Basics of

Linear Programming (LP) Problems

Notes for AGEC 622

Bruce McCarl

Regents Professor of Agricultural Economics

Texas A&M University

Basic LP Problem

McCarl and Spreen Chapter 2

A LP problem is a linear form of mathematical programming

$$
\begin{array}{llllllll}
Max & c_1 X_1 & + c_2 X_2 & + c_3 X_3 & \ldots & + c_n X_n & & \\
s.t. & a_{11} X_1 & + a_{12} X_2 & + a_{13} X_3 & \ldots & + a_{1n} X_n & \leq & b_1 \\
 & a_{21} X_1 & + a_{22} X_2 & + a_{23} X_2 & \ldots & + a_{2n} X_n & \geq & b_2 \\
 & a_{21} X_1 & + a_{32} X_2 & + a_{33} X_3 & \ldots & + a_{3n} X_n & = & b_3 \\
 & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
 & a_{m1} X_1 & + a_{m2} X_2 & + a_{m3} X_3 & \ldots & + a_{mn} X_n & \leq & b_m \\
 & X_1 & X_2 & X_3 & & X_n & \geq & 0
\end{array}
$$

This formulation may also be expressed in matrix notation.

$$
\begin{array}{lll}
\text{Max} & CX & \\
\text{Subject to} & AX & \leq b \\
 & X & \geq 0
\end{array}
$$

Basic LP Application
Joe's Van Shop

Suppose Joe converts plain vans into custom vans and produces two grades, fine and fancy.

Joe makes money from this. Both types require a \$25,000 plain van. After conversion, fancy vans sell for \$37,000. Joe must use \$10,000 in parts, and there is a profit margin of \$2,000 per van. A fine van uses \$6,000 in parts and sells for \$32,700, yielding a profit of \$1,700.

Joe wishes to maximize his profits. Assuming that profits are constant per van, this can be written mathematically as:

Maximize $Z = 2000\ X_{fancy} + 1700\ X_{fine}$

Basic LP Application
Joe's Van Shop

Joe is limited to the number of vans he can convert. The shop can work on no more than 12 vans in a week, so:

$$X_{fancy} + X_{fine} \leq 12$$

Labor is also limiting. Joe has seven employees who work eight hours per day, five days a week. Therefore, he has at most 280 hours of labor available in a week. Joe has found that a fancy van will take 25 labor hours to make, while a fine van takes 20.

$$25\ X_{fancy} + 20\ X_{fine} \leq 280$$

Joe can also only convert nonnegative numbers of vans. So:

$$X_{fancy}\ ,\ X_{fine} \geq 0$$

Basic LP Application
Joe's Van Shop

So Joe's LP problem is:

$$\begin{aligned}
\text{Maximize } \; & 2000\, X_{fancy} + 1700 X_{fine} \\
\text{s.t. } \; & X_{fancy} + X_{fine} \leq 12 \\
& 25\, X_{fancy} + 20\, X_{fine} \leq 280 \\
& X_{fancy} ,\; X_{fine} \geq 0
\end{aligned}$$

Such a problem can be solved a number of ways.

The answer is:

Z=profits=$ 22800

$X_{fancy} = 8$

$X_{fine} = 4$

This solution uses all of our labor and capacity. We also get shadow prices.

Capacity value = $500 per van

Labor value = $60 per hour

Basic LP Application
Joe's Van Shop

Specific properties of the solution.

How much profit is produced is the objective value.
Z=profits=$ 22800

How much of each good is made. These are the decision variables.
$X_{fancy} = 8$, $X_{fine} = 4$

How much the resources are worth in their current application are found in the shadow prices or Lagrange multipliers since we use all of our labor and capacity.

Capacity value = $500 per van
Labor value = $ 60 per hour

A **shadow price** is an important LP concept and is an estimate of how much the objective function will change with a one unit change in the right hand side of an equation

Assumptions of LP

Attributes of model
- objective function appropriateness
- decision variable appropriateness
- constraint appropriateness

Math in model
- proportionality
- additivity
- divisibility
- certainty

Assumptions - Attributes of model

Objective Function Appropriateness

The objective function is the proper and sole criteria for choosing among the feasible values of the decision variables.

Decision Variable Appropriateness

The decision variables are all fully manipulatable within the feasible region and are under the control of the decision maker.

All appropriate decision variables have been included in the model.

Assumptions - Attributes of model

Constraint Appropriateness

The constraints fully identify the bounds placed on the decision variables by resource availability, technology, the external environment, etc. Any selection of decision variables, which simultaneously satisfies all the constraints, is admissible.

The resources used and/or supplied within any single constraint are homogeneous items that can be used or supplied by any decision variable appearing in that constraint.

Constraints do not improperly eliminate admissible values of the decision variables.

The constraints are inviolate. No considerations involving model variables other than those included in the model can lead to the relaxation of the constraints.

Assumptions - Math in model

Proportionality

The contribution per unit of a variable to any equation is constant. There are no economies of scale

Additivity

Contributions of variables to an equation are additive. The objective function and resource use equals the sum of contributions of each variable times levels. This rules out interaction or multiplicative terms in the objective function or the constraints.

Divisibility

All decision variables can take on any nonnegative value including fractional ones. In other words, the decision variables are continuous.

Certainty

All parameters are known constants. The optimum solution is predicated on perfect knowledge of all parameter values.