

## 641 FINAL EXAM

1. Discuss the qualifications you would place on the use of shadow prices from
  - a. Linear programming
  - b. Quadratic programming
  - c. Risk programming
  - d. Multiple objective programming under a weighted tradeoff objective function
  - e. Pure integer programming
  - f. Mixed integer programming
2. Suppose you were helping someone decide how to set up a new enterprise and had formulated the model

$$\begin{aligned} \max \quad & \sum_j \sum_k c_{jk} x_{jk} \\ \text{s.t.} \quad & \sum_j \sum_k a_{ijk} c_{jk} \leq b_i \quad \text{for all } i \\ & \sum_j x_{jk} \leq \text{cap}_k \quad \text{for all } k \\ & x_{jk} \geq 0 \end{aligned}$$

where  $j$  is activity type

$k$  is activity type of new enterprises

and you now need to reflect

- a. the fixed costs of the enterprises ( $F_k$ ) as well as the fact that you only get capacity ( $\text{cap}_k$ ) if the fixed cost is incurred.
- b. the fact that across all the possible new enterprises ( $k$ ) only 3 of them can be chosen with the rest zero.

Modify the model accordingly.

3. Suppose you have a problem with two objectives: why might you use a lexicographic or a weighted tradeoff model?
4. Given the quadratic program

$$\begin{aligned} \max \quad & cx + 1/2c'Qx \\ & Ax \leq b \\ & x \geq 0 \end{aligned}$$

- Give and explain the Kuhn Tucker conditions.
- Tell when the problem will lead to a guarantee that the solution satisfying the Kuhn Tucker conditions will be optimal.

5. Given the linear programming problem:

$$\begin{aligned} \max \quad & cX && - gZ & - rQ \\ & X - dY && + fZ & - Q \leq 0 \\ & bY && + hZ & \leq k \\ & X, Y, Z, Q && \geq 0 \end{aligned}$$

where X, Y, and Z are vectors.

- What is the nature of the demand and supply curves in the model for X, Q and the resources in the second constraint set.
  - Modify the model so it includes linear downward sloping demand curves for X, as well as upward sloping supply curves for Q and the resources in the second constraint.
  - Explain the consequences of the integrability assumptions as they affect the exogenous demand curve for X and supply curves that can be specified for Q in your answer to part b.
6. Suppose you have the problem

$$\begin{aligned} \max \quad & c_1x_1 + c_2x_2 \\ & -a_1x_1 + a_2x_2 \leq 0 \\ & x_1 + x_2 \leq 100 \\ & x_1, x_2 \geq 0 \end{aligned}$$

- Structure a model under the conditions that  $c_1, c_2, a_1, a_2$  are uncertain and each have 3 possible values ( $c_{ik} \ k=1,2,3$ ) and ( $a_{ik} \ k=1, 2, 3$ ). Assume all activities are decided on now but the uncertainty is resolved later.

- b. When  $c_1$  is known but  $a_1$  can take on the values 3 or 4 with probabilities .2 and .8 and the distribution of  $c_2$  and  $a_2$  is as below if  $a_2 = 3$

event	prob.	$c_2$	$a_2$
1	.3	4	2
2	.7	3	1

but equals the following if  $a_2 = 4$

event	prob.	$c_2$	$a_2$
1	.7	7	2
2	.3	4	4

and  $x_2$  is decided on after  $a_1$  becomes known.

7. State the assumptions of linear programming, 2 ways to relax each of them (if there are that many) and qualifications you might place on whether the assumptions are truly relaxed by the ways you identify.