Basic Optimization Problem

Notes for AGEC 622

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Basic Optimization Problem

McCarl and Spreen Chapter 1

Optimize	$F(X)$		
Subject To (s.t.)	$G(X)$	ε	S_1
	X	ε	S_2

X is a vector of **decision variables**. X is chosen so that the objective F(X) is optimized.

F(X) is called the **objective function**. It is what will be maximized or minimized.

In choosing X, the choice is made, subject to a set of **constraints,** $G(X)\ \varepsilon S_1$ and $X \varepsilon S_2$ must be obeyed.

Basic Optimization Problem

Optimize **F(X)**

Subject to (s.t.)	**G(X)**	**ε**	S_1
	X	**ε**	S_2

A program is a linear programming problem when **F(X)** and **G(X)** are linear and X's ≥ **0**

When **X** εS_2 requires **X's to take on integer values,** you have an integer programming problem**.**

It is a quadratic programming problem **where G(X)** is **linear and F(X) is quadratic.**

It is a nonlinear programming problem **when F(X) and G(X)** are **general nonlinear functions.**

Decision Variables

They tell us how much of something to do:

acres of crops
number of animals by type
truckloads of oil to move

They are generally assumed to be nonnegative.

They are generally assumed to be continuous.

Sometimes they are problematic. For example, when the items modeled can not have a fractional part and integer variables are needed

They are assumed to be manipulatable in response to the objective.

This can be problematic also.

Constraints

Restrict

how much of a resource can be used

what must be done

For example

acres of land available

hours of labor

contracts to deliver

production requirements

nutrient requirements

They are generally assumed to be an inviolate limit.

They can be combined with variables to allow the use of more resources at a specific price or a buy out at a specified level.

Nature of Objective function

A decision maker is assumed to be interested in optimizing a measure(s) of satisfaction by selecting values for the decision variables.

This measure is assumed to be quantifiable and a single item.
For example:
Profit maximization
Cost minimization

It is the function that, when optimized, picks the best solution from the universe of possible solutions.

Sometimes, the objective function can be more complicated. For example, when dealing with profit, risk or leisure.

Example Applications

A firm wishes to develop a cattle feeding program.

Objective - minimize the cost of feeding cattle
Variables - quantity of each feedstuff to use
Constraint- non negative levels of feedstuffs
nutrient requirements so the animals don't starve.

A firm wishes to manage its production facilities.

Objective - maximize profits
Variables - amount to produce
inputs to buy
Constraints- nonneg production and purchase
resources available
inputs on hand
minimum sales per agreements

Example Applications

A firm wishes to move goods most effectively.

Objective - minimize transportation costs
Variables - amount to move from here to there
Constraints- nonnegative movement
available supply by place
needed demand by place

A firm is researching where to locate production facilities.

Objective - minimize production + transport cost
Variables - where to build
amount to move from here to there
amount to produce by location
Constraints- nonnegative movement, construction, production
available resources by place
products available by place
needed demand by place

This mixes a transport and a production problem.

Approach of the Course

Users generally know about the problem and are willing to use solvers as a “black box.”

We will cover:

appropriate problem formulation

results interpretation

model use

We will treat the solution processes as a **"black box."**

Algorithmic details and explanations will be left to other texts and courses such as industrial engineering.

Fundamental Types of Uses

Mathematical programming is way to develop the optimal values of decision variables.

However, there are a considerable number of other potential usages of mathematical programming.

Numerical usage is used to determine exact levels of decision variables is probably the least common usage.

Types of usage:

- problem insight construction
- numerical usages which find model solutions
- solution algorithm development and investigation

We discuss the first two types of use.

Problem Insight Construction

Mathematical programming usage requires a rigorous problem statement.

One must define:

- the objective function
- the decision variables
- the constraints
- complementary, supplementary and competitive relationships among variables

The data must be consistent.

A decision maker must understand the problem interacting with the situation thoroughly, discovering relevant decision variables and constraining factors in order to select the appropriate option.

Frequently, resultant knowledge outweighs the value of any solutions.

Numerical Mathematical Programming

Three main subclasses:

prescription of solutions
prediction of consequences
demonstration of sensitivity

Prescriptive usually involves addressing What should I do? (or normative) type questions.

For example: What decision should be made, given a particular specification of objectives, variables, and constraints?

It is probably the model in least common usage over universe of models.

Do you think that many decision makers yield decision making power to a model?

Very few circumstances deserve this kind of trust.

Models are an abstraction of reality that will yield a solution suggesting a practical solution, not always one that should be implemented.

Predictive

Most models are used for decision guidance or to predict the consequences of actions.

They are assumed to adequately and accurately depict the entity being represented.

They are used to predict in a conditional normative setting. In other words, if the firm wishes to maximize profits, then this is a prediction of what they should do, given particular stimulus.

In business settings models predict consequences of investments, acquisition of resources, drought management, and market price conditions.

In government policy settings models predict the consequences of:

policy changes
regulations
actions by foreign trade partners
public service provision (weather forecasting)
environmental change (global warming)

Sensitivity Demonstration

Many firms, researchers and policy makers would like to know what would happen if an event occurs.

In these simulations, solutions are not always implemented. Likewise, the solutions may not be used for predictions.

Rather, the model is used to demonstrate what might happen if certain factors are changed.

In such cases, the model is usually specified with a "realistic" data set. It is then used to demonstrate the implications of alternative input parameters and constraint specifications.