case above have not omitted any major choices like additional feedstuffs.

Constraint Appropriateness
In case above have not omitted any major restrictions like other nutrien max and min.

Proportionality
In case above there might be increasing prices the more of a good you feed

Additivity
In case above there might be interactions between feeds

Divisibility
What if we always require feeds in amounts no smaller than 1/10 of a kilogram

Certainty
Do we really know the nutrient requirements

Answer #3

2 for stating each relation (12)
1 for each conclusion (6)
2 for showing up

old problem

max	cx	
	ax	<= b
	x	>= 0

NEW problem

max	Ecx	
	ax	<= b
	x	>= 0

where E is pesos per dollar

so new c = E * old c
and new c_B = E* old c_b

a. Cause the current solution (basis) to cease to be optimal.

Optimality when

$-(C_b B^{-1} A_j - C_j) < 0$ for all j

Now becomes

$-E(C_b B^{-1} A_j - C_j) < 0$ for all j

just multiplied by E so basis is still optimal

b. Cause the current solution (basis) to cease to be feasible.

$X_b = B^{-1} b$ is unchanged and remains ≥ 0

c. Alter the value of the decision variables.

$X_b = B^{-1} b$ is unchanged and remains ≥ 0

d. Alter the value of the objective function.

$C_b X_b = C_b B^{-1} b$ becomes

$EC_b X_b = EC_b B^{-1} b$ multiplied by exchange rate

e. Alter the value of the shadow prices.

$C_b B^{-1}$ becomes

$EC_b B^{-1}$ multiplied by exchange rate

f. Alter the value of the reduced costs.

$-(C_b B^{-1} A_j - C_j) < 0$ for all j

Now becomes

$-E(C_b B^{-1} A_j - C_j) < 0$ for all j

just multiplied by E

Answer #4

2 points for concept , 4 for row numerator, 4 for column denominator.

Homogeneity of units test

Rows must have constant numerator columns denominator.

Answer #5

6 points

a. amount of each variable in the solution and a one sentence interpretation one would place on one nonzero and one zero variable

Final value column under adjustable cell section below

Those numbers give amount of feedstuffs to use. Solution says feed zero slurry and 86.81 kg corn.

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Corn	86.61526027	86.61526027
$C$3	Hay	11.85594856	11.85594856
$D$3	Soybean	0	0
$E$3	Urea	0.02879104	0.02879104
$F$3	Dical	0	0
$G$3	Salt	1.5	1.5
$H$3	Vitamin A	0	0
$I$3	Slurry	0	0

6 points

b. shadow prices on the resources and a one sentence interpretation one

would place on one of those shadow prices which is zero and one which is non zero

Shadow price is Lagrange multiplier column in second part below

Says volume (in row k32) cost 0.58 $/kg o the margin, calorie min (row 7) worth 0.03587 on margin.

Constraints

Cell	Name	Final Value	Lagrange Multiplier
$K$21	Protein Max	8.077282512	0
$K$22	Fat Max	3.35299566	0
$K$23	Salt Max	1.5	0
$K$24	Calcium Max	0.321429878	0
$K$25	Phospohrous Max	0.35	0
$K$32	Volume	99.99999987	0.05849562
$K$26	Calories Min	134	0.035876104
$K$27	Protein Min	8.077282512	0
$K$28	Vitamin A Min	655199.8188	0
$K$29	Salt Min	1.5	0.05150438
$K$30	Calcium Min	0.321429878	0
$K$31	Phosphorous Min	0.35	0.402212306

3 points

c. reduced costs for variables not produced and a one sentence interpretation one would place on one of those reduced costs

Reduced cost is Reduced gradient column in table just below

Says forcing in soybeans would cost $0.19 per kg on the margin.

Adjustable Cells

Cell	Name	Final Value	Reduced Gradient
$B$3	Corn	86.61526027	0
$C$3	Hay	11.85594856	0
$D$3	Soybean	0	0.19220763
$E$3	Urea	0.02879104	0
$F$3	Dical	0	0.364491785
$G$3	Salt	1.5	0
$H$3	Vitamin A	0	0.227504397
$I$3	Slurry	0	0.090671288

3 points

d. objective function value and the one sentence interpretation one would place on that value

Use objective row sum from table below = $10.87 and that is firm cost per 100 kg of feed.

Target Cell (Min)

Cell	Name	Original Value	Final Value
$K$20	Cost	10.87499108	10.87499108