AGEC 641 Final

Answer 5 of the first 6 questions take number 7 home and 1 of the other 6 of your choice.

1. Suppose you have an optimal solution to the problem

$$\begin{aligned} \text{Max} \quad & cx \\ & Ax = b \\ & x \geq 0 \end{aligned}$$

where A is 2 x N
b is 2 x 1

and the second constraint is replaced by the sum of the first and second constraints. What will happen to the optimal primal solution, shadow prices and objective function?

Hint: Premultiplying a 2 row system of equations by

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

results in the second row being an addition of the first two before multiplication.

2. Given the problem

$$\begin{aligned} \text{Max} \quad & 3x_1 & & & - & x_3 & \\ & x_1 & - & x_2 & & \leq 10 & \\ & & + & x_2 & - & x_3 & \leq 5 \\ & & & & & x_3 & \leq 2 \end{aligned}$$

if it is stipulated that x_1, x_2, and x_3 are nonzero and positive then what are the optimal values of the shadow prices?

3. If you wish to restrict the difference of two shadow prices, say $u_1 - u_2$, to be no more than \$10 and no less than \$5 what types of variables would you add to the primal?

4. Explain the following briefly:

   a) the relation between primal degeneracy and dual variables

b) the dual constraint arising from

$$\begin{aligned} \text{Max} \quad & -cx \\ & Ax \leq b \\ & x \geq 0 \end{aligned}$$

c) signing conventions

d) the considerations involved when modeling using summation notation rather than a numerically specific tableau

e) equilibrium, and disequilibrium dynamic models

f) accounting equations

5) Set up an LP of the following

$$\begin{aligned} \text{Max} \quad & 4X^{1/2} - (3 + 2Y)Y \\ \text{s.t.} \quad & 0.3X - 10Y \leq 0 \\ & X + Y \leq 30 \\ & X, Y \geq 0 \end{aligned}$$

and tell whether this model relaxes any of the LP assumptions

6. Suppose you have a problem where you

a) Invest in a set of alternative investments of which there are N where x_j gives the amount invested in the jth type and c_j the hours of operation that may be employed if the item is bought

b) Obtain returns on these investments from production alternatives where alternative k (y_k) uses a respecified number of hours of the capacity acquired when the jth investment is purchased

c) Have other resource constraints which limit production

d) The annual interest cost of the investments is a fixed percentage of the amount invested

Formulate a general model of this process maximizing the average rate of return first in fractional then in linear terms.

7. The Peck family is studying whether to make an offer on some property. Currently, th e Peck's own two farms and a feedlot in Illinois. The option they are considering is th e purchase of some land in Montana. The land in Montana would be used to raise cattle which would be in part fed to slaughter age in Montana and in part shipped to Illinois for feeding. Technical data follows:

Farm Data		Farm 1	Farm 2
Plowing	Cost/hour	15	12
	Acres/hour	3	2.5
	Labor /hour plowing	1	1
Planting Corn	Cost/hour	13	12
	Acres/hour	5	6
	Labor/hr of planting	1	1
Planting Soybeans	Cost/hour	15	12
	Acre/hour	6	8
	Labor/hr of planting	1	1
Harvesting Corn			
	Cost/hour	45	45
	Acre/hour	3	3
	Labor/hour	1.5	1.5
Harvesting Soybeans	Cost/hour	45	45
	Acres/hour	5	5.5
	Labor/hour	1.5	1.5

Corn yields in bushels on Farm 1 are:

		Planting Date	
		May	June
Harvest Date	October	120	110
	November	140	130

Corn yields on Farm 2 are 90% of those on Farm 1.

Soybean yields in bushels (bu) on Farm 1 are:

		Planting Date	
		May	June
Harvest Date	September	40	38
	October	45	42

Soybean yields on Farm 2 are 110% of those on Farm 1. Plowing is done in December , March, April or May. Farm labor is share d between the farms and there are 60 hours in each month. Farm 1 has 300 acres, Farm 2 has 200 acres.

Feed Lot Data	Characteristics of Alternative Feeding Systems per Calf Fed			
Method Number	1	2	3	4
Final Animal Weight/lbs.	900	870	900	920
Bushels of Corn	40	45	30	42
Bushels of Soybeans	10	5	20	6
Other Costs	20	25	15	30

Market Data:

Purchase cost of calves	$125/head
Market price of corn (same to both farms)	$3.00/bu
Market price of soybeans (same to both farms)	$6.75/bu
Cost of purchasing feed	120% of market price
Price of fed beef applicable to final animal weight	$50/100 lbs

Transport Costs:

Farm 1 to Feedlot	
Corn	$0.05/bu
Soybeans	$0.12/bu
Farm 2 to Feedlot	
Corn	$0.10/bu
Soybeans	$0.05/bu

Assume purchased feed and crops sold to market have a zero transport cost.

<u>Montana Proposal</u>

Land required/cow unit(cow and calf)	5 acres
Number of calves raised in year per cow unit	0.7 calves/cow
Amortized Cost to buy and maintain a cow unit	$55/year
Land utilized per calf raised to sale	5 acres
Cost/calf to ship to Illinois	$10/calf
Weight of calves if raised to slaughter in Montana	800 lbs.
Cost/calf raised to maturity in Montana	$250
Annual cost of acquiring land in Montana	$5/acre

Sale price of calves raised to slaughter in Montana	$.50/lb.
Maximum slaughter market potential in Montana	500 head calves

a. Formulate a model for optimum firm size including both farms, the feedlot, and the Montana option.

b. Discuss how you would use this model to determine whether to invest in Montan a land.

c. What types of formulations are implicit in this solution relati ve to those in the chapters we studied (resource allocation, etc.)?