Including Risk

Notes for AGEC 622

Bruce McCarl

Regents Professor of Agricultural Economics

Texas A&M University

Including Risk

McCarl and Spreen Chapter 14

Where does it occur

Maximize	CX		
Subject to	AX	$\leq$	b
	X	$\geq$	0

Objective function returns - C

Variability in prices, production quantities, costs, and market sales

Resource usages - A

Variability in raw input quality, working conditions, intermediate product yields, and product requirements

Resource endowments - b

Variability in demand firm faces, resources available, and working conditions

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When incorporating risk there are three big issues

1. What is the nature of risk?

    a. What parameters of the model are uncertain?

    b. How do we describe their distribution?

2. When are risk outcomes revealed?

    Does the producer receive information about uncertain events and will make adaptive decisions?

3. How do we model behavioral reaction to risk?

    Is expected profit maximization not the proper objective but rather some degree of aversion to the variation caused by risk?

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Forms of assumed reaction to risk

Non-Recourse or non adaptive decision making

- Decisions made now consequences felt later
- No additional decisions made until consequences felt

Example – Buy stock now make no decisions for one year

Recourse or adaptive decision-making

- Decisions made now consequences arise over time
- Later time additional decisions made.
- Later one knows what happened between first decision and now.
- Cannot reverse prior actions but can adjust ie can be make adjustments -- phenomena called irreversibility and recourse

Example – Buy stock now, review decisions quarterly possibly selling and buying other stocks

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Forms of assumed reaction to risk

Expected Value Maximization

$$\begin{array}{llll} \max & \bar{c}X & & \\ s.t. & \bar{A}X & \leq & \bar{b} \\ & X & \geq & o \end{array}$$

Conservative - Fat or thin coefficients

where

$$\tilde{c} = \bar{c} - \text{Risk Discount}$$

$$\tilde{a} = \bar{a} + \text{Risk Discount}$$

$$\tilde{b} = \tilde{b} - \text{Risk Discount}$$

E- V

Maximize E(income) - RAP * Variance(income)

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Forms of assumed reaction to risk

Our risk treatment will be limited because of time constraints (see newbook.pdf chapter 14 for more extensive treatment)

1. Will treat risk in objective function coefficients. Will not discuss how to form such probability distributions

2. Will only cover the expected value variance formulation (this is the two main one used in practice -- see newbook.pdf chapter 14 for more extensive treatment)

3. We will treat non adaptive behavior

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Why model risk

Why not just solve for all values of risky parameters

Curses of dimensionality and certainty

Dimensionality -Number of possible plans

(3 possible values for 5 parameters $3^5 = 243$)

Certainty Each would be certain of data so we would have

243 different things we could do –

Which one would we do?

General Risk Modeling Aim

Generate a plan which is Robust in the face of the Uncertainty

Not best performer necessarily in any setting, but a good performer across many or most of the uncertainty spectrum

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First Risk Model

Markowitz mean-variance portfolio choice formulation

Given Problem

max sum(invest, moneyinvest(invest)*avgreturn(invest))

s.t sum(invest,moneyinvest(invest)*price(invest))$\leq$ funds

moneyinvest(invest) $\geq$ 0

Markowitz observed not all money is invested in the highest valued stock

Inconsistent with LP formulation- Why? Not a basic solution

Markowitz posed the hypothesis that average returns and the variance of returns were important

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Given a linear objective function

$$Z = c_1X_1 + c_2X_2$$

Where X_1, X_2 are decision variables

c_1 , c_2 are uncertain parameters

Then Z is distributed with mean and variance

$$Z = \hat{c}_1X_1 + \hat{c}_2X_2$$

$$S_z = s_1^2X_1^2 + s_2^2X_2^2 + 2 * s_{12}X_1X_2$$

Where $\hat{c}_1\ \hat{c}_2$ are the means of c_1, c_2

$s_1^2\ s_2^2$ are the variances of c_1, c_2

s_{12} is the covariance of c_1, c_2

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Defining terms

s_i^2 is the variance of objective function coefficient of X_i, which is calculated using $\Sigma_k (c_{ik} - \hat{c}_i)^2/N$ where c_{ik} is the k^{th} observation on the objective value of X_i and N the number of observations.

s_{ij} for $i \neq j$ is the covariance of the objective function coefficients between X_i and X_j, calculated by the formula $s_{ij} = \Sigma_k (c_{ik} - \hat{c}_i)(c_{jk} - \hat{c}_j)/N$.

Note $s_{ij} = s_{ji} = r * \text{sqrt}(s_i^2 * s_i^2)$ where r is correlation

$\hat{c}_i$ is the mean value of the objective function coefficient associated with X_i, calculated by $= \Sigma_k c_{ik}/N$.

(Assuming an equally likely probability of occurrence.)

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E-V Model – Statistical Background

In matrix terms the mean and variance of Z are

$$\overline{Z} = \overline{c}\, X$$

$$\sigma_Z^2 = X'\, S\, Z$$

where in the two by two case

$$\overline{Z} = E = \begin{bmatrix} \overline{c}_1 & \overline{c}_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$\sigma_Z^2 = \begin{bmatrix} X_1 & X_2 \end{bmatrix} \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

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E-V Model – Statistical Background

Programming Formulation

$$Max \quad E - \phi \sigma_Z^2 \quad = \quad \bar{c}\, X - RAP * X'\, S\, Z$$

$$= \quad \sum_j \bar{c}_j X_j - RAP \sum_j \sum_k s_{jk} X_j X_k$$

$$s.t. \qquad \sum_j X_j \quad \leq funds$$

$$X_j \quad \geq 0 \; for\, all \; j$$

or Max E - RAP * Variance

RAP equals dollars of income forgone per dollar of variance reduced

It is used by varying RAP from

- 0 (profit max solution)
- infinity (minimum variance solution)

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Data for E-V Example -- Returns by Stock and Event

----Stock Returns by Stock and Event----

	Stock1	Stock2	Stock3	Stock4
Event1	7	6	8	5
Event2	8	4	16	6
Event3	4	8	14	6
Event4	5	9	-2	7
Event5	6	7	13	6
Event6	3	10	11	5
Event7	2	12	-2	6
Event8	5	4	18	6
Event9	4	7	12	5
Event10	3	9	-5	6
Stock Price	22	30	28	26

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Mean Returns and Variance Parameters for Stock Example

	Stock1	Stock2	Stock3	Stock4
Mean Returns	4.70	7.60	8.30	5.80

Variance-Covariance Matrix

	Stock1	Stock2	Stock3	Stock4
Stock1	3.21	-3.52	6.99	0.04
Stock2	-3.52	5.84	-13.68	0.12
Stock3	6.99	-13.68	61.81	-1.64
Stock4	0.04	0.12	-1.64	0.36

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EV Model Example

Max E - ☠ ☞^{2} = E - RAP * Variance

$$
\begin{aligned}
&Max \quad \sum_j \bar{c}_j X_j \quad - \phi \sum_j \sum_k s_{jk} X_j X_k \\
&s.t. \quad \sum_j X_j \qquad\qquad \leq funds \\
&\qquad X_j \qquad\qquad \geq 0 \ for\, all \ j
\end{aligned}
$$

or for the example

$$
Max \begin{bmatrix} 4.70 & 7.60 & 8.30 & 5.80 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} - \phi \begin{bmatrix} X_1 & X_2 & X_3 & X_4 \end{bmatrix} \begin{bmatrix} +3.21 & -3.52 & +6.99 & +0.04 \\ -3.52 & +5.84 & -13.68 & +0.12 \\ +6.99 & -13.68 & +61.81 & -1.64 \\ +0.04 & +0.12 & -1.64 & +0.36 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}
$$

$$
s.t. \ \ 22X_1 + \ 30X_2 + \ 28X_3 + 26X_4 \leq 500
$$

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EV Model Example

$$Max\ [4.70\ \ 7.60\ \ 8.30\ \ 5.80]\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} - \phi\ [X_1\ \ X_2\ \ X_3\ \ X_4]\begin{bmatrix} +3.21 & -3.52 & +6.99 & +0.04 \\ -3.52 & +5.84 & -13.68 & +0.12 \\ +6.99 & -13.68 & +61.81 & -1.64 \\ +0.04 & +0.12 & -1.64 & +0.36 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

$$s.t.\ \ 22X_1 + 30X_2 + 28X_3 + 26X_4 \leq 500$$

or

$$Max\ 4.70X_1 + 7.60X_2 + 8.30X_3 + 5.80X_4$$

$$-\varphi\begin{bmatrix} +3.21X_1^2 & -3.52X_1X_2 & +6.99X_1X_3 & +0.04X_1X_4 \\ -3.52X_1X_2 & +5.84X_2^2 & -13.68X_2X_3 & +0.12X_2X \\ +6.99X_1X_3 & -13.68X_2X_3 & +61.81X_3^2 & -1.64X_3X_4 \\ +0.04X_1X_4 & +0.12X_2X_4 & -1.64X_3X_4 & +0.36X_4^2 \end{bmatrix}$$

$$s.t.\quad 22X_1 + 30X_2 + 28X_3 + 26X_4 \leq 500_4$$

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Solutions for Alternative Risk Aversion Parameters

RAP	**0**	**0.00025**	**0.0005**	**0.00075**	**0.001**
BUYSTOCK2			1.263	5.324	7.355
BUYSTOCK3	17.857	17.857	16.504	12.152	9.977
OBJ	148.214	143.287	138.444	135.688	134.245
MEAN	148.214	148.214	146.581	141.331	138.705
VAR	19709.821	19709.821	16274.764	7523.441	4460.478
STD	140.392	140.392	127.573	86.738	66.787
SHADPRICE	0.296	0.277	0.261	0.260	0.260

RAP	**0.011**	**0.012**	**0.015**	**0.025**	**0.050**
BUYSTOCK1			1.273	4.372	4.405
BUYSTOCK2	12.893	12.960	12.420	11.070	8.188
BUYSTOCK3	4.043	3.972	3.550	2.561	1.753
BUYSTOCK4					4.168
OBJ	125.441	124.614	123.380	120.375	116.805
MEAN	131.545	131.459	129.839	125.939	121.656
VAR	554.929	547.587	430.560	222.576	97.026
STD	23.557	23.401	20.750	14.919	9.850
SHADPRICE	0.239	0.236	0.234	0.230	0.224

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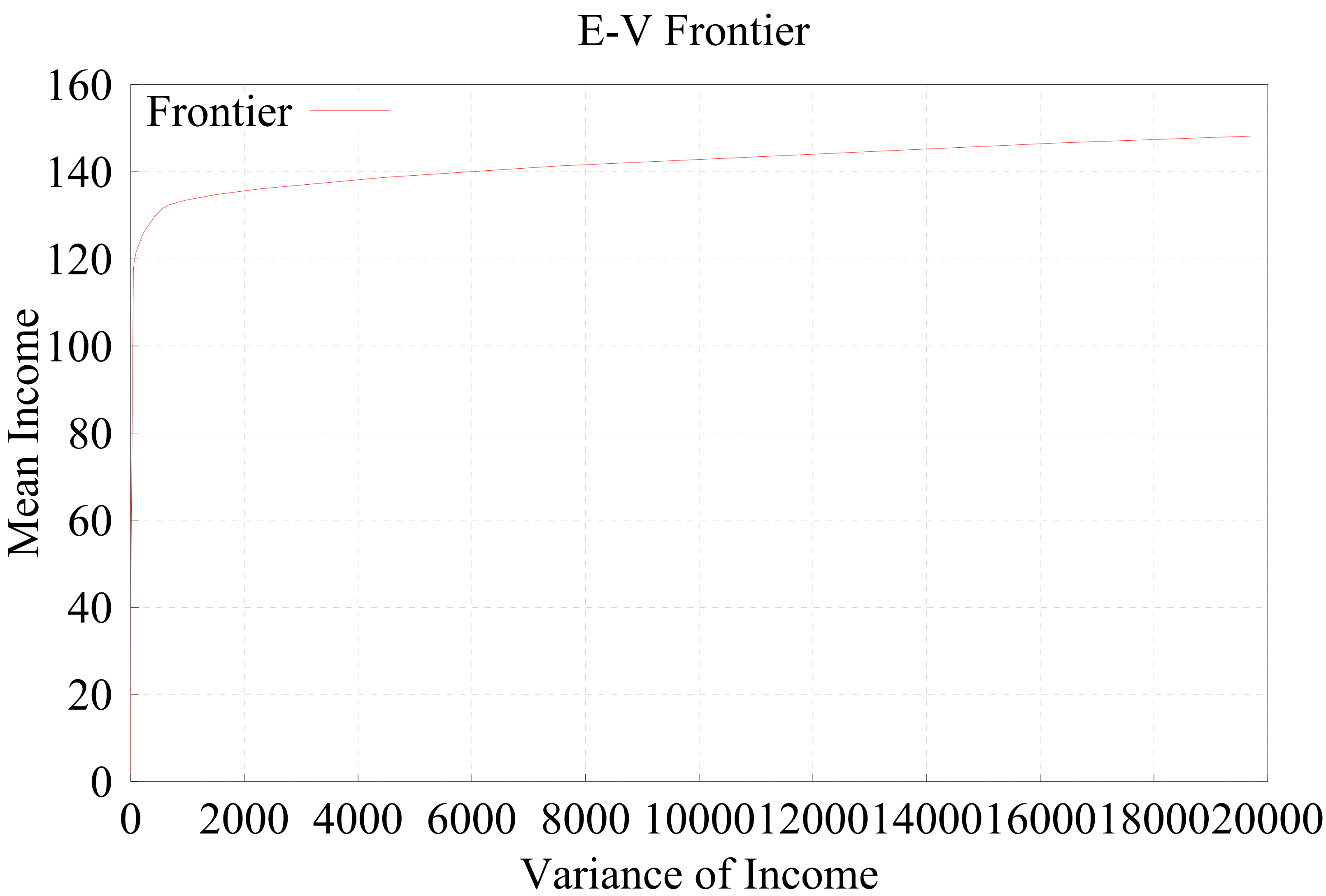
EV Example Frontier



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Shadow prices

$$\begin{aligned} &Max \quad \bar{C}X - \Phi \; X'SX \\ &s.t. \quad AX = b \end{aligned}$$

Lagrangian

$$\ell(X,\mu) = \bar{C}X - \Phi X'SX - \mu(AX - b)$$

$$\partial\ell / \partial X = \bar{C} - 2\Phi X'S - \mu A = 0$$

$$\partial\ell / \partial\mu = -(AX - b) = 0$$

implies roughly

$$\mu = (\bar{C} - 2\Phi X'S) / A$$

So u = risk adjusted average returns

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Forming Probability Distributions

Probability distributions state the relative frequency of occurrence of a set of mutually exclusive events.

Desirable Characteristics

1) each of the states of nature must be mutually exclusive;

2) probability of occurrence of each of the states of nature must be an unbiased measure of the current probability of that state of nature occurring;

3) the sum of the probabilities across the states of nature must equal one

Second property is the most troubling in when using objective, historical data. trends, events

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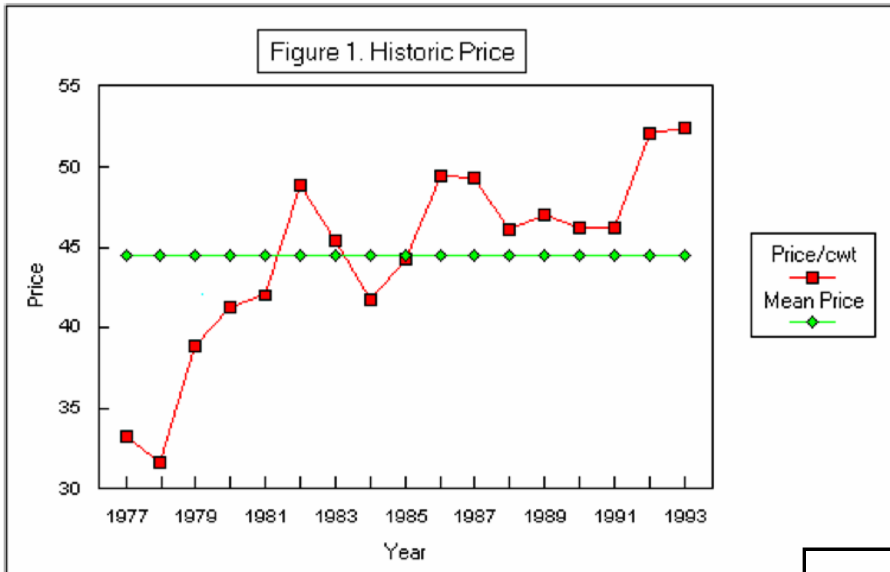
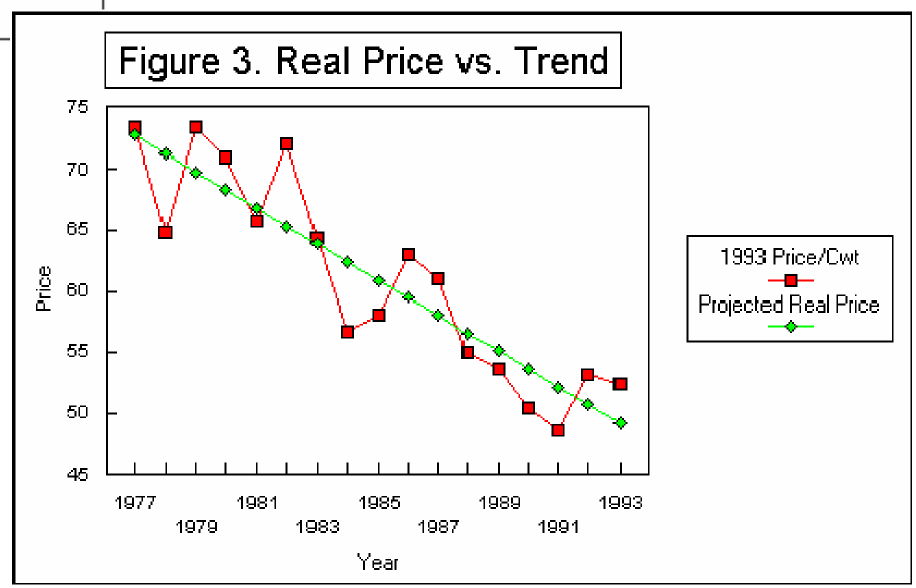
Forming Probability Distributions





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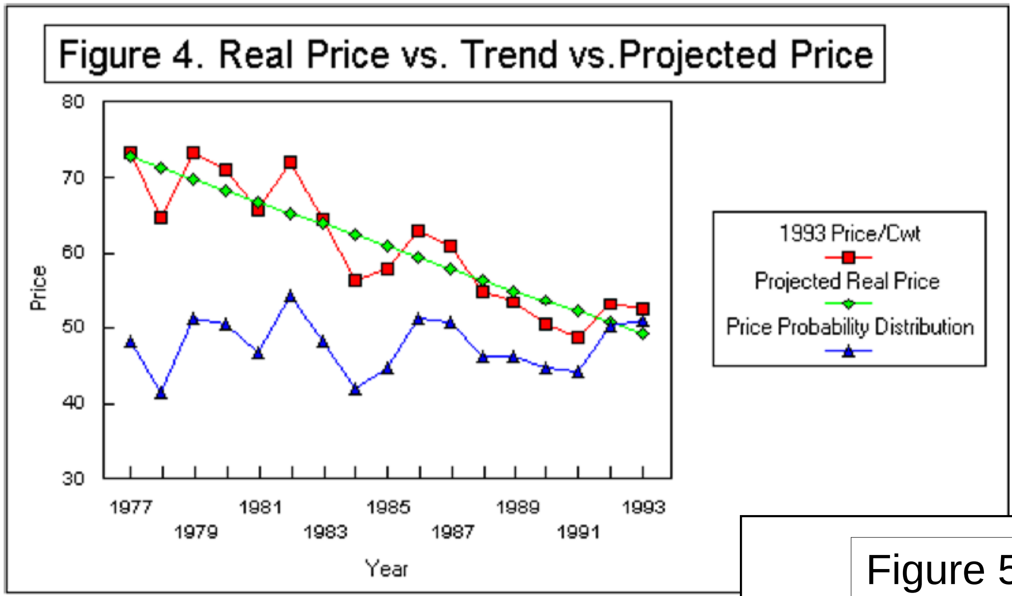
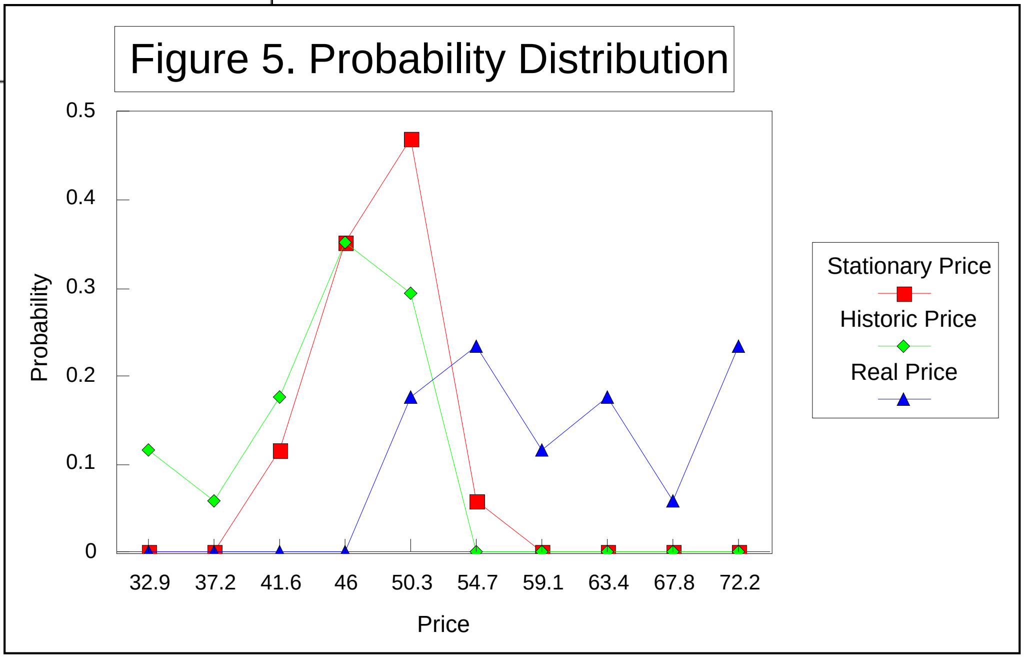
Forming Probability Distributions





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General Lessons Learned on Objective Probabilities

Use objective data – trends and other systematic effects can bias

Use a procedure like regression to develop values expected

One may find residual terms are hetereoskedastic