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IP is a powerful technique for the formulation of a wide variety of problems. This section presents a number of common formulations.

16.1 Knapsack - Capital Budgeting Problem

The knapsack problem, also known as the capital budgeting or cargo loading problem, is a famous IP formulation. The knapsack context refers to a hiker selecting the most valuable items to carry, subject to a weight or capacity limit. Partial items are not allowed, thus choices are depicted by zero-one variables. The capital budgeting context involves selection of the most valuable investments from a set of available, but indivisible, investments subject to limited capital availability. The cargo loading context involves maximization of cargo value subject to hold capacity and indivisibility restrictions.

The general problem formulation assuming only one of each item is available is

$$\begin{array}{lrcl} \text{Max} & \sum_j v_j X_j & & \\ \text{s.t.} & \sum_j d_j X_j & \leq & W \\ & X_j & = & 0 \text{ or } 1 \quad \text{for all } j \end{array}$$

The decision variables indicate whether the j^{th} alternative item is chosen (X_j=1) or not (X_j=0). Each item is worth v_j. The objective function gives the total value of all items chosen. The capacity used by each X_j is d_j. The constraint requires total capacity use to be less than or equal to the capacity limit (W).

16.1.1 Example

Suppose an individual is preparing to move. Assume a truck is available that can hold at most 250 cubic feet of items. Suppose there are 10 items which can be taken and that their names, volumes and values are as shown in Table 16.1. The resultant formulation is

$$\begin{array}{llllllllllll} \text{Max} & 17x_1 & + 5x_2 & + 22x_3 & + 12x_4 & + 25x_5 & + x_6 & + 15x_7 & + 21x_8 & + 5x_9 & + 20x_{10} & \\ \text{s.t.} & 70x_1 & + 10x_2 & + 20x_3 & + 20x_4 & + 15x_5 & + 5x_6 & + 120x_7 & + 5x_8 & + 20x_9 & + 20x_{10} & \leq 250 \\ & & x_j = 0 \text{ or } 1, \text{ for all } j & & & & & & & & & \end{array}$$

The GAMS formulation is called KNAPSACK. The optimal objective function value equals 128. The values of the variables and their respective reduced costs are shown in Table 16.2. This solution indicates that all items except furniture, X_7, should be taken.

There are a couple of peculiarities in this solution which should be noted. First, the constraint has 65 units in slack (250-185) and no shadow price. However, for practical purposes the constraint does have a shadow price as the X_7 variable would come into the solution if there were 120 more units of capacity, but slack is only 65. Further, note that each of the variables has a non-zero reduced cost. This is because this particular problem was solved with the GAMS version of OSL, a branch and bound type algorithm and each of these variables was bounded at

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one. Thus, they have reduced costs reflecting bounds requiring the variables to equal either zero or one. These data are misleading as indicated in the discussion in the previous chapter on IP shadow prices.

16.1.2 Comments

The knapsack problem has been the subject of considerable theoretical interest and several applications (see von Randow; Salkin, 1975a). Armstrong, Sinha, and Zoltners provide a recent application. The capital budgeting problem context has been extensively studied (Weingartner 1963, 1966, von Randow). Variants include the cutting stock problem, where one explores the best way to cut up items such as logs, sheets of veneer, and plywood, (Eisemann and Golden). Knapsack problems also commonly appear as subproblems in algorithmic approaches to problems as shown by Williams (1978a) and Geoffrion and McBride.

The knapsack formulation contains a number of simplifying assumptions. First, the formulation permits no more than one unit of any item. This assumption could be relaxed by changing from zero-one to integer variables with constraints on item availability. Second, the value and resource usage of the items are assumed independent of the mix of items chosen. However, there may be interactions where the value of the one item is increased or decreased when certain other items are also chosen. Thus, one might need to include formulation features involving multiplication of zero-one variables. Third, capacity available is assumed independent of the value of the resource. One could relax this assumption and put in a supply curve representation.

16.2 Warehouse Location

Warehouse location problems are commonly formulated as integer programs. They involve location of warehouses within a transportation system so as to minimize overall costs. The basic decision involves tradeoffs between fixed warehouse construction costs and transportation costs. In agriculture, this formulation has been used in the location of high volume grain handling facilities (Hilger, McCarl and Uhrig) and agricultural processing facilities (Fuller, Randolph and Klingman; Faminow and Sarhan). The plant, store and distribution center location problems are closely related (von Randow). A general warehouse location problem formulation is as follows:

$$
\begin{array}{llccccccccll}
\text{Min} & \sum_k F_k V_k & + & \sum_i \sum_k C_{ik} X_{ik} & + & \sum_k \sum_j D_{kj} Y_{kj} & + & \sum_i \sum_j E_{ij} Z_{ij} & & & \\
\text{s.t.} & & & \sum_k X_{ik} & + & & & \sum_j Z_{ij} & \leq & S_i & \text{for all } i \\
 & & & & & \sum_k Y_{kj} & + & \sum_i Z_{ij} & \geq & D_j & \text{for all } j \\
 & & & -\sum_i X_{ik} & + & \sum_j Y_{kj} & & & \leq & 0 & \text{for all } k \\
 & -CAP_k V_k & & & + & \sum_j Y_{kj} & & & \leq & 0 & \text{for all } k \\
 & \sum_k A_{mk} V_k & & & & & & & \leq & b_m & \text{for all } m \\
 & V_k = 0 \text{ or } 1, & & X_{ik}, & & Y_{kj}, & & Z_{ij} & \geq & 0 & \text{for all } i,\ j,\ k
\end{array}
$$

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This is an extension of the basic transportation problem containing intermediate shipments (transshipments) into warehouses from supply points (X_{ik}) and from warehouses to demand points (Y_{kj}). The formulation also contains fixed cost and new warehouse capacity considerations. The variables in the formulation are:

V_k - a zero-one indicator variable indicating whether the k^{th} warehouse is constructed;

X_{ik} a continuous variable indicating the quantity shipped from supply point i to warehouse k;

Y_{kj} - a continuous variable indicating the quantity shipped from warehouse k to demand point j;

Z_{ij} - a continuous variable indicating the quantity shipped from supply point i directly to demand point j.

The problem is also characterized by a number of parameters.

F_k - the fixed cost associated with construction of the k^{th} warehouse. This cost should be developed so that it represents the cost incurred during the period of time represented by the supply and demand constraints;

CAP_k - the capacity of the k^{th} warehouse during the time frame leading to the supply and demand quantities;

A_{mk} - the amount of the m^{th} configuration constraint used when constructing the k^{th} warehouse;

C_{ik} - the cost of shipping from supply point i to warehouse k;

D_{kj} - the cost of shipping from warehouse k to demand point j;

E_{ij} - the cost of shipping from supply point i to demand point j;

D_j - the amount of demand which must be filled at the j^{th} demand point in the time period modeled;

S_i - the amount of supply available at i^{th} supply point in the time period modeled;

b_m - the upper limit on the m^{th} configuration constraint.

The objective function depicts total cost minimization where total cost includes warehouse construction plus shipping costs for shipments a) to warehouses, b) from warehouses, and c) directly to final demand points. The first constraint equation balances outgoing shipments with available supply for a supply point. The second constraint gives the demand requirements by demand location and requires a minimum level of incoming shipments from warehouses and supply locations. The third constraint requires outgoing shipments at a warehouse location not to exceed incoming shipments to that warehouse. The next constraints both involve our zero-one warehouse variables imposing prospective warehouse capacity using the modeling approach in the fixed cost discussion in chapter 15. Outgoing shipments are balanced with constructed warehouse capacity. When the warehouse is not constructed then outgoing shipments must equal

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zero. Thus, warehouses can only be used when constructed. The last constraint limits warehouse construction through configuration constraints. Many different types of constraints could be included here, dependent on the problem setting. An example is given below.

16.2.1 Example

Suppose a firm can construct a warehouse at one of three sites (A,B,C). Currently, the firm has two supply points and ships to two demand points with annual demand requirements and supply capacity given in Table 16.3. Further suppose that the potential warehouses have annual capacity and fixed cost as shown in Table 16.4. If warehouse B were constructed its annual capacity would be 60, it would cost $720 for the 12 year life or, assuming straight line depreciation, $60 per year. Suppose that the firm has developed a transport cost matrix as shown in Table 16.5. Finally suppose only one warehouse can be built.

This leads to the formulation shown in Table 16.6. The objective function minimizes the annual fixed cost of warehouses plus the annual variable cost of shipping. The constraints impose maximum supply constraints at two locations, minimum demand constraints at two locations, supply/demand balances at three warehouses, balances between capacity and warehouse use at three warehouses, and a constraint that requires only one of the three warehouses be constructed (i.e., a configuration constraint). Warehouse 1 capacity is set to 9999 which effectively makes its capacity unlimited if it is constructed. The GAMS formulation is called WAREHOUS.

In the solution to this model, the objective function value equals 623, and the variable and equation solutions are shown in Table 16.7. This solution corresponds to the company constructing warehouse C. The shipment pattern involves shipping 70 units from supply point 2 to warehouse C, 20 units from warehouse C to demand point 1, and 50 units from C to demand point 2. In addition, 5 units are shipped directly from supply point 2 to demand point 1 while 50 units are shipped from supply point 1 to demand point 1. The shadow prices reflect demand at point 1 costing 7 units on the margin and a cost of 5 units at demand point 2. Additional supply is worth $3 a unit at the first supply point and $0 a unit at the second supply point.

16.2.2 Comments

This formulation is simplified. One could have a number of complications such as cost-volume relationships, or multiple warehouse alternatives at a site. Those interested in related work and extensions should see the papers by Geoffrion (1975); Francis and Goldstein; Francis, McGinnis, and White; McGinnis; Fuller, Randolph, and Klingman; Hilger, McCarl, and Uhrig; or Geoffrion and Graves.

16.3 Traveling Salesman Problem

Another common IP formulation is the "Traveling Salesman Problem" (Burkard; Bellmore and Nemhauser). This problem involves developing a minimum cost route for a salesman visiting N cities then returning home. The basic problem involves selection of a route visiting all cities which minimizes the total travel cost. The machine shop scheduling may also be formulated as a travelling salesman problem (Pickard and Queyranne).

The basic problem formulation is much like the assignment problem and is:

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$$\text{Min} \quad \sum_{i}\sum_{\substack{j \\ i \neq j}} d_{ij} X_{ij}$$

$$\text{s.t.} \quad \sum_{\substack{j \\ i \neq j}} X_{ij} = \quad 1 \qquad \text{for all } i$$

$$\sum_{\substack{j \\ i \neq j}} X_{ij} = \quad 1 \qquad \text{for all } j$$

$$X_{ij} = \quad 0 \text{ or } 1 \qquad \text{for all } i \text{ and } j \text{ where } i \neq j$$

The decision variable (X_{ij}) equals one if the salesman goes from city i to city j, and zero otherwise. The possibility of moving from any city to itself is precluded. There is a known cost of moving from city i to city j (d_{ij}). The objective function gives the total cost of completing the route which will be minimized. The first constraint states that the salesman must leave each city once. The second constraint states that the salesman must enter each city once. All decision variables are restricted to equal either zero or one.

The above formulation is that of the classical assignment problem (Wagner); however, it is not yet a complete traveling salesman formulation. There is a difficulty that often arises, known as a subtour. Consider a 5-city problem in which the optimum solution consists of X_{12}=1, X_{23}=1, X_{31}=1, X_{45}=1 and X_{54}=1. This solution is feasible in the above formulation and could be minimum distance. However, it reflects a disjointed trip in which one salesman goes from city 1 to city 2 to city 3 and back to city 1 without visiting cities 4 and 5, while another salesman goes from city 4 to city 5 and back to city 4. This solution exhibits so-called subtours, disjoint loops of a size less than the number of cities. Such subtours can be of any size involving two, three, four, or any number of cities up to the number in the problem minus two, although empirical evidence (cited in Garfinkel and Nemhauser; Bellmore and Nemhauser) indicates that subtours of more than four or five cities do not appear in practice. The prohibition of subtours requires additional constraints. The subtours could be eliminated by the imposition of the following constraints:

Two City

$$X_{ij} + X_{ji} \leq 1 \qquad \text{for all } i \text{ and } j \qquad \text{where } i \neq j$$

Three City

$$X_{ij} + X_{jk} + X_{ki} \leq 2 \qquad \text{for all } i, j, \text{ and } k \qquad \text{where } i \neq j \neq k$$

Four City

$$X_{ij} + X_{jk} + X_{kL} + X_{Li} \leq 3 \qquad \text{for all } i, j, k, \text{ and } L \qquad \text{where } i \neq j \neq k \neq L$$

The first set of constraints renders all two-city subtours infeasible enforcing mutual exclusivity between the variables representing travel from city i to city j and travel from city j to city i. The next constraint set precludes three city subtours prohibiting travel from i to j then on to k, finally from k back to i. Here only two of the three activities are allowed in the solution. Similarly, the four-city subtour constraints prevent one from traveling from city i to city j, then j to k, and on from k to L, and from L back to i.

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In a practical problem this way of dealing with subtours would produce a very large constraint set. For example, with 30 cities there would be 870 constraints for the prevention of the two city subtours alone. In general, constraints would be required precluding subtours from size 2 up through the greatest integer number not exceeding half the number of cities. Other formulations exist which preclude subtours in a more compact fashion. Miller, Tucker, and Zemlin show that the following constraints eliminate subtours in an N city problem,

$$U_i - U_j + NX_{ij} \leq N - 1 \quad i = 2, \ldots, N; \quad j = 2 \ \ldots, N; \quad i \neq j$$
$$U_i \geq 0 \quad i = 2, \ldots, N$$

where new continuous variables (U) are introduced. Dantzig, Fulkerson, and Johnson (1954) give yet another method.

16.3.1 Example

Consider a salesman that has to visit six cities. Suppose these cities are separated by the distances in Table 16.8 and the salesman wants to minimize total distance traveled. The example formulation appears in Table 16.9. The objective function minimizes the sum of the distance times zero-one variables indicating whether the salesman travels between cities i and j, X_{ij}. The first six constraints require that each city be left and the next six constraints require that each city be visited. Subtours are prevented by the last 20 constraints following Miller, Tucker, and Zemlin (containing the 6s in the matrix and 5s on the right-hand sides). The GAMS formulation is called TRAVEL. The solution to this problem is shown in Table 16.10.

This solution reflects the traveling salesman traveling 46 miles going from city 1 to city 2, to city 3, to city 6, to city 5, to city 4 and back to city 1, completing a loop. Subtours are not present.

16.3.2 Comments

This problem has been extensively studied (see reviews by Bellmore and Nemhauser; Golden and Assad; Laporte and Lawler et. al.). Unfortunately, solving this problem is very difficult because of the number of possible feasible solutions (e.g., in the six-city problem there are five factorial possible solutions). Several heuristics have been developed for this problem. It is not recommended that it be directly solved with an IP algorithm, rather heuristics are usually used. A variant of this problem involves scheduling problems (Eilon).

16.4 Decreasing Costs

Models may need to depict situations where volume increases lead to either marginal cost decreases or marginal revenue increases. For example such situations would occur when: a) the purchase of transportation services involves volume discounts, or b) production exhibits positive economies of scale when cost drops as more units are produced. LP cannot satisfactorily model these situations. A separable LP formulation would use the cheapest cost activity first ignoring the volume requirements necessary to incur such a cost (i.e., using the activity with lowest transportation cost at less than the required volume rather than using more expensive transport rate relevant at that lower volume). Thus, another modeling approach is required. One could use the nonlinear form of separable programming, but this would yield local optimal solutions. Alternatively, a mixed IP formulation can be used. This will be explained herein.

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The basic problem in matrix form is

$$
\begin{array}{llllllll}
\text{Max} & eY & & & - & f(Z) & & \\
\text{s.t.} & Y & - & \sum_m G_m X_m & & & \le & 0 \\
 & & & \sum_m A_m X_m & - & Z & \le & 0 \\
 & & & \sum_m H_{im} X_m & & & \le & b_i \quad \text{for all } i \\
 & Y, & & X_m, & & Z & \ge & 0
\end{array}
$$

where Z is the quantity of input used, f(Z) is the total cost of acquiring the input which exhibits diminishing marginal cost (i.e., the per unit cost of Z falls as more is purchased); e is the sale price for a unit of output (Y); G_m is the quantity of output produced per unit of production activity X_m; A_m is the amount of the resource which is used per unit of X_m; and H_{im} is the number of units of the i^{th} fixed resource which is used per unit of X_m.

In this problem the objective function maximizes total revenue from product sale (eY) less total costs (f(Z)). The first constraint balances products sold (Y) with production ($\sum G_mX_m$). The second constraint balances input usage ($\sum A_mX_m$) with supply (Z). The third constraint balances resource usage by production ($\sum H_{im}X_m$) with exogenous supply (b_i). This problem may be reformulated as an IP problem by following an approximation point approach.

$$
\begin{array}{llllllllll}
\text{Max} & eY & & & - & \sum_k f'(Z_k^*) & R_k & & & \\
\text{s.t.} & Y & - & \sum_m G_m X_m & & & & & \le & 0 \\
 & & & \sum_m A_m X_m & - & \sum_k & R_k & & \le & 0 \\
 & & & \sum_m H_{im} X_m & & & & & \le & b_i \quad \text{for all } i \\
 & & & & & & R_k & - (Z_k^* - Z_{k-1}^*)D_k & \le & 0 \quad \text{for all } k \\
 & & & & & - & R_k & + (Z_k^* - Z_{k-1}^*)D_{k+1} & \le & 0 \quad \text{for all } k \text{ but last} \\
 & Y, & & X_m, & & & R_k & & \ge & 0 \quad \text{for all } m \text{ and } k \\
 & & & & & & & D_k = 0 \text{ or } 1 & & \quad \text{for all } k
\end{array}
$$

The variables are Y and X_m, as above, but the Z variable has been replaced with two sets of variables: R_k and D_k. The variables R_k which are the number of units purchased at cost $f'(Z_k*)$; Z_k* are a set of approximation points for Z where $Z_0* = 0$; where $f'(Z_k*)$ is the first derivative of the f(Z) function evaluated at the approximation point Z_k^*. While simultaneously the data for D_k is a zero-one indicator variable indicating whether the k^{th} step has been fully used.

The formulation insures that the proper total cost is incurred, and that the decreasing per unit costs are only used when the proper quantities are purchased. The last two constraints enforce this restriction, requiring R_k to equal Z_k - Z_{k-1} before R_{k+1} can be non-zero (i.e., the k^{th} increment must be paid for before the $k+1^{st}$ increment can be purchased). The first three equations are as defined above. Notice that the k^{th} step variable can be no larger than D_k times the difference between Z_k and Z_{k-1}. Thus, R_k is prevented from being non-zero unless the indicator variable D_k

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is also non-zero. However, the last constraint imposes a relationship between the k^{th} step variable and the indicator variable for step k+1. Consequently, R_k must equal its maximum value (Z_{k+1} - Z_k) if the k+1st indicator is non-zero. Similarly, R_1 through R_{k-1} must equal their upper limits in order that R_k can be non-zero. Consequently, this only permits input purchases at the lower cost exhibited under the higher volumes, only if inputs have been purchased at all volumes previous to those.

16.4.1 Example

Consider a problem in which total cost of the input Z and the production relationships are given by

$$
\begin{array}{lllllllll}
\text{Max} & 4Y & & & - & (3-.125Z) & Z & & \\
 & Y & - & 2X & & & & \le & 0 \\
 & & & X & - & & Z & \le & 0 \\
 & & & X & & & & \le & 5 \\
 & Y, & & X, & & & Z & \ge & 0
\end{array}
$$

Suppose we approximate Z at 2, 4, 6, 8 and 10. The formulation becomes

$$
\begin{array}{lllllllllllllll}
\text{Max} & 4Y & & & - & 2.50R_1 & - & 2.00R_2 & - & 1.50R_3 & - & 1.00R_4 & - & 0.50R_5 & & & \\
 & Y & - & 2X & & & & & & & & & & & & \le & 0 \\
 & & & X & - & R_1 & - & R_2 & - & R_3 & - & R_4 & - & R_5 & & \le & 0 \\
 & & & X & & & & & & & & & & & & \le & 5 \\
 & & & & & R_1 & & & & & & & & & -2D_1 & \le & 0 \\
 & & & & & & & R_2 & & & & & & & -2D_2 & \le & 0 \\
 & & & & & & & & & R_3 & & & & & -2D_3 & \le & 0 \\
 & & & & & & & & & & & R_4 & & & -2D_4 & \le & 0 \\
 & & & & & & & & & & & & & R_5 & -2D_5 & \le & 0 \\
 & & & & - & R_1 & & & & & & & & & +2D_2 & \le & 0 \\
 & & & & & & - & R_2 & & & & & & & +2D_3 & \le & 0 \\
 & & & & & & & & - & R_3 & & & & & +2D_4 & \le & 0 \\
 & & & & & & & & & & - & R_4 & & & +2D_5 & \le & 0
\end{array}
$$

where the variables D_1 through D_5 are zero-one indicator variables and the variables X, Y, and R are continuous. Note that before R_2 can be nonzero, the variable D_2 must be nonzero because of the equation relating R_2 and D_2. However, if D_2 is nonzero, R_1 must be in the solution equaling 2, because of the equation relating R_1 and D_2. The other constraints also require that D_1 be one. Consequently, in order to purchase inputs at the second cost step, the first cost step must be fully utilized. In general for R_n to be non-zero then r_1 through r_{n-1} must be in solution at their upper limits. Thus, one must use the higher cost (lower revenue) activities before the lower cost (higher revenue) activities can be undertaken. The GAMS formulation is called DECOST. The

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solution to this problem is given in Table 16.11 and shows that Y = 10, X = 5, and Z = 5 based on the r values (R_1=R_2=2 and R_3=1). Note that the first three indicator variables are in the basis at 1, and that the last two are in at zero. Thus, the values of the variables R_1 and R_2 must equal their upper limit, and R_3 is between zero and its upper limit. In this case, it is equal to 1 because of the constraint $X \geq 5$.

16.4.2 Comments

This problem depicts minimization of a non-convex phenomena. However, a global optimum solution will be found because of the enumerative nature of IP algorithms. The objective function approximates total revenue minus total cost by accumulating the total cost approximation as the sums of derivatives at the approximating points times the associated quantities.

16.5 Machinery Selection

IP is often used to formulate investment problems (Weingartner [1963, 1966]). The machinery selection problem is a common investment problem. In this problem one maximizes profits, trading off the annual costs of machinery purchase with the extra profits obtained by having that machinery. A general formulation of this problem is

$$
\begin{array}{llllllll}
\text{Max} & - & \sum_k F_k Y_k & + & \sum_j \sum_m C_{jm} X_{jm} & & & \\
\text{s.t.} & - & Cap_{ik} Y_k & + & \sum_j \sum_m A_{ijkm} X_{jm} & \leq & 0 & \text{for all } i \text{ and } k \\
 & & & & \sum_j \sum_m D_{njm} X_{jm} & \leq & b_n & \text{for all } n \\
 & & \sum_k G_{rk} Y_k & & & \leq & e_r & \text{for all } r \\
 & & Y_k \text{ is a} & & \text{nonnegative integer,} \quad X_{jm} & \geq & 0 & \text{for all } j, k, \text{and } m
\end{array}
$$

The decision variables are Y_k, the integer number of units of the k^{th} type machinery purchased; X_{jm}, the quantity of the j^{th} activity produced using the m^{th} machinery alternative. The parameters of the model are: F_k, the annualized fixed cost of the k^{th} machinery type; Cap_{ik}, the annual capacity of the k^{th} machinery type to supply the i^{th} resource; G_{rk}, the usage of the r^{th} machinery restriction when purchasing the k^{th} machinery type; C_{jm}, the per unit net profit of X_{jm}; A_{ijkm}, the per unit use by X_{jm} of the i^{th} capacity resource supplied by purchasing machine k; D_{njm}, the per unit usage of fixed resources of the n^{th} type by X_{jm}; b_n, the endowment of the n^{th} resource in the year being modeled; and e_r, the endowment of the r^{th} machinery restriction.

The objective function maximizes profits from machinery operation less the fixed costs of acquisition. The first constraint balances the capacity of the machinery purchased with the use of that capacity. These constraints preclude machinery from being used unless it is purchased. The second constraint imposes constraints on resources other than machinery. The third constraint imposes configuration constraints on machinery purchases.

16.5.1 Example

Assume that a farm is considering the purchase of equipment involving a choice of two tractors,

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two plows, two discs, two planters and two harvesting units. The working rates and costs are given in Table 16.12. Time available by period is given in Table 16.13. The farm has 600 acres. Machinery resource calculations are shown in Table 16.14. Yields, prices, and costs are given in Table 16.15.

Three operations are done on the farm: plowing, simultaneous discing and planting, and harvesting; plowing is done in time periods 1-2; disc-planting in period 2 and harvesting in period 3. In addition, when buying the equipment, one must match the disc and the planter; disc number one an be purchased only with planter number one and disc number two only with planter number two. The formulation is given in Table 16.16 and in file MACHSEL. The solution to this IP problem yields an IP objective function of 116,100 when it is solved as an LP its objective function equals 124,301. The values of the solution variables are given in Table 16.17.

16.5.2 Comments

This formulation has been used in agricultural economics. For example see the machinery selection work by Danok, McCarl, and White (1978, 1980); Clayton and McCarl; or Baker, Dixit, and McCarl.

16.6 Other Formulations

While several classes of formulations were addressed above, there are numerous other formulations which could have been included. Here we mention networks, dynamic programming, scheduling, and combinational problems.

The vast majority of network problems are integer by nature. Many of them yield integer solutions because of the structure of the basis (Wagner, 1969). These types of problems are the assignment, transportation, transshipment, shortest path, maximal flow, and minimum spanning tree. A general presentation can be seen in Kennington and Helgeson; Bazaraa, Jarvis and Sherali; or Jensen and Barnes.

A second related class of problems are dynamic programming problems. Many dynamic programming algorithms involve integer valued variables. Many common IP problems have been cast as dynamic programming problems; e.g., Nemhauser mentions network, traveling salesmen and scheduling problems as places where dynamic programming has been applied.

There is also a large class of integer scheduling problems. One such problem is the vehicle scheduling problem where buses, aircraft, or ships are routed to places where items need to be delivered. Wagner (1969), and Markowitz and Manne give early developments and references to solve this class of problems. While Assad and Golden give more recent references there have been a vast number of machine scheduling applications involving assembly line balancing, flow shop scheduling, batch sizing, etc. Eilon reviews this topic, and von Randow gives 13 pages of references. Project scheduling problems have also been formulated (Davis, Patterson).

Another class of integer problems are the combinational problems, most of which can be formulated as IP problems. These include network type problems such as maximum flow problems, set covering, matching problems, weighted matching problems, spanning trees, and

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traveling salesmen problems.   Many of these problems are classed as very difficult to solve. The book by Papadimitriou and Steiglitz gives background and formulations.

Finally, we should mention that new applications of IP are developed virtually every day. For example, von Randow, in a bibliography of studies between 1978 and 1981, gives 130 pages of citations to IP relating mainly to that time period. Thus, there are many classes of problems that we have not covered above.

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Table 16.1. Items for the Knapsack Example Problem

Variable	Item Name	Item Volume (Cubic feet)	Item Value ($)
X_1	Bed and mattress	70	17
X_2	TV set	10	5
X_3	Turntable and records	20	22
X_4	Armchairs	20	12
X_5	Air conditioner	15	25
X_6	Garden tools and fencing	5	1
X_7	Furniture	120	15
X_8	Books	5	21
X_9	Cooking utensils	20	5
X_{10}	Appliances	20	20

Table 16.2. Solution to the Knapsack Example Problem

Obj = 128

Variable	Value	Reduced Cost
X_1	1	17
X_2	1	5
X_3	1	22
X_4	1	12
X_5	1	25
X_6	1	1
X_7	0	15
X_8	1	21
X_9	1	5
X_{10}	1	20

Constraint	Activity	Shadow Price
Space	185	0

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Table 16.3. Supply/Demand Information for Warehouse Location Example

Total Supply		Total Demand	
Point	Units	Point	Units
1	50	1	75
2	75	2	50

Table 16.4. Warehouse Capacities and Costs for the Warehouse Location Example

Warehouse	Annual Capacity	Fixed Cost/Life ($)	1 -Year Cost
A	Unlimited	500/10 years	$50
B	60	720/12 years	$60
C	70	680/10 years	$68

Table 16.5. Transport Costs (in $/unit) for Warehouse Location Example

		Shipping Point Supply		Warehouse		
		1	2	A	B	C
Warehouse	A	1	6	-	-	
	B	2	3	-	-	-
	C	8	1	-	-	-
Demand	1	4	7	4	3	5
	2	8	6	6	4	3

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Table 16.6. Formulation of the Warehouse Location Example Problem

V_A	V_B	V_C	X_{1A}	X_{1B}	X_{1C}	X_{2A}	X_{2B}	X_{2C}	Y_{A1}	Y_{A2}	Y_{B1}	Y_{B2}	Y_{C1}	Y_{C2}	Z_{11}	Z_{12}	Z_{21}	Z_{22}	RHS	
50	60	68	1	2	8	6	3	1	4	6	3	4	5	3	4	8	7	6	Min	
									1		1		1		1		1		≤	75
										1		1		1		1		1	≤	50
			1	1	1										1	1			≤	50
						1	1	1									1	1	≤	75
			-1			-1			1	1									≤	0
				-1			-1				1	1							≤	0
					-1			-1					1	1					≤	0
-9999									1	1									≤	0
	-60										1	1							≤	0
		-70											1	1					≤	0
1	1	1																	≤	1
		V_A, V_B, V_C			≤	(0,1)		x_{ik},							Y_{kj},		Z_{ij}		≤	0 for all i, j, k

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Table 16.7. Solution Results for the Warehouse Location Example

Obj = 623

Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
V_A	0	0	1	0	-3.00
V_B	0	2	2	0	0
V_C	1	0	3	0	7.00
X_{1A}	0	0	4	0	5.00
X_{1B}	0	2.00	5	0	-4
X_{1C}	0	10.00	6	0	-3.00
X_{2A}	0	2	7	0	-1.00
X_{2B}	0	0	8	0	-0.05
X_{2C}	70	0	9	0	-1.00
Y_{A1}	0	1.052	10	0	-1.00
Y_{A2}	0	5.052	11	0	-2
Y_{B1}	0	0			
Y_{B2}	0	3.00			
Y_{C1}	20	0			
Y_{C2}	50	0			
Z_{11}	50	0			
Z_{12}	0	6.00			
Z_{21}	5	0			
Z_{22}	0	1.00			

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Table 16.8. Distances Between Cities for the Travelling Salesman Problem

	1	2	3	4	5	6
1	--	11	7	6	8	14
2	11	--	7	9	12	13
3	7	7	--	3	7	8
4	6	9	3	--	4	8
5	8	12	7	4	--	10
6	14	13	8	8	10	--

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Table 16.9. Formulation of the Traveling Salesman Problem

	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X						
	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	6	U	U	U	U	U	
	2	3	4	5	6	1	3	4	5	6	1	2	4	5	6	1	2	3	5	6	1	2	3	4	6	1	2	3	4	5	2	3	4	5	6	
Min	11	7	6	8	14	11	7	9	12	13	7	7	3	7	8	6	9	3	4	8	8	12	7	4	10	14	13	8	8	10						
	1	1	1	1	1																															= 1
						1	1	1	1	1																										= 1
											1	1	1	1	1																					= 1
																1	1	1	1	1																= 1
																					1	1	1	1	1											= 1
																										1	1	1	1	1						= 1
						1					1					1					1					1										= 1
	1											1					1					1					1									= 1
		1					1											1					1					1								= 1
			1					1					1											1					1							= 1
				1					1					1					1											1						= 1
					1					1					1					1					1											= 1
							6																								1	-1				≤ 5
								6																							1		-1			≤ 5
									6																						1			-1		< 5
										6																					1				-1	< 5
												6																			-1	1				< 5
													6																			1	-1			< 5
														6																		1		-1		< 5
															6																	1			-1	< 5
																	6														-1		1			< 5
																		6														-1	1			< 5
																			6														1	-1		< 5
																				6													1		-1	< 5
																						6									-1			1		< 5
																							6									-1		1		< 5
																								6									-1	1		< 5
																									6									1	-1	< 5
																											6				-1				1	< 5
																												6				-1			1	< 5
																													6				-1		1	< 5
																														6				-1	1	< 5

X_{ij} $U_i > 0$

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Table 16.10. Solution to the Travelling Salesman Example

Obj = 46

Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
X_{12}	1.00	11	Leave 1	0	0
X_{13}	0	7	Leave 2	0	0
X_{14}	0	6	Leave 3	0	0
X_{15}	0	8	Leave 4	0	0
X_{16}	0	14	Leave 5	0	0
X_{21}	0	11	Leave 6	0	0
X_{23}	1.00	7	Enter 1	0	0
X_{24}	0	9	Enter 2	0	0
X_{25}	0	12	Enter 3	0	0
X_{26}	0	13	Enter 4	0	0
X_{31}	0	7	Enter 5	0	0
X_{32}	0	7	Enter 6	0	0
X_{34}	0	3	Subtour 23	4	0
X_{35}	0	7	Subtour 24	8	0
X_{36}	1.00	8	Subtour 25	7	0
X_{41}	1.00	6	Subtour 26	0	0
X_{42}	0	9	Subtour 32	0	0
X_{43}	0	3	Subtour 34	9	0
X_{45}	0	4	Subtour 35	8	0
X_{46}	0	8	Subtour 36	7	0
X_{51}	0	8	Subtour 42	2	0
X_{52}	0	12	Subtour 43	1	0
X_{53}	0	7	Subtour 45	4	0
X_{54}	1.00	4	Subtour 46	3	0
X_{56}	0	10	Subtour 52	3	0
X_{61}	0	14	Subtour 53	2	0
X_{62}	0	13	Subtour 54	0	0
X_{63}	0	8	Subtour 56	4	0
X_{64}	0	8	Subtour 62	4	0
X_{65}	1.00	10	Subtour 63	3	0
U_3	1	0	Subtour 65	0	0
U_4	4	0	Subtour 64	7	0
U_5	3	0			
U_2	0	0			

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Table 16.11. Solution to the Decreasing Costs Example

Objective function = 29.50

Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
Y	10	0	Y balance	0	4.0
X	5	6.5	Z balance	0	1.5
R_1	2	0	R_1D_1	0	0
R_2	2	0	R_2D_2	0	0
R_3	1	0	R_3D_3	1	0
R_4	0	0	R_4D_4	0	0.5
R_5	0	0	R_5D_5	0	1.0
D_1	1	0	R_1D_2	0	1.0
D_2	1	-2	R_2D_3	0	0.5
D_3	1	-1	R_3D_4	1	0
D_4	0	1	R_4D_5	0	0
D_5	0	2			

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Table 16.12. Data for the Machinery Selection Problem

Equipment	Annualized Fixed Cost	Cost/Hour of Operation	Hrs. of Labor Used/Hr. of Operation	Acres Treated/Hour
Tractor 1	5,000	10.00	1.00	-
Tractor 2	9,000	10.00	1.00	-
Plow 1	1,000	2.00	0.20	5*
Plow 2	1,200	2.00	0.20	10*
Disc 1	1,000	1.20	0.10	10*
Disc 2	1,200	1.20	0.10	12**
Planter 1	2 ,000	3.40	0.10	--**
Planter 2	2,100	3.40	0.22	--**
Harvester 1	1,000	23.0	1.00	3***
Harvester 2	12,000	28.0	1.00	4***

* Requires a tractor. Working rates are given for tractor 1; tractor 2 is twice as fast.

** Has the same working rate as that of the disc that the planter is used with.

*** Uses one hour of tractor time/hour of harvesting.

Table 16.13. Hours Available for the Machinery Selection Problem

Time Period	Hours of Labor	Hours for Machinery
1	200	160
2	210	180
3	250	200

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Table 16.14. Machinery Usage Computations

Operation	Tractor Used	Cost/Acre ($)	Hrs. of Tractor/Acre	Plow Used	Hrs. Plow Use/Acre	Planter Used	Hrs. Planter Use/Acre	Disc Used	Hrs. Disc Used/Acre	Harvester Used	Hrs. Harvester Used/Acre
Plow	1	2.40	0.2	1	0.2	--	--	--	--	--	--
Plow	1	1.20	0.1	2	0.1	--	--	--	--	--	--
Plow	2	1.20	0.1	1	0.1	--	--	--	--	--	--
Plow	2	0.60	0.05	2	0.05	--	--	--	--	--	--
Plant-disc	1	1.46	0.1	--	--	1	0.1	1	0.1	--	--
Plant-disc	1	1.22	0.0833	--	--	2	0.0833	2	0.0833	--	--
Plant-disc	2	0.73	0.05	--	--	1	0.05	1	0.05	--	--
Plant-disc	2	0.61	0.04167	--	--	2	0.04107	1	0.0417	--	--
Harvest	1	11	0.333	--	--	--	--	--	--	1	0.333
Harvest	2	11	0.333	--	--	--	--	--	--	1	0.333
Harvest	1	9.5	0.25	--	--	--	--	--	--	2	0.25
Harvest	2	9.5	0.25	--	--	--	--	--	--	2	0.25

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Table 16.15. Yields, Prices, and Costs

Non-machinery cost per acre	110
Price per unit of yield	2.5
Yield per acre	140

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Table 16.16. Formulation of the Machinery Selection Problem

		Machinery Acquisition Integer Variables										Machinery Use Continuous Variables																		
		Tractor		Plow		Planter		Disc		Harvester		Plow with Tractor 1				Plow with Tractor 2				Plant Disc 8				Harvest with				Crop Sales	Input Purchases	
												and Plow 1 in Period		and Plow 2 in Period		and Plow 1 in Period		ans Plow 2 in Period		Tractor 1 Planter		Tractor 2 Planter		Tractor 1 Harvester		Tractor 2 Harvester				
		1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2			
Objective (max)		-5000	-9000	-1000	-1200	-2000	-2100	-1000	-1200	-10000	-12000	-2.4	-2.4	-1.2	-1.2	-1.2	-1.2	-0.6	-0.6	-1.46	-1.22	-0.73	-0.61	-9.33	-8.35	-9.33	-8.25	2.5	-110	
Trac 1 Cap in Period	1	-160										2		.1																<0
	2	-180											.2		.1					.1	.0833									<0
	3	-200																						.33	.25					<0
Trac 2 Cap in Period	1		-160													.1		.05												<0
	2		-180														.1		.05			.05	.04167							<0
	3		-200																							.33	.25			<0
Plow 1 Cap in Period	1			-160								.2				.1														<0
	2			-180									.2				.1													<0
Plow 2 Cap in Period	1				-160									.1				.05												<0
	2				-180										.1				.05											<0
Cap Planter	1					-180														.1		0.05								<0
	2						-180														0.083		.0417							<0
Cap Disc	1							-180												.1		0.05								<0
	2								-180												0.083		.0417							<0
Cap Harvester	1									-200														.33		.33				<0
	2										-200														.25		.25			<0
Labor Avai Period	1											.24		.12		.12		.06												<200
	2												.24		.12		.12		.06	.12	.11	.06	.055							<210
	3																							.5	.375	.5	.375			<250
Plow-Plant Seq												-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1							<0
Plant-Harvt Seq																				-1	-1	-1	-1	1	1	1	1			<0
Land Available												1	1	1	1	1	1	1	1											<60
Mutual Exclu	Planters					1	1																							<1
	Discs							1	1																					<1
Disc-Planter	1-1					-1		1																						<0
	2-2						-1		1																					<0
Yield Balance																								-140	-140	-140	-140	1		<0
Input Balance																				1	1	1	1						-1	<0

Table 16.17. Solution for the Machinery Selection Problem

obj = 116,100

Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
Buy Plow 1	0	0	Trac 1 capacity in Period 3	50	0
Buy Trac 2	0	0	Trac 1 capacity in Period 2	130	0

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Buy Trac 1	1	-5,000	Trac 1 capacity in Period 1	100	0
Buy Plow 2	1	-1,200	Trac 2 capacity in Period 1	0	12
Buy Planter 1	0	0	Trac 2 capactiy in Period 2	0	14.6
Buy Planter 2	1	-3300	Trac 2 capacity in Period 3	0	22.26
Buy Disc 1	0	0	Plow 1 capacity in Period 1	0	6.25
Buy Disc 2	1	0	Plow 1 capacity in Period 2	0	0
Buy Harvester 1	0	0	Plow 2 capacity in Period 1	100	0
Plow with Trac 1 and Plow 1 in Period 2	0	-1.20	Planter 2 capacity	130	0
Buy Harvestor 2	1	0	Plow 2 capacity in Period 2	180	0
Plow with Trac 1 and Plow 1 in Period 1	0	-2.45	Planter 1 capacity	0	0
Plow with Trac 1 and Plow 2 in Period 1	600	0	Disc 1	0	0
Plow with Trac 1 and Plow 2 in Period 2	0	0	Disc 2	130	0
Plow with Trac 2 and Plow 1 in Period 1	0	-1.825	Harvester 1	0	50
Plow with Trac 2 and Plow 1 in Period 2	0	-1.46	Harvester 2	50	0
Plow with Trac 2 and Plow 2 in Period 1	0	0	Labor available in Period 1	128	0
Plow with Trac 2 and Plow 2 in Period 2	0	0.13	Labor available in Period 2	144	0
Plant with Trac 1 and Planter 1	0	-1.91	Labor available in Period 3	25	0
Plant with Trac 1 and Planter 2	0	0	Plow Plant	0	230.533
Plant with Trac 2 and Planter 1	0	-1.077	Plant Harvester	0	341.75
Plant with Trac 2 and Planter 2	0	0	Land	0	229.333
Harvest with Trac 1 and Harvester 1	0	-17.75	One Planter	0	0
Harvest with Trac 1 and Harvester 2	600	0	One Disc	0	0
Harvest with Trac 2 and Harvester 1	0	-25.17	Planter 1 to Disc 1	0	0
Harvest with Trac 2 and Harvester 2	0	-5.565	Planter 2 to Disc 2	0	0
Sell Crop	84,000	0	Yield Balance	0	2.5
Purchase Inputs	600	0	Input Balance	0	110

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