(Draft)

Notes on a GAMS adaptation of the SGMCGE model

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During the DOE integrated assessment project a GAMS version of the SGM model has been built (SGMGAMS for short). SGMGAMS is a static single year (1990), single country (US) representation which uses the data from the total SGM model. The data used in SGMGAMS are mainly from the ****usa90cal22j.xls**** obtained from the SGM developers (Sands).

SGMGAMS includes 23 sectors. Within these sectors subsectors are included in the electricity generation sector based on the type of fuel source (generation using oil, coal, gas, biomass, nuclear, and hydropower). On the factor side SGMGAMS depicts use and productivity of 3 traditional factors (labor, capital, and land) and 22 intermediate inputs. In addition, there are 4 vintages of capital stocks represented.

While we have made efforts to completely mimic we have not fully included the full SGM features on the carbon market, the biomass subsector, and agricultural lands as we have not fully tried to incorporate the AGLU features due to it's potential differences with our sector based estimation approach. We feel a need to have more interaction with the SGM developers to finish this part.

Overview of SGMGAMS in GAMS

The basic components of a GAMS model consist of SETS, PARAMETERS, VARIABLES, and EQUATIONS.

Sets

Sets are the basic building a block of GAMS and define the model scope. SGMGAMS contains a number of SETS. The most important SETS mainly used in the model are

Sector	which defines the 23 producing sector with the elements {OtherAg, CrudeOil, NatGas, Coal, Electricity, … , FoodProcess}
SubSector	which defines the fuel based subsectors {Oil, Gas, Coal, Biomass, Nuclear, Hydro}
Activity	which identifies the type of inputs that can be used {OtherAg, Service, CrudeOil, … , Labor, Land, Capital}
Vintage	which identifies the number of periods ago that the item was installed {-3, -2, -1, 0}

In addition, the SET called "AllSet" is used to insure proper ordering and includes the elements from all other SETS used in the model.

The complete SETS definition and declaration are shown from lines 63 to 1214 in the **allsgm.lst** file.

Parameters

Input data can be derived directly from the given data entry (i.e. Tables, Parameters, or Scalars as shown from lines 1246 to 1961) or from the direct assignment through a calculation within the model (lines 3603 to 4711, and lines 4926 to 7073).

Direct assignments are mostly used in calibrating parameters needed in the model. The following is an example of CES value-added share factor input parameter calculation:

DeltaVa(Factor2,AllSecAllSub)
 $((QF0(Factor2,AllSecAllSub) ne 0)
   and YesCESVa(AllSecAllSub)
)
 = (FactPriceDist0(Factor2,AllSecAllSub)
     * FactPricNation(Factor2)
     * (QF0(Factor2,AllSecAllSub))**(1 + RhoVa(AllSecAllSub))
) /SUM(Factor2P,
        FactPriceDist0(Factor2P,AllSecAllSub)
         * FactPricNation(Factor2P)
           *(QF0(Factor2P,AllSecAllSub))**(1 + RhoVa(AllSecAllSub))
) ;

The parameters used in the calibration process are defined in **sgmcalib.gms** and are shown from line 4792 to 4921 in the **allsgm.lst** file.

Variables

The decision variables or endogenous variables can be unrestricted in sign (i.e. government saving that can be negative depicting government borrowing) or constrained to be nonnegative (or in the GAMS language POSITIVE). These variables are declared in **sgmmodel.gms** and in the lines listed as 7278 to 7344 in the **allsgm.lst** file and use the ALLSET and it's aliases here we include more information on what goes in the index positions as follows

VARIABLES
    GovSaving   Government saving or borrowing

POSITIVE VARIABLES
* #### Quantity block
    FactorQ(Factor,Sector,SubSector)   Factor use by a producing sector for CES bottom level
    Production(Sector,SubSector,Vintage)   Production quantity
    QValAdd(Sector,SubSector)   Quantity value-added
    QintA(Sector,SubSector)   Quantity intermediate inputs
    QIntC(Activity,Sector,SubSector)   Intermediate demand
    QExport(Sector,SubSector)   US export quantity
    QImport(Sector,SubSector)   US import quantity

* #### Price block
    FactPriceNation(Factor)   Average economy-wide wage average
    FactPriceSec(Factor,Sector,SubSector)   Factor price by sector and subsector
    ComPrice(Sector)   Prices of commodities
    PricePaid(Activity,AllSecAllSub)   Prices paid to input i for output j
    PriceReceive(AllSecAllSub)   Prices received from output j
    PValAdd(Sector,SubSector)   Value-added price
    PIntA(Sector,SubSector)   Intermediate inputs price
    PExp(Commodity)   Price received by producers for exported goods
    PImp(Commodity)   Price paid by consumer for imported goods
    PCarb(Region)   Carbon price

* #### Household consumption information
    GoodsDemand(Commodity,HouseholdH) Commodity demand by households
    HhExpend(HouseholdH)   Household expenditure
    HHIncome(HouseholdH)   Household income

* #### Investment, expected profit rate, and capital stock information
    AlphaInv1(Activity,AllSecAllSub)   Dynamic calculation for expected profit rate
    AlphaInv2(AllSecAllSub)   Dynamic calculation for expected profit rate
    AlphaInv3(Activity,AllSecAllSub)   Dynamic calculation for expected profit rate
    AlphaInv(Activity,AllSecAllSub)   Dynamic calculation for expected profit rate
    IBigAij(Activity,AllSecAllSub)   Dynamic calculation for expected profit rate
    QInvest(AllSecAllSub)   Quantity investment
    ExpProfSec(Sector)   Expected profit rate for investment by Sector
    ExpProfSub(SubSector)   Expected profit rate for investment by SubSector
    Zvalue(AllSecAllSub)   Aggregate calculation for Z
    SubInvSh1(AllSecAllSub)   Aggregate calculation for share investment page 25
    SubInvSh3(AllSecAllSub)   Aggregate calculation for share investment page 25
    KStock(AllSecAllSub,Vintage)   Implied capital

* #### Government information
    GovExpend   Government expenditure
    GovTrnsfPaymt   Government transfer payments
    GovDemand(HouseholdG)   Government demands for goods and services
    GovIncome   Total government tax revenue
    GovConsumption(Sector,HouseHoldG) Government consumption

* #### Greenhouse gas information
    AgSinkQ(Ghg,Vintage)   Agricultural sinks using response functions
    AgEmitQ(Ghg,Vintage)   Agricultural sources using response functions
    EnergyEmitQ(Sector,Ghg,Vintage)   Energy emissions
    NetEmiTQ(Region,Ghg)   Net emissions
    CarbTaxRate(Sector,Ghg)   Carbon tax rate
    CarbTaxTot(Sector,Ghg)   Total Carbon tax revenues collecting from sectors

Equations

Equations represent groups of relationships in the model more than one equation depending on how many domains (indices) are in equations. The equations are defined as follows:

* #### Quantity block
    CESQVAEq(Sector,SubSector)   Quantity value-added for the bottom level CES
    CESQVAFOC(Factor)   Factor demand from the first order condition
    FactorMkt(Factor,Sector,SubSector)   Factor market balances
    QValAddEq(Sector,SubSector)   Quantity value-added for the top level Leontief
    QintAEq(Sector,SubSector)   Quantity intermediate inputs for the Leontief
    QIntCEq(Activity,Sector,SubSector)   Intermediate demand
    ImpBal(Commodity)   Import balance
    ExpBal(Commodity)   Export balance
    ExportEq(Sector,SubSector)   US export quantity in billion
    ImportEq(Sector,SubSector)   US import quantity in billion

* #### Price block
    FactPriceSecEq(Factor,Sector,SubSector) Factor price at sector and subsector levels
    PVaEq(Sector,SubSector)   Activity revenue and costs where Revenue is exhausted by payments for value-added and intermediate inputs
    PIntAEq(Sector,SubSector)   Aggregate intermediate inputs price
    PricePaidEq(Activity,AllSecAllSub)   Price paid to input i equation
    PriceReceivEq(AllSecAllSub)   Price received from output j

* #### Household consumption information
    GoodsDemandEq(Commodity,HouseholdH) Commodity demand by Households
    HhExpendEq(HouseholdH)   Household expenditure balance
    IncomeEq(HouseholdH)   Household budget constraint

* #### Investment, expected profit rate, and capital stock information
    AlphaInv1Eq(Activity,AllSecAllSub)   Dynamic calculation for expected profit rate
    AlphaInv2Eq(AllSecAllSub)   Dynamic calculation for expected profit rate
    AlphaInv3Eq(Activity,AllSecAllSub)   Dynamic calculation for expected profit rate
    AlphaInvEq(Activity,AllSecAllSub)   Dynamic calculation for expected profit rate
    IBigAijEq(Activity,AllSecAllSub)   Dynamic calculation for expected profit rate
    InvestDem(AllSecAllSub)   Investment demand balance
    ExpProfSecEq(Sector)   Expected profit rate for investment by Sector
    ExpProfSubEq(SubSector)   Expected profit rate for investment by SubSector
    ZvalueEq(AllSecAllSub)   Aggregate calculation for Z
    SubInvSh1Eq(AllSecAllSub)   Aggregate calculation for share investment page 25
    SubInvSh3Eq(AllSecAllSub)   Aggregate calculation for share investment page 25
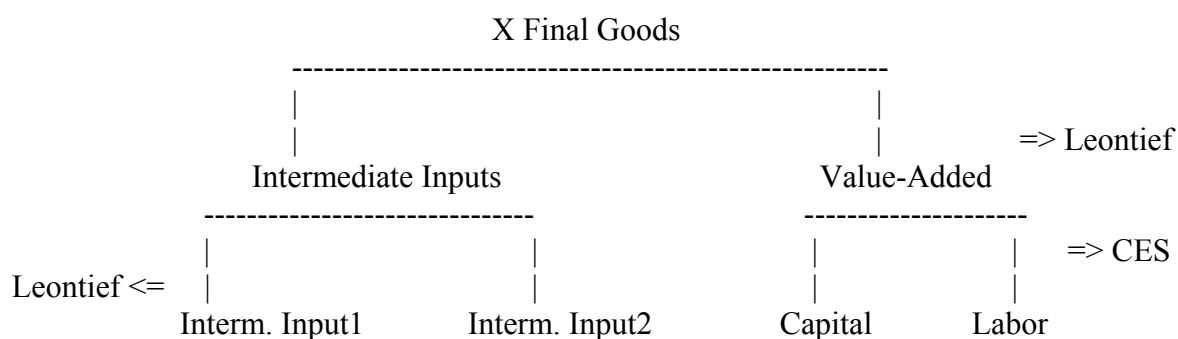
KStockEq(AllSecAllSub,Vintage)   Implied capital balance

* #### Equilibrium conditions
    Profit(Sector,SubSector,Vintage)   Zero profit condition
    SupplyDemandEq(Sector)   Commodity market balance

* #### Government information
    GovDemandEq(HouseHoldG)   Government demands for goods and services
    GovTrnsfEq   Government transfer payment equation
    GovIncomeEq   Government net tax revenue equation
    GovBudgetEq   Government budget constraint
    GovConsumpEq(Sector,HouseHoldG)   Government consumption equation
    GovExpendEq   Government expenditure

* #### Greenhouse gas emissions and sinks information
    AgEmitQEq(Ghg,Vintage)   Agricultural source emissions
    AgSinkQEq(Ghg,Vintage)   Agricultural sinks
    EnergyEmitqEq(Sector,Ghg,Vintage) Energy emissions
    NetEmisQEq(Region,Ghg)   Net emissions
    EmisQLimitEq(Region)   Emission limits constraint
    CarbTaxRateEq(Sector,Ghg)   Carbon tax rate
    CarbTaxTotEq(Sector,Ghg)   Total Carbon tax revenues collecting from sectors

Model Structure

In general, the structure of SGMGAMS mirrors the SGM model structure. Details follow:

Production Sector

When intermediate inputs are used in the production of goods, the production process is modeled as 2-level nested employing CES and Leontief production functions. The top level including the aggregated intermediate inputs and value-added is represented using a Leontief production process whereas the bottom level models the aggregated intermediate inputs as the Leontief technology of all intermediate inputs but the value-added is represented by the CES production function of the production factors (labor and capital).

```
                        X Final Goods
          ------------------------------------------------------
              |                                  |
              |                                  |          => Leontief
              |                                  |
        Intermediate Inputs                 Value-Added
        -------------------------           ---------------------
          |              |                   |           |       => CES
Leontief <=  |           |                   |           |
        Interm. Input1   Interm. Input2   Capital      Labor
```

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Five equations involved in the production process and input use are:

Quantity Value-Added for the CES Bottom Level Technology
      The value-added is represented by the CES production function of the production factors (labor and capital) for the bottom level.

CESQVAEq(Sector,SubSector)
  $(TisMap(Sector,SubSector)
    and YesCESVa(Sector)
)..

  QValAdd(Sector,SubSector)
  =E=
  AlphaVa1(Sector,SubSector)
  *(SUM(Factor2,
      DeltaVa1(Factor2,Sector,SubSector)
      * FactorQ(Factor2,Sector,SubSector) **(-RhoVa1(Sector,SubSector))
)
)**(-1/RhoVa1(Sector,SubSector)) ;

Factor Demand

      This first order condition implies that the marginal factor cost (labor and capital) is equal to the marginal revenue product (net of intermediate input costs) of the factor. This equation follows equation (16) in Lofgren 2001.[1]

CESQVAFOC(Factor2)
  $(sum(Sector$YesCESVa(Sector),
     sum(MapSecSub(Sector,SubSector),
      FactPriceDist1(Factor2,Sector,SubSector)))
)..

  FactPriceNation(Factor2)
  =E=
  sum(Sector$YesCESVa(Sector),
   sum(MapSecSub(Sector,SubSector),
     (PValAdd(Sector,SubSector)*(1 - ValAddTax1(Sector,SubSector))
      *QValAdd(Sector,SubSector)
       * SUM(Factor2P,
         DeltaVa1(Factor2P,Sector,SubSector)
         * FactorQ(Factor2P,Sector,SubSector)**(-RhoVa1(Sector,SubSector))
)**(-1)
         * DeltaVa1(Factor2,Sector,SubSector)

[1] Lofgren, H., R. L. Harris, S. Robinson, M. Thomas, and M. El-Said, "A Standard Computable General Equilibrium (CGE) Model in GAMS, TMD Discussion Paper No. 75, IFPRI, May 2001.

$$* \text{FactorQ}(\text{Factor2},\text{Sector},\text{SubSector})**(-\text{RhoVa1}(\text{Sector},\text{SubSector})-1)$$
```
      )
   ))
 /sum(Sector$YesCESVa(Sector),
   sum(MapSecSub(Sector,SubSector),
    FactPriceDist1(Factor2,Sector,SubSector) ))
 ;
```

Quantity Value-Added for the Leontief Top Level Technology

The value-added is represented by the Leontief technology for the top level.

```
 QValAdd(Sector,SubSector)
   =E=
 iva1(Sector,SubSector)
  *sum(Vintage,Production(Sector,SubSector,Vintage))
   ;
```

Quantity Intermediate Inputs

When the intermediate inputs are used in the production of goods, it is modeled as a Leontief function at the lower level. IntA1(Sector,SubSector) is the input-output coefficient which is calculated as the quantity of input divided by the quantity of output.

```
QintAEq(Sector,SubSector)
 $TisMap(Sector,SubSector)..

 QintA(Sector,SubSector)
   =G=
 IntA1(Sector,SubSector)
  * sum(Vintage,Production(Sector,SubSector,Vintage) )
   ;
```

Quantity Intermediate Input Demand

The demand for disaggregated intermediate inputs is modeled as a Leontief production. ica1 represents a fixed intermediate input coefficient and is calculated within the model (see lines 5214 to 5223 and lines 6053 to 6056).

```
QIntCEq(Activity,Sector,SubSector)
 $(  TisMap(Sector,SubSector)
   and ica1(Activity,Sector,SubSector)
  )..

 QIntC(Activity,Sector,SubSector)
   =G=
```

ica1(Activity,Sector,SubSector)
   * QIntA(Sector,SubSector)   ;

Household Sector

   Factors such as population, labor and capital endowment, personal saving, and transfer payments are involved in modeling the household consumption. The population data is given by SGM data.

<u>Household Income</u>

   The household income is the sum of retained earning income, labor income, government transfer payments minus personal income tax and saving. These household income and saving equations follow equations (33) – (40) on pages 28-30 in the SGM model documentation. Because of space limitation, these equations are not presented here but the completed expression of these equations involved the household consumption can be looked up from lines 7771 to 7857.

<u>Household Budget</u>

   The household budget is used to make sure that the incomes after tax minus personal saving plus the government transfer payments are exhausted (see lines 7714 to 7769).

<u>Household Consumption</u>

   The household consumption is allocated across different commodities according to the Linear Expenditure System (LES):

$$P_i Q_i = P_i \gamma_i + \beta_i \left(Y - \sum_i P_i \gamma_i \right)$$

where P_i is i commodity price, Q_i is i commodity quantity, $P_i Q_i$ is the expenditure on i commodity, β_i is the marginal budget share, γ_i is the minimum requirements subsistence quantity, and Y is the income. Below shows how this LES is incorporated in the model

GoodsDemandEq(Commodity,HouseholdH)
 $(Beta(Commodity,HouseHoldH)
   and ComPrice0(Commodity)
   and SAM(Commodity,HouseholdH)
)..

  ComPrice(Commodity)
     *GoodsDemand(Commodity,HouseholdH)
   =G=
  ComPrice(Commodity)
   *AdjConsumerQ(Commodity)
    *AddAdjCP(Commodity)

+ ComPrice(Commodity)
    *AddAdjCP(Commodity)
      *(Gamma(Commodity,HouseholdH))
  + (Beta(Commodity,HouseholdH)
     *(HhExpend(HouseholdH)
      - SUM(CommodityP,ComPrice(Commodity)
          *AddAdjCP(Commodity)
            *Gamma(CommodityP,HouseholdH))
)) ;

Government Sector

Six equations contribute to the government sector are the government demands for goods and services, consumption, income, expenditure, transfer payments, and saving.

<u>Government Demand</u>

The government quantity or value of production in government sector followed equation (32) on page 28 in SGM model documentation is shown below:

GovDemandEq(HouseHoldG)..
  GovDemand(HouseHoldG)
   =E=
  (GovIncome
   - GovSaving
   - GovTrnsfPaymt
)*GovShare0(HouseholdG)
    *(Pgss0(HouseholdG)
      **(GammaGov-1))
        *(sum(HouseholdGP,
          (GovShare0(HouseholdGP)
              *Pgss0(HouseholdGP)
                (GammaGov))))(-1) ;

<u>Government Consumption by Sector</u>

For the government consumption by sector, the fixed weights for the government expenditure and the share of tax revenue among sectors and households are used.

GovConsumpEq(Sector,HouseHoldG)..
  GovConsumption(Sector,HouseHoldG)
   =E=
  GovFixWeight(Sector,HouseHoldG)*GovShareTR0("GoToSector")
   * (GovIncome - GovSaving)
    /Comprice(Sector)   ;.

Government Income

The government income is a sum of corporate tax, personal income tax, social security tax, indirect business tax, and carbon tax. Because of space limitations the expression of this equation is shown in lines 8308 to 8415.

Government Budget

The government budget is used to ensure that the government income and expenses are exhausted (balance). This constraint is expressed as:

GovBudgetEq..
  GovIncome
   =E=
  GovSaving
 + GovTrnsfPaymt
 + sum((Sector,HouseHoldg),
      GovConsumption(Sector,HouseHoldG)
       *Comprice(Sector)
)   ;

Government Transfer Payments

The government transfer payments can be modeled in two ways. As shown in equation (28) in SGM documentation, the transfer payments are assumed to be a function of population
$$Transfer = \beta_0 Population^{\beta_1}$$
where β_0 and β_1 are given. Because the current SGMGAMS is a static model (populations remains constant), with this function the government transfer payments will be exogenous regardless of changes in the government consumption. Therefore, the second alternative model is used. This alternative the government transfer payments is modeled as a function of expenditure and consumption:

GovTrnsfEq..
  GovTrnsfPaymt
   =E=
  GovExpend
 - sum((Sector,HouseHoldg),
      GovConsumption(Sector,HouseHoldG)*Comprice(Sector)   )
   ;
Government Expenditure

The government expenditure is expressed as:

GovExpendEq..
  GovExpend
   =E=

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```
    GovTrnsfPaymt
  + sum((Sector,HouseHoldg),
        GovConsumption(Sector,HouseHoldG)
          *Comprice(Sector)
    )
    ;
```

Investment Sector

      The investment demand function for sectors follows equation (21) on page 24 in the SGM model documentation while the investment demand function for subsector using the logit function follows equation (22) on page 25 in the SGM model documentation. Note that crude oil and natural gas sectors have exogenous investment.

      The model will solve for investment based on the expected profit rate. The expected profit rate is a function of prices and quantities as described in equations (7) to (15) in the SGM model documentation:

$$\pi = \alpha_0 p_j \left[1 - (\alpha_0 p_j)^{-r} \sum_{i=1}^{N-M} \left(\frac{p_i}{\alpha_{ij}} \right)^r \right]^{1/r} \left(\sum_{i=N-M+1}^{N} \left(\alpha_{ij} x_{ij} \right)^{\rho} \right)^{1/\rho}$$

where α_0, and α_{ij} are technical coefficients $i=1,\dots,N$ and

$$\alpha_{ij} = \alpha_0^{-\rho} (A_{ij})^{1-\rho} \left(\frac{p_i}{p_j} \right)^{\rho}.$$

A_{ij} is a set of input-output coefficients. N is the number of inputs to production. ρ is a parameter that controls the elasticity of substitution, $r = \rho/(\rho-1)$, p_i is the price of the ith input, p_j is the price of output j, and x_{ij} is the amount of input i used in the production of output j.

$$x_{ij} = \alpha_{0j}^{1-r} \alpha_{ij}^{-r} \left(\frac{p_j}{p_i} \right)^{1-r} \left(\frac{Y}{Z} \right)^{1/\rho}$$

$$Y = \sum_{i=N-M+1}^{N} (a_{ij} x_{ij})^{\rho}$$

$$Z = 1 - (\alpha_{0j} p_j)^{-r} \sum_{i=1}^{N-M} \left(\frac{p_i}{\alpha_{ij}} \right)^r$$

      A set of equations involves with the investment sectors are shown from lines 7858 to 8206.

Expected Profit Rate

      Since the calculation of the expected profit rate is complicated, only main equations are presented below. The rest of the equations are shown from lines 7858 to 8206. Information to calculate the present values used in the model including investment lifetime, real interest rate, sector interest rate, and wedge rate are directly obtained from SGM.

* #### Sector expected profit rate

ExpProfSecEq(AllSecAllSub)
  $(YesInv(AllSecAllSub)
    and OnlySector(AllSecAllSub)
    and (1 - InvTaxCredit(AllSecAllSub))
)..

   ExpProfSec(AllSecAllSub)
     =E=
  (TechAlpha0(AllSecAllSub)
     * PriceReceive(AllSecAllSub)
       * PresentValue(AllSecAllSub)
         *(Zvalue(AllSecAllSub)**(1/RRho(AllSecAllSub))
)
          * ((AlphaInv("Capital",AllSecAllSub)
)
               **(1/Rho(AllSecAllSub)))
)/(1 - InvTaxCredit(AllSecAllSub))
    ;

* #### SubSector expected profit rate

ExpProfSubEq(AllSecAllSub)
  $(YesInv(AllSecAllSub)
    and OnlySubSector(AllSecAllSub)
    and (not sameas(AllSecAllSub,"SElecBiom"))
    and (1 - InvTaxCredit(AllSecAllSub))
)..

   ExpProfSub(AllSecAllSub)
    =E=
  (

   ((((1/sum(SubSectorP
         $((not sameas(SubSectorP,"SElecNucl"))
           and (not sameas(SubSectorP,"SElecHydro"))
           and (not sameas(SubSectorP,"SElecBiom"))
           ),
       SubInvSh3(SubSectorP)
)
)
)*
      (SubInvSh1(AllSecAllSub)
        /SubInvSh1("SElecOil"))**LamdaInv   )

```
        $(not sameas(AllSecAllSub,"SElecOil"))
   + ( 1/sum(SubSectorP
            $(    (not sameas(SubSectorP,"SElecNucl"))
              and (not sameas(SubSectorP,"SElecHydro"))
              and (not sameas(SubSectorP,"SElecBiom"))
              ),
        SubInvSh3(SubSectorP)
        )
     )$sameas(AllSecAllSub,"SElecOil")

 )/(1 - InvTaxCredit(AllSecAllSub) )
   ;
```

Investment Demand

      The investment takes place at the sector and subsector. Also, some sectors (crude oil and natural gas) have fixed investment. The investment is defined as a function of last period's investment, a base rate, the growth in working population, expected profit rate, and elasticity of excess profit rate. These parameters are given in SGM, except for the expected profit rate that is determined within the model as described above.

```
InvestDem(AllSecAllSub)
 $ ( (    YesInv(AllSecAllSub)
      or OnlySector(AllSecAllSub)
      or OnlySubSector(AllSecAllSub)
      )
   and (not sameas(AllSecAllSub,"SElecBiom"))
   )..

   QInvest(AllSecAllSub)
     =E=

* #### this is for Sector

   ( QInvest0("1985",AllSecAllSub)
     * BaseRate
       * Population0("1990","WorkRate")
        * ( ExpProfSec(AllSecAllSub))**ElasInv(AllSecAllSub)
   )$( OnlySector(AllSecAllSub) and not ExoInv(AllSecAllSub) )


* #### this is for SubSector using logit sharing structures

 + ( QInvest0("1985",AllSecAllSub)
     * BaseRate
       * Population0("1990","WorkRate")
```

```
        * (ExpProfSub(AllSecAllSub))**ElasInv(AllSecAllSub)
    )$(      OnlySubSector(AllSecAllSub)
        and (not ExoInv(AllSecAllSub))
        and YesInv(AllSecAllSub)
      )
```

* #### this is for exogenous investment

```
 + (QExogInv(AllSecAllSub))$( ExoInv(AllSecAllSub)  )
      ;
```

Capital Stock

     The investment will be converted into a capital stock for the next time period. Capital stocks are modeled as falling into 4 vintage classes. For the next time period, the new vintage of capital stocks operates under the long-run elasticity of substitution while the old vintage of capital stocks operates under the short-run elasticity of substitution.

```
KStockEq(AllSecAllSub,Vintage2)
 $(not sameas(AllSecAllSub,"SElecBiom"))..

  KStock(AllSecAllSub,Vintage2)
    =E=
 (  2*QInvest0("1985",AllSecAllSub)
   + 3*QInvest(AllSecAllSub)
 )$(sameas(Vintage2,"0"))

 + KStock0(AllSecAllSub,"-1")
     $(sameas(Vintage2,"-1"))
 + KStock0(AllSecAllSub,"-2")
     $(sameas(Vintage2,"-2"))
 + KStock0(AllSecAllSub,"-3")
     $(sameas(Vintage2,"-3"))
 ;
```

Exports and Imports

     Because the current model is a single country operation (only the US), the quantity of exports and imports are defined as a fixed proportion to the production. The proportion values are calculated from quantity exports and imports given in SAM table.

```
ImportEq(Sector,SubSector)
 $( YesImport(Sector)
   and TisMap(Sector,SubSector)
 )..
```

```
  QImport(Sector,SubSector)
   $TisMap(Sector,SubSector)
   =E=
 ( QImportPct(Sector)
   *sum(Vintage,Production(Sector,SubSector,Vintage))
  )$(not sameas(Sector,"ElecGen"))

 + ( QImportPct(SubSector)
   *sum(Vintage,Production(Sector,SubSector,Vintage))
  )$(sameas(Sector,"ElecGen"))
    ;

ExportEq(Sector,SubSector)
 $( YesExport(Sector)
   and TisMap(Sector,SubSector)
  )..

  QEXport(Sector,SubSector)
   $TisMap(Sector,SubSector)
   =E=
 ( QExportPct(Sector)
   *sum(Vintage,Production(Sector,SubSector,Vintage))
  )$(not sameas(Sector,"ElecGen"))

 + ( QExportPct(SubSector)
   *sum(Vintage,Production(Sector,SubSector,Vintage))
  )$(sameas(Sector,"ElecGen"))
    ;
```

Note that when the current model moves from the single region model to global model, these exports and imports will be modified to be traded at world market prices.

Greenhouse Gas Modeling

Modeling greenhouse gasses involves portrayal of both emissions and sinks from all sectors. We discuss separately the agricultural sector and from energy sectors as well as non-energy sectors. The greenhouse gas limitation is used to cap on the carbon net emission under the Kyoto.

Agricultural Emissions and Sinks

Agricultural emissions and sinks are included in the model using response functions estimated using data generated from repeated runs of the U.S. Agricultural Sector Model greenhouse gas version (ASMGHG). These are used to model agricultural emissions and sink reactions to changes in the general economy. Conceptually, these functions are modeled as follows:

Agricultural Emissions = exp(a_1
                    + $a_2 * \ln(e_1 + \text{Carbon price})$
                    + $a_3 * \ln(e_2 * (1 + \Delta \text{ in US agricultural demand}))$
                    + $a_4 * \ln(e_3 * (1 + \Delta \text{ in Energy price}))$
                    + $a_5 * \ln(e_4 * (1 + \Delta \text{ in US agricultural export demand})))$

Ln(Agricultural Sinks)   = exp(b_1
                    + $b_2 * \ln(e_1 + \text{Carbon price})$
                    + $b_3 * \ln(e_2 * (1 + \Delta \text{ in US agricultural demand}))$
                    + $b_4 * \ln(e_3 * (1 + \Delta \text{ in Energy price}))$
                    + $b_5 * \ln(e_4 * (1 + \Delta \text{ in US agricultural export demand})))$

where a_i and b_i are estimated parameters associated with carbon price, US agricultural demand, Energy price, and US exports; and e_i are the base values (0 for carbon price and 100 for the others). These estimated parameters (a_i and b_i) are previously estimated in the response function with a log-linear function, $\ln(\mathbf{Y}) = \mathbf{A} + \boldsymbol{\beta} * \ln(\mathbf{x})$ where $\mathbf{A}$ and $\boldsymbol{\beta}$ are a vector of intercept terms and a vector of estimated parameters associated with a vector of $\mathbf{Y}$ and $\mathbf{x}$, respectively. The base functions with all of the independent variables held at the base level depict the ASMGHG output under a zero carbon price, and the1997 levels of energy price, domestic agricultural product demand, and agricultural export demand. The completed expression of these equations is presented in lines 8503 to 8597.

Energy/non-energy Sector Emissions

       To calculate emissions by the energy sectors, the production levels within the sectors using energy inputs are converted into physical energy units (joules) using the conversion from SGM and then these physical energy units are multiplied by GHG emission coefficients as shown below:

       Emissions = $c_i * g_{ig} * \text{values of production}$

where c_i is the physical energy conversion and g_{ig} is the emission coefficient by Ghg and energy input types (see lines 8598 to 8632).

	Conversion (c_i)	GHG emission coefficients (g_{ig})		
		CO_2	CH_4	N_2O
Crude oil	0.000436	18.81	0.187	0.010
Natural gas	0.000436	14.28	0.187	0.007
Coal	0.001045	23.74	0.354	0.014

Net Emissions

       This equation implies that the net emissions in the region are equal to the sum of the agricultural and energy emissions minus the agricultural sinks.

NetEmisQEq(Region,Ghg)$sameas(Region,"US")..
* #### Sources and Sinks from Ag sector

   + sum(Vintage,
         AgEmitQ(Ghg,Vintage)
        - AgSinkQ(Ghg,Vintage)$(not sameas(Ghg,"N2O"))
)

* #### Sources from Energy sector

   + sum((Sector,Vintage)
     $TisEnergy(Sector),
        EnergyEmitq(Sector,Ghg,Vintage)
           $(CEmission(Sector,Ghg)   )
)
   =G=
   NetEmiTQ(Region,Ghg)
    ;

GHG Emissions Limitation

At the base, the amount of greenhouse gas is not limited and therefore the US net emissions are in the neighborhood of the 1990 net emissions and the carbon price is zero. When the amount of greenhouse gas is limited at some levels (i.e. Kyoto), the model will search for a carbon price at the equilibrium to ensure that this greenhouse gas constraint is binding. We have done trial runs at the Kyoto limits and get a carbon price from $12 to $50 per a metric ton of carbon equivalent (see **output.txt** file).

EmisQLimitEq(Region)$sameas(Region,"US")..
 sum(TisLevel,
     sum(Ghg,GhgLimitQ(Region,Ghg,TisLevel))   )
 + 10*(1-LimitType)
  =G=
 sum(Ghg,NetEmiTQ(Region,Ghg))
  ;

Carbon Tax Rate

Imposing a carbon net emission cap imposes a cost on sectors that use inputs involving with the greenhouse gas. Therefore, the carbon tax rate is first calculated and then it is used to calculate the total carbon tax value which is considered as revenues to the government but costs to sectors. This total carbon tax values is then included into zero profit condition as costs.

CarbTaxRateEq(Sector,Ghg)
 $TisEnergy(Sector)..

```
   CarbTaxRate(Sector,Ghg)
    =E=
   Pcarb("US")
    * PricToPhysic(Sector)
      * CEmission(Sector,Ghg)
        *GWP(Ghg)
          *Exchange0
   ;

* #### Carbon tax revenues by users

CarbTaxTotEq(Sector,Ghg)
  $TisEnergy(Sector)..

  CarbTaxTot(Sector,Ghg)
    =E=
  CarbTaxRate(Sector,Ghg)
      * (
          + sum(MapSecSub(OthSector,SubSector),
              QIntC(Sector,OthSector,SubSector)
                $ica1(Sector,OthSector,SubSector))
                  *ComPrice(Sector)

          + sum(HouseholdH,
              GoodsDemand(Sector,HouseholdH)
                $SAM(Sector,HouseholdH)  )
                  *ComPrice(Sector)

          + sum(HouseHoldG,
              GovConsumption(Sector,HouseHoldG)
                $GovConsumption0(Sector,HouseHoldG)
            )*ComPrice(Sector)

          + sum(MapSecAllSub(Sector,AllSecAllSub),
              QInvest(AllSecAllSub)$YesInv(AllSecAllSub))
                  *ComPrice(Sector)
          + QStockChnge(Sector)*ComPrice(Sector)

          + sum(MapSecSub(Sector,SubSector)$YesExport(Sector),
              QExport(Sector,SubSector)
                *PExp(Sector))
        )
    ;
```

Equilibrium Conditions

A set of non-zero prices in output and input markets, production, consumption levels, production levels, and factor usages constitutes an economic equilibrium solution and a solution to a CGE of the situation if the following conditions are satisfied.

Factor Market Balance

Total factors usage in production is less than or equal to the total supply in every factor input markets (labor and capital) which is composed from the household endowments.

FactorMkt(Factor2,Sector,SubSector)
 $TisMap(Sector,SubSector)..

 sum(HouseholdH,
   Endowment1(Factor2,Sector,SubSector,HouseholdH))
     $sameas(Factor2,"Labor")
 + sum(Vintage2,KAPrior0(Sector,SubSector,Vintage2))
   $sameas(Factor2,"Capital")
    =G=
  FactorQ(Factor2,Sector,SubSector)
  ;

Commodity Market Balance

This condition implies that the total demand in every output market including consumer and intermediate production usage is less than or equal to total supply in that market. In other words, the excess demand in each output market is less than or equal to zero:

SupplyDemandEq(Sector)
 $(COMPRICE0(Sector)   )..

 sum(MapSecSub(Sector,SubSector),
   sum(Vintage,Production(Sector,SubSector,Vintage)))
 =G=
 sum(HouseholdH,
    GoodsDemand(Sector,HouseholdH)
      $SAM(Sector,HouseholdH))

 + sum(HouseHoldG,
     GovConsumption(Sector,HouseHoldG)
)

 + sum(MapSecSub(OthSector,SubSector)
      $ica1(Sector,OthSector,SubSector),
       QIntC(Sector,OthSector,SubSector))

+ sum(MapSecAllSub(Sector,AllSecAllSub),
      QInvest(AllSecAllSub)$YesInv(AllSecAllSub)   )

+ sum(MapSecSub(Sector,SubSector)
      $YesExport(Sector),
       QExport(Sector,SubSector))

- sum(MapSecSub(Sector,SubSector)
      $YesImport(Sector),
       QImport(Sector,SubSector))

+ QStockChnge(Sector)

   ;

Zero Profit Condition

For each production sector revenues are less than or equal to costs with in effect all rents allocated to factors. Thus the total revenue is less than factor usage costs plus costs of intermediate products obtained plus other tax payments and plus carbon costs when the carbon emission limits is imposed.

Profit(Sector,SubSector,Vintage)
  $TisMap(Sector,SubSector)..

* #### Factor costs
    sum(Factor2,
        FactorQ(Factor2,Sector,SubSector)
          * FactPriceSec(Factor2,Sector,SubSector)
)

* #### Intermediate inputs costs
  + ComPrice(Sector)
      * sum(Activity
          $ica1(Activity,Sector,SubSector),
             QIntC(Activity,Sector,SubSector))

* #### corporate taxes
  + sum(MapSecSubSub(Sector,AllSecAllSub,SubSector),
        1 - PriceReceive(AllSecAllSub)
)
       * Production(Sector,SubSector,Vintage)

* #### Carbon costs from Agricultural sector from ASM
  + sum(Region$sameas(Region,"US"),

```
    sum(Ghg,
         PCarb(Region)*PricToPhysic("OthAg")*
           (  AgEmitQ(Ghg,Vintage)
            - AgSinkQ(Ghg,Vintage)$(not sameas(Ghg,"N2O"))
            )
         )
      )$AgGroup(Sector)

* #### Carbon costs from Energy sector
    +  sum(ghg,
          sum(MapSecSub(OthSector,SubSector)$TisEnergy(OthSector),
              CarbTaxRate(OthSector,Ghg)*QIntC(OthSector,Sector,SubSector)
                $ica1(OthSector,Sector,SubSector)
                  *ComPrice(OthSector))
          )
    +  sum(Ghg,
          CarbTaxRate(Sector,Ghg)$TisEnergy(Sector)
            *  (  QInvest(Sector)
               + QStockChnge(Sector)
              )$YesInv(Sector)
                *ComPrice(Sector)
          )
    +  sum(Ghg,
          sum(MapSecSub(Sector,SubSector)$YesExport(Sector),
              CarbTaxRate(Sector,Ghg)$TisEnergy(Sector)
               * QExport(Sector,SubSector)
                 *PExp(Sector)  )
          )
  =G=
  ComPrice(Sector)
    * Production(Sector,SubSector,Vintage)
     ;
```

Price Block

Below equations represent the price system of the model. Some of these price equations are linked to other prices variables while some are linked to non-price variables in the model.

Intermediate Input Prices

The intermediate input prices show costs of disaggregated intermediate inputs.

PIntAEq(Sector,SubSector)
 $TisMap(Sector,SubSector)..

 PIntA(Sector,SubSector)

```
 =G=
SUM(Activity,
   ica1(Activity,Sector,SubSector)
     *ComPrice(Activity)      )
 ;
```

<u>Value-Added Prices</u>

This value-added price indicates that the total revenue (net of taxes) is fully exhausted by payments for value-added and intermediate inputs.

```
PVAEq(Sector,SubSector)
  $TisMap(Sector,SubSector)..

  sum(MapSecSub(Sector,SubSector),
   ComPrice(Sector)
   )
      *  sum(Vintage,Production(Sector,SubSector,Vintage)  )
  =G=
  PValAdd(Sector,SubSector)* QValAdd(Sector,SubSector)
 + PIntA(Sector,SubSector)*  QIntA(Sector,SubSector)
   ;
```

<u>Price Paid</u>

The price paid represents the price in which sectors paid for the use of input i.

```
PricePaidEq(Activity,AllSecAllSub)
 $( (not sameas(AllSecAllSub,"SElecBioM"))
   and (not sameas(Activity,"IBT"))
   and (not sameas(Activity,"Land"))
 )..

  PricePaid(Activity,AllSecAllSub)
   =E=
  ComPrice(AllSecAllSub)
    $(   (not sameas(Activity,"Labor") )
      and (not sameas(Activity,"Capital") )
      and (not sameas(Activity,"Land") )
       and OnlySector(AllSecAllSub)
      )
 + ComPrice("ElecGen")
    $(   (not sameas(Activity,"Labor") )
      and (not sameas(Activity,"Capital") )
      and (not sameas(Activity,"Land") )
       and OnlySubSector(AllSecAllSub)
```

```
        )
  + sum(MapSecSub(AllSecAllSub,SubSector)$TisMap(AllSecAllSub,SubSector),
        FactPriceSec(Activity,AllSecAllSub,SubSector)
      $(  (  sameas(Activity,"Labor")
            or sameas(Activity,"Capital")
          )
        and OnlySector(AllSecAllSub)
        and YesCESVa(AllSecAllSub)
    )   )
  + FactPriceSec(Activity,"ElecGen",AllSecAllSub)
      $(  (  sameas(Activity,"Labor")
            or sameas(Activity,"Capital")
          )
        and OnlySubSector(AllSecAllSub)
        and YesCESVa(AllSecAllSub)    )
  + ( sum(Ghg$(TisEnergy(Activity)   ),
        CarbTaxRate(Activity,Ghg))     )
      $  (  sameas(Activity,"Labor")
            or sameas(Activity,"Capital")
            or TisEnergy(Activity)           )
  + ( sum(Ghg$(TisEnergy(Activity)
            and OnlySubSector(AllSecAllSub)            ),
        CarbTaxRate("ElecGen",Ghg))     )
      $  (  sameas(Activity,"Labor")
            or sameas(Activity,"Capital")           )
    ;
```

Price Received

The price received represents the price in which producers received from the sales of output j excluding indirect business taxes.

```
PriceReceivEq(AllSecAllSub)
 $(not sameas(AllSecAllSub,"SElecBiom"))..

  PriceReceive(AllSecAllSub)
   =E=
  sum(MapSecAllSub(Activity,AllSecAllSub),
     PricePaid(Activity,AllSecAllSub)
      *(1 - IBTTaxRate0(AllSecAllSub))
  )
   ;
```

Factor Price by Sector

This equation represents a factor price by sector and subsector. Note that the average nation wage factor price (FactPriceNation) is an endogenous variable where the activity specific "wage-distortion factor (FactPriceDist1) is exogenous.

```
FactPriceSecEq(Factor,Sector,SubSector)
 $(    TisMap(Sector,SubSector)
   and YesCESVa(Sector)
   )..

  FactPriceSec(Factor,Sector,SubSector)
  =E=
  FactPriceNation(Factor)
   * FactPriceDist1(Factor,Sector,SubSector)
 ;
```

Technical Change

We assume no technical change for the current model but can add this when we add a dynamic updating procedure in a multi-time period version.

Model Solve Statements

The class of models that this SGMGAMS falls into is the so-called mixed complementarity class (an MCP) in GAMS. To solve such models we must have a model with complementarity requirements associated with each equation as shown in the table below. The complementary relationships are expressed in the model statement.

Complementary Items

Variable Name	Equation Name
Quantity block:	
QValAdd(Sector,SubSector)	CESQVAEq(Sector,SubSector)
FactPriceNation(Factor)	CESQVAFOC(Factor)
QintA(Sector,SubSector)	QintAEq(Sector,SubSector)
QIntC(Activity,Sector,SubSector)	QIntCEq(Activity,Sector,SubSector)
PImp(Commodity)	ImpBal(Commodity)
PExp(Commodity)	ExpBal(Commodity)
QImport(Sector,SubSector)	ImportEq(Sector,SubSector)
QExport(Sector,SubSector)	ExportEq(Sector,SubSector)
Price block:	
FactPriceSec(Factor,Sector,SubSector)	FactPriceSecEq(Factor,Sector,SubSector)
PValAdd(Sector,SubSector)	PVAEq(Sector,SubSector)
PIntA(Sector,SubSector)	PIntAEq(Sector,SubSector)
PricePaid(Activity,AllSecAllSub)	PricePaidEq(Activity,AllSecAllSub)
PriceReceive(AllSecAllSub)	PriceReceivEq(AllSecAllSub)

Household consumption:	
HhExpend(HouseholdH)	HhExpendEq(HouseholdH)
Hhincome(HouseholdH)	IncomeEq(HouseholdH)
GoodsDemand(Commodity,HouseholdH)	GoodsDemandEq(Commodity,HouseholdH)
Investment:	
QInvest(AllSecAllSub)	InvestDem(AllSecAllSub)
AlphaInv1(Activity,AllSecAllSub)	AlphaInv1Eq(Activity,AllSecAllSub)
AlphaInv2(AllSecAllSub)	AlphaInv2Eq(AllSecAllSub)
AlphaInv3(Activity,AllSecAllSub)	AlphaInv3Eq(Activity,AllSecAllSub)
AlphaInv(Activity,AllSecAllSub)	AlphaInvEq(Activity,AllSecAllSub)
IBigAij(Activity,AllSecAllSub)	IBigAijEq(Activity,AllSecAllSub)
Zvalue(AllSecAllSub)	ZvalueEq(AllSecAllSub)
SubInvSh1(AllSecAllSub)	SubInvSh1Eq(AllSecAllSub)
SubInvSh3(AllSecAllSub)	SubInvSh3Eq(AllSecAllSub)
ExpProfSec(AllSecAllSub)	ExpProfSecEq(AllSecAllSub)
ExpProfSub(AllSecAllSub)	ExpProfSubEq(AllSecAllSub)
Kstock(AllSecAllSub,Vintage)	KstockEq(AllSecAllSub,Vintage)
Government:	
GovSaving	GovBudgetEq
GovIncome	GovIncomeEq
GovTrnsfPaymt	GovTrnsfEq
GovDemand(HouseHoldG)	GovDemandEq(HouseHoldG)
GovConsumption(Sector,HouseHoldG)	GovConsumpEq(Sector,HouseHoldG)
GovExpend	GovExpendEq
Greenhouse gas:	
AgEmitQ(Ghg,Vintage)	AgEmitQEq(Ghg,Vintage)
AgSinkQ(Ghg,Vintage)	AgSinkQEq(Ghg,Vintage)
EnergyEmitQ(Sector,Ghg,Vintage)	EnergyEmitQEq(Sector,Ghg,Vintage)
NetEmiTQ(Region,Ghg)	NetEmisQEq(Region,Ghg)
PCarb(Region)	EmisQLimitEq(Region)
CarbTaxRate(Sector,Ghg)	CarbTaxRateEq(Sector,Ghg)
CarbTaxTot(Sector,Ghg)	CarbTaxTotEq(Sector,Ghg)
Equilibrium Condition:	
FactorQ(Factor,Sector,SubSector)	FactorMkt(Factor,Sector,SubSector)
ComPrice(Sector)	SupplyDemandEq(Sector)
Production(Sector,SubSector,Vintage)	Profit(Sector,SubSector,Vintage)

Model Files

To run the model, please open the integrating file called **allsgm.gms**. **Allsgm.gms** includes

> **sgmdat.gms**   containing the SGMGAMS sets, parameters, scalars, and data

sgmcalib.gms	calculating some predetermined parameters needed to calibrate the model and other exogenous values
sgmmod.gms	containing the SGMGAMS equation structure
sgmparm.gms	containing declaration of sets and parameters used for report writing
sgmreport.gms	containing report writing code
sgmcompr.gms	containing report writing code for comparative analysis
sgmloop.gms	containing codes for comparative analysis and writing out comparative results. Here, emission reductions from the agriculture are considered.
sgmloopfix.gms	containing codes for comparative analysis and writing out comparative results. Here, emission reductions from the agriculture are not considered.

Algorithm Used

The SGMGAMS is best solved with the PATH solver. Details on that solver can be found at http://www.gams.com/solvers/solvers.htm#PATH. So we choose PATH as the solver and solve with

OPTION MCP = PATH;

SOLVE SGMCGE USING MCP;

Resultant Solution

In turn, a solution arises and it can be used in the normal report writing, graphics etc. as with any other model (see **sgmgams.out** file)