CHAPTER VIII: MULTI-YEAR DYNAMICS AND LINEAR PROGRAMMING

Many problems contain multiple year dynamic elements (hereafter called dynamics). This is especially true in agricultural problems covering perennial crops, livestock and/or facility investments. Dynamic concerns arise when decision makers face: a) binding financial constraints which change with time (i.e. cash flow considerations); b) a production situation in which current actions impact the productivity of future actions (e.g., crops in rotation, livestock breeding or equipment purchases); c) a need to adjust over time to exogenous stimulus (e.g., altering production as prices or resource endowments change); d) future uncertainty and/or e) an exhaustible resource base. This section presents modeling techniques for incorporating dynamic considerations into linear programs. The discussion will be limited to the modeling of multiple-period situations. Within-period (year) dynamics are covered in the sequencing example of the previous chapter. Before beginning a discussion of methods, we first present background principles.

8.1 Dynamics Background

A number of key questions are involved with the modeling of dynamic situations. Fundamentally, one must ask whether an explicit multiple time period representation is necessary. If so, a number of other questions are relevant. First, the length of the total time period and the starting date must be determined. Second, the length of the time intervals explicitly represented within the total time period must be determined. Third, initial and final inventory conditions must be specified. Fourth, one must decide on activity life; i.e., when a particular activity is begun and how long it lasts. Fifth, the rate of time preference must be determined; i.e., one needs the discount rate at which future returns are considered when compared with current returns. Sixth and finally, one must decide whether to include uncertainty. The sections below present discussion on each of these topics.

8.1.1 Should Dynamics be Explicit?

Dynamic situations may not require multi-period dynamic models. Some dynamic questions must be explicitly modeled, allowing the solution to change over time. On the other hand, other questions may be adequately depicted by a steady state equilibrium model. In an equilibrium model the same decision is assumed to be repeatedly made in all time periods and thus a "representative" single period representation is used.

Choice between these two modeling alternatives depends on a number of considerations. First, one must ask whether modeling adaptation is important. This depends upon whether the modeled entity is likely to experience growth, development/exhaustion of its resource base, and/or dynamic changes in model parameters. Second, one must be interested in the time path of adjustment and must not be content to solve a model for an optimal final state with the adjustments required to attain that state determined exogenously. Simultaneously, one must ask whether the data are present in sufficient detail to support a dynamic model. Finally, the multi-period dynamic analysis must be affordable or practical given the model size and data required.

Dynamic equilibrium models may be used when one is willing to assume: a) the resource, technology and price data are constant; and b) a long-run "steady state" solution is acceptable. Disequilibrium models are used when these assumptions do not hold. Often reliance on equilibrium models is stimulated by the absence of data on parameter values over time.

The decision on whether or not to assume equilibrium needs to be addressed carefully. Two common errors occur in the context of dynamic models: unnecessarily entering explicit dynamics into a model and improperly omitting them. Naturally, the proper dynamic assumption depends upon the problem. Treating dynamics as an equilibrium does not imply ignoring dynamics, but rather assumes repetitive decision making with equal initial and final inventory, a zero growth rate and a constant resource base.

8.1.2 How Long?

Given that a dynamic model is to be used, one must determine the length of time that the model covers. There are trade-offs between time coverage and size. Longer lengths generally add variables and constraints along with data needs.

Determination of the model time frame involves many considerations. One of the simplest, yet powerful statement comes from Modigliani and Cohen, who stated that the time horizon must be long enough so that alterations in its duration do not impact the initial period solution. However, this statement is based on the assumption that the first period variables are the main focus of interest. When more periods are of interest (e.g., first five years), an obvious restatement is that the time frame should be long enough so that its extension does not alter the variables of interest. Such a criteria, while appealing, may not be terribly practical. Model size may limit its attainment. An alternative modeling strategy is to specify a number of periods explicitly, then introduce terminal conditions for in-process inventories. Specification of terminal conditions is discussed below. Resolution of the model duration question is well beyond the scope of this effort. The questions of time preference and uncertainty are intimately related. Resolution of the question, however depends upon the problem at hand. Theoretical investigation may be found in Arrow and Kurtz; Graff; and Boussard. Also, a review of the literature may be found in Nuthall.

8.1.3 Time Interval Representation

A discrete time representation is used in all models in this book. Within such a setting one must define time sub-intervals, hereafter called periods. One may specify periods of uniform or non-uniform duration. Basically, the specification depends upon the nature of the decisions to be made and the length of the production cycle.

Annual time disaggregation is most common. However, decades, half years, quarters and months have been used depending upon the problem structure. Model size usually increases rapidly as more periods are specified. Further, it is possible to have fine breakdowns in resource availability within time periods, for example, having weekly detail on some variables (e.g., product scheduling), but coarser detail on others (i.e., machinery investment). The breakdown used must be determined by the elasticity of substitution (η) between resources at different times; i.e., when labor in two weeks is perfectly substitutable (η=1) then include these weeks in one larger period. Otherwise, multiple periods are defined.

8.1.4 Initial and Terminal Conditions

Often initial and terminal conditions are specified when formulating a multi-period dynamic model. In such a model it is important to adequately reflect the in-process inventory and previously made commitments (e.g., trees planted in the past, animals on hand, or partially depreciated investments) as initial conditions.

Most models do not contain either an infinite time horizon or conditions where all dynamic enterprises stop at the end of the horizon. Consequently, terminal conditions are important. Terminal conditions reflect the value of in-process inventory beyond the final period explicitly modeled and should either value or require a minimum level of inventory. When used, terminal values should reflect the net present value of the future income stream earned by ending the time horizon with a unit of the in-process inventory. Such conditions can insure that model activity will be reasonable up until the final year. The error created by ignoring terminal conditions can be illustrated through example. Suppose a firm uses trucks which must be replaced after 6 years. Given a model with a 7 year horizon and no terminal conditions, there would be little incentive to buy new trucks toward the end of the model time horizon, say in the 6th year. The model would simply not reflect gains from the future availability after the 7 year horizon. Thus in a seven year model, a new truck acquired in the 6th year would need a terminal condition reflecting the returns to having the truck in its 5 years of existence after the explicit model time frame. The specification of terminal values can be difficult, particularly in the case of investments which enhance production and therefore alter the income stream.

Initial and terminal conditions may be specified in several ways, as always dependent upon the problem. The most common specification of initial conditions has the initial quantities specified as an exogenous limit (i.e. that there are 40 acres of existing trees that are 20 years old). Initial conditions may also allow inventory to be purchased at a price or according to a price quantity schedule. The same basic modeling alternatives are available for depicting terminal conditions. The terminal in-process inventory may be required to equal or exceed a fixed quantity, valued at a fixed price, or valued according to a price quantity demand schedule. It is usually improper to omit terminal conditions. Terminal values sometimes can be inferred by observing, for example, the model shadow prices on intermediate inventory in early time periods. Terminal values are much more difficult to specify than initial conditions (due to the element of forecasting involved). However, these parameters are key to the meaningful modeling of multi-period dynamic situations.

8.1.5 Enterprise Life

Enterprise life refers to the number of years that an activity lasts and may be assumed to be known or unknown. If enterprise life is unknown and is to be determined in the model, then constraints and activities must be present in the model to keep track of age of the items on hand. If not, a single activity can be used. Models using alternative enterprise life assumptions are given below.

8.1.6 Time Preference

Time preference is an important dynamic modeling concern. Given that returns and costs occur over time and alternative streams must be compared, these must be placed on common footing, usually using the concepts of discounting and present value. The <u>present value</u> of a dollar earned in year t, given a constant opportunity cost of funds of r is $(1+r)^{-t}$. Thus, the current value of a flow of funds which varies from year to year (c_t) over T years is

$$PV = \sum_{t=0}^{T} (1+r)^t C_t$$

Once time preference is considered, the question of specifying the appropriate cost of funds, (also called the discount rate) arises. Many considerations enter, such as whether the rate should be in real or nominal terms, how to determine the opportunity cost of capital, how to include the cost of borrowing funds, how to consider the value of alternative investments, and how to take risk into account. It is beyond the scope of this book to cover how to determine such a rate. Rather, the reader is referred to the discussion in Bussey relative to rates in private firms or Mishan for discussion of rates in social choice situations.

8.1.7 Risk

Risk is definitely a factor in dynamic situations. Risk is almost always present as situations evolve over time. This chapter only treats certainty models and unique aspects of risk in the context of dynamics; The majority of the risk techniques which have been used are discussed in the risk programming chapter below.

8.2 Dynamic Linear Programming

The most straight forward linear programming model of a dynamic situation contains multi-year enterprises of known life with multiple years modeled. This is where our discussion of dynamic models will begin. Subsequently, models with unknown life will be discussed, followed by sections on the equilibrium, typical single period model with known and unknown life. For economy of description, we will refer to multi-period models as disequilibrium models, and typical period representations as equilibrium models.

8.2.1 Disequilibrium - Known Life

Consider a problem involving decisions on how to commit resources over a number of time periods considering items that, once begun, will require resources for a known number of periods. A formulation of this problem must consider resource availability, choice of variables and continuing resource usage during the life of multi-period enterprises. Time preference of income and initial and terminal conditions must also be incorporated. Assume all enterprises have a K year life so that an enterprise undertaken in year t lasts until t+K. A model of this situation, assuming resource use, yields and returns are independent of the year in which the activity begins is

$$\text{Max} \quad \sum_t \sum_j (1+r)^{-t} C_{jt} X_{j,t} \quad + \quad (1+r)^{-T} \sum_j \sum_{\substack{e \\ e<K_j}} F_{je} I_{je}$$

$$\begin{array}{llcccl}
\text{s.t.} & \sum_j \sum_{e<K_j} A_{ije} X_{j,t-e} & & \leq & b_{it} & \text{for all i and t} \\
& X_{j,-e} & & = & X^*_{j,-e} & \text{for all } e>0 \text{ and } e<K_j \text{ and all j} \\
& -\sum_j X_{j,T-e} & + \; I_{je} & = & 0 & \text{for all } e<K_j \text{ and all j} \\
& X_{j,t}, & I_{je} & \geq & 0 & \text{for all j, t and } e<K_j
\end{array}$$

Here t is the year identifier and begins at zero for the first year; T is the last year in the model; j identifies alternative variables; e indexes the elapsed age of an enterprise; K_j is the life of enterprise j (i.e., the number of periods after starting enterprise j that resources are used); i indexes resources; r is the discount rate; C_{jt} is the profit from initiating variable X_{jt} in year t; X_{jt} is the number of units of enterprise j initiated in year t; F_{je} is the terminal value of the jth enterprise when it is e years old after the explicit time period in the model lapses; I_{je} is the number of units of the jth enterprise which are e years old at the model completion; A_{ije} is the usage of resource i by the jth alternative when it is e years old; b_{it} is the endowment of resource i in year t; and $X^*_{j,-e}$ is the initial condition giving the amount of X_j done e years before the model begins.

The model maximizes net present value of activity from years 0 through T plus the terminal value of in-process inventory. Annual profits are converted to present value by multiplying through by the discount factor. The model assumes the profits (C_{jt}) are collapsed into the period when the X_j variable is initiated. This is done by calculating the term C_{jt} as

$$C_{jt} = \sum_e (1+r)^{-(t+e)} h_{je}$$
$$\text{where } e \leq \min\{K_j, T-t\}$$

where h_{je} is the net profit from the jth enterprise when it is e years old. The inequality for e accounts for cases where the life of the enterprise is both in process and possibly extends beyond the time horizon.

The second term of the objective function gives the terminal value of enterprises not entirely completed during the years the model explicitly covers. For example, enterprises begun in the last year explicitly modeled are valued at F_{j1}, since they will be one year old in the period following the model. The F_{je} terms depict a future stream of profits to activities lasting beyond the model horizon discounted back to year T.

The first model constraint requires resource use to be less than or equal to resource availability in that year. Annual resource use depends upon enterprises begun in the current time period as well as items that began up to K_j years in the past. Thus, the summation subscripting adds all variables that are of age 0 to K_j years old in the current year (t). Resource use is assumed to depend only on the elapsed age (e) of the activity. Each alternative (j) once committed will always exist for years t+0 to t+K_j using an a priori specified amount of resources (A_{ije}) each year. The next constraint set specifies the initial conditions: i.e., the amount of enterprises undertaken before the model was started which use resources during the time period covered by the model. The last constraint set adds up in process inventory at the end of the explicit time horizon setting the variable I_{je} equal to the amount of the ith enterprise which is e years old in the final model year and will extend into future years, thus the $e=K_j$ case is not included.

The formulation assumes equal length periods.

8.2.1.1 Example

Suppose a farmer wishes to establish a five-year crop plan. Two crops are under consideration: wheat and strawberries. Constraints restricting the choice of plans include quantities of land and water. Assume wheat has a one year life and requires one acre of land while an acre uses $60 of variable costs per acre using 1 acre-ft/acre of water and yielding 100 bushels of wheat which sell for $4.00 each. Assume strawberries have a 3 year life.

The Strawberry costs, resource usage and yields are

	Year 0	Year 1	Year 2
Cost/Acre	150	280	300
Yield/acre in tons	0	7	7
Water/acre in acre-feet	.8	4.5	4.5

The price per ton of strawberries is $140. The farm has 700 acres, 50 planted in 0 year old strawberries and 10 in 1 year old strawberries. Water available consists of 1200 acre ft. per year.

We need terminal conditions to value those items which are carried into the fifth and beyond years. Assume that the following values have been derived.

Product Terminal Value	
New strawberries (0 years old the year after the model)	$160/acre
1 year old strawberries	$110/acre

The initial tableau of this formulation appears in Table 8.1. The tableau reflects the known life assumption in that, under the strawberry activities, when the strawberries are begun, they commit resources both in the year they are begun and in later years. Thus, while one acre of land in year one is used for year zero strawberries, it is also matched by an acre in years one and two. This reflects the assumption that strawberries are always kept three years, i.e. that they have a known life. The initial conditions on the problem are reflected by the right-hand sides. Notice the year one land constraint has a resource availability of 640, which is the 700 acres of available land less the sixty acres previously planted to strawberries. The water constraint is similarly adjusted. This is also reflected in the second year where the resource endowments reflect continuation of the ten acre patch previously existing. Terminal conditions in the model are reflected on the strawberries begun in years three and four, which are not fully completed during the model time frame.

The solution is shown in Table 8.2. The GAMS file DISEQK depicts this problem. The solution reflects the disequilibrium nature of the model. Wheat acreage varies from year to year as the acres of strawberries are established. Note that no strawberries are established in the last couple of years, and therefore there are no terminal quantities of strawberries. This is due to relatively low terminal conditions on strawberries. The solution indicates that it would cost as much as $330 in year five to plant strawberries. This means that the terminal conditions would have to be at least $490 before it would be optimal in the model to do that.

8.2.2 Disequilibrium - Unknown Life

A second possibility involves modeling of situations wherein the exact life of activities is to be endogenously determined (although a maximum life is known) in a multi-period context. A formulation of this problem must include the considerations in the known life model as well as variables depicting the decision on whether to continue or terminate an activity. Adopting the same assumptions as above, a general formulation of this model is

$$
\begin{array}{llllll}
\text{Max} & \sum_t \sum_j \sum_{e \le K_j} (1+r)^{-t} C_{je} X_{j,t,e} & + (1+r)^{-T} \sum_j \sum_{e<K_j} F_{je} I_{je} & & & \\
\text{s.t.} & \sum_j \sum_{e \le K_j} A_{ije} X_{j,t,e} & & \le & b_{it} & \text{for all i and t} \\
 & X_{j,0,e} & & = & X^*_{j,0,e} & \text{for all j and } e < K_j \\
 & -X_{j,T,e} & + I_{je} & = & 0 & \text{for all j and } e < K_j \\
 & -X_{j,t-1,e-1} + X_{j,t,e} & & \le & 0 & \text{for all t, j and } e > 0 \text{ and } e \le K_j \\
 & X_{j,t,e} \;, & I_{je} & \ge & 0 & \text{for all j, t, e}
\end{array}
$$

Where t is the time index; T is the length of the total planning horizon; j is an index for identifying alternative variables; e is an index identifying the elapsed age of a variable; K_j is the maximum age of a variable; i is an index which identifies resources; r is the discount rate; C_{je} is the per unit profit from variable j when it is of age e; $X_{j,t,e}$ is the number of units of alternative j on hand in period t which are of elapsed age e; F_{je} is the terminal value of incomplete units of enterprise j which are of elapsed age e; A_{ije} is the resource i usage by one unit of the production represented of enterprise j when it is of elapsed age e; b_{it} is the endowment of resource i in period t; and $X_{j,o,e}$ is the initial amount of enterprise j of elapsed age e before the model begins (in time period 0).

This model has many common features with the known life model. In particular, the objective function interpretation is virtually identical. However, there are several distinctive features that should be brought out.

Since the life of a production variable may be terminated, the decision variable now is $X_{j,t,e}$ which allows for the possibility of employing variable j in year t when it is of elapsed age e. The constraint before the nonnegativity conditions relates the amount of e year old variable in year t to the amount of e-1 year old variable in year t-1. The inequality on this constraint permits the predecessor to be nonzero and the successor to be zero (thus depicting selections less than maximum life for the variable).

This model is probably more realistic than the earlier one, but this has a price. The number of variables has been multiplied by the number of years a production activity could exist $\sum_J (K_j + 1)$ and the number of constraints has expanded. This is potentially a much larger model.

8.2.2.1 Example

For this example we use the data above plus longer retention of strawberries. Assume they may be kept up to 4 years and that in the fourth year the planting costs \$400 with the yield being 5, and water use being 5.7 acre feet. We will also assume that the terminal value of 3-year old strawberries is \$20/acre.

The resultant model is given in Table 8.3. The model shows several things. First the initial endowments of strawberries, are reflected on the right-hand side in the first year. Here there are 50 acres of zero year old strawberries, 10 of one year old strawberries, and zero of two year old strawberries. Second, the unknown life assumption is reflected in the linkage constraints that are labeled straw 0-1 year one etc. These constraints require that the strawberries in year one that are zero years old be a prerequisite for strawberries that are one year old in the second year, and similarly two year old strawberries in year two are a prerequisite for three year old strawberries in year three. Third, the model thus has its choice of whether to continue or not continue the strawberries throughout their life. This is reflected in the solution (Table 8.4), which shows that three year old strawberries are never used, indicating their life is always terminated, except that the two year old strawberries are kept in the terminal condition in the final year. Fourth, disequilibrium behavior is reflected in that different amounts of strawberries are established in each year, as are different amounts of wheat.

8.2.2.2 Comments

An alternative form of the disequilibrium unknown life model merges the concepts of the known and unknown life models. In this model the linkage equations above are dropped and the X variables are assumed to have known life wherein the alternative variables are formulated for each possible activity age. Thus in our example, we would have first year variables for strawberries planted in the first year and kept with a known life of 1, 2, 3, and 4 years (4 variables). Such an example will be illustrated in the equilibrium unknown life section.

8.2.3 Equilibrium - Known Life

Disequilibrium models are usually relatively large. Further, the specification of the initial and terminal conditions is of key importance. However, such conditions can be difficult to specify. Equilibrium models provide an alternative approach. In an equilibrium model, the firm is assumed to operate in a steady state manner, repeatedly making identical decisions year after year. In such a case, initial and final inventories of in-process goods will be equal; thus, the initial and final conditions may simply be set equal and their optimal levels determined by the model. This leads to a smaller model depicting a representative equilibrium year. Equilibrium solutions, however, do not portray growth situations or time paths of adjustment; only final equilibriums are created.

Equilibrium models are developed as follows: assume we have a variable with life of 4 periods and resource use, yield, etc., equal to A_e, where e is the elapsed age of the activity (0-3). Let us (assuming we start with zero initial activity) portray the resource use over several periods.

	Begin Activity in Period						
	1	2	3	4	5	6	7
Period 1 Resource Usage	A_0						
Period 2 Resource Usage	A_1	A_0					
Period 3 Resource Usage	A_2	A_1	A_0				
Period 4 Resource Usage	A_3	A_2	A_1	A_0			
Period 5 Resource Usage		A_3	A_2	A_1	A_0		
Period 6 Resource Usage			A_3	A_2	A_1	A_0	
Period 7 Resource Usage				A_3	A_2	A_1	A_0

Once the activity pattern enters period 4 we have resource use by each age of the activity. In this and subsequent periods, resource use in any period may be written as

$$R_t = A_3 X_{t-3} + A_2 X_{t-2} + A_1 X_{t-1} + A_0 X_t$$

where X_t is the quantity of the activity begun in period t.

Under an equilibrium assumption the same thing is done each year in a continually repeated sequence. Thus, $X_t = X_{t-1} = X_{t-2} = X_{t-3}$ and the resource use may be rewritten as

$$R_t = A_3 X_t + A_2 X_t + A_1 X_t + A_0 X_t = (A_0 + A_1 + A_2 + A_3) X_t = X_t \sum_e A_e$$

In an equilibrium model, then, the resource use for a variable is the sum of its resource usages over its whole life. (At this juncture we should observe that these assumptions imply that initial and final inventories are equal.)

Thus, a general formulation of the equilibrium model with known life is

$$\begin{aligned} \text{Max} \quad & \sum_j C_j X_j & & \\ \text{s.t.} \quad & \sum_j A_{ij} X_j & \leq \; b_i & \quad \text{for all j} \\ & X_j & \geq \; 0 & \quad \text{for all j} \end{aligned}$$

where: X_j is the quantity of the jth enterprise produced in equilibrium.

C_j is the revenue per unit of X_j and equals the sum of the returns to the activity over the periods (e) of its life, which is assumed to be known;

A_{ij} is the use of resource i per unit of X_j and equals A_{ije} or the sum of the resource usages over the years of the enterprise=s life; and

b_i is the amount of resource i available.

8.2.3.1 Example 1

Two examples will be shown for this particular formulation to illustrate its features. The first continues the example used throughout the chapter; in the second, initial and final inventory are explicitly handled.

The equilibrium known life nature of this model is reflected in Table 8.5 and GAMS formulation EQUILK. The model reflects the known life assumption under the strawberry activity in that the objective function of the strawberry activity equals the sum of the objective function values over all three years of its life. Similarly, land use equals three, indicating that when one acre is started in the first year that implicitly an acre is committed a year from now and two years from now. Under the equilibrium assumption for each acre started now, there will be two additional acres; one that was started one year ago and one that started two years ago, for a total of three acres of land. Initial and terminal conditions are not reflected. The solution to this model is given in Table 8.6, which shows that the equilibrium solution is to plant 479 acres of wheat each and every year and 74 acres of strawberries. This model can also be re-expressed in terms of average resource use. This is done in Table 8.7, where an average of one acre of land is used every year generates an average of $410 and the usage of 3.27 acre feet of water. The solution for this model essentially identical to the solution for the previous model, but the strawberry variable equals 221. This indicates that the equilibrium solution averages 221 acres of strawberries. Thus, in the strawberry rotation, one-third of the 221 acres (or 74 as in the earlier model) would be first year, one-third second year and one-third third year.

8.2.3.2 Example 2

The above example does not explicitly show the initial and final inventory situations. Therefore, we wish to construct another example that demonstrates this point more explicitly.

Suppose a farmer wishes to establish a crop rotation. Two crops are planted: corn and soybeans. Assume crop yield varies with dates of planting and harvest, thus the yields are dependent upon time of planting. Time is disaggregated into five annual periods: one in the fall after harvest through spring before planting, two during planting, and two during harvest. Cultural operations of the crop are plowing, planting and harvesting. Plowing may be done any time after a crop is harvested (during or after the harvest period) and before planting (during or before the planting period). Also assume that corn yields depend upon whether corn follows corn or soybeans. Land and labor are the limiting resources.

Technical data pertinent to the model are:

	Corn	Soybeans
Prices/Unit	$2.50	$6.50
Production Cost/ Acre	100	50
Labor Use in hrs/acre		
Plowing	.4	.3
Planting	.15	.15
Harvesting	.35	.17

Yields:

		Corn after Corn Planting Period		Corn After Soybeans Planting Period		Soybeans (after Anything) Planting Period	
		Pd2	Pd3	Pd2	Pd3	Pd2	Pd3
Harvest	Pd4	130	120	145	133	35	45
Period	Pd5	125	110	137	129	33	42

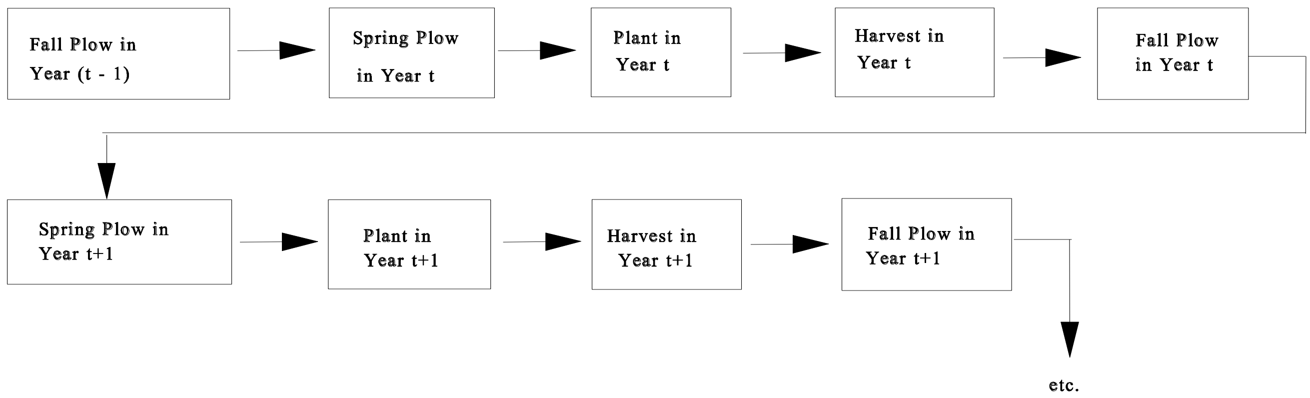
Labor Availability (hrs)

Period	Available Labor
Post harvest/Preplant (Pd1)	80
Plant (Pd2)	65
Plant (Pd3)	75
Harvest (Pd4)	100
Harvest (Pd5)	80

The farm has 400 acres.

Before formulating this model, its dynamic nature should be explored. Specifically, we must address this question: what are the items which constitute the initial and terminal conditions?

A diagram of the dynamic process appears below:



This dynamic process reflects linkages between last year's fall plowing with this years activities. The equilibrium assumption then is that this year's fall plowing and crop mix equals last year's. Therefore, we will require this to be true by using only one set of variables for fall plowing which by nature assumes equality. The

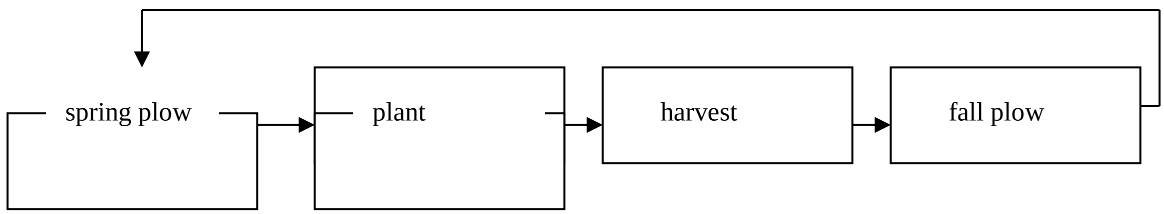


diagram of the resulting dynamic process is which implies that planting precedes harvest which precedes planting; thus, planting precedes itself in an assumed equilibrium situation.

The model formulation' reflecting this is shown in Table 8.8. Note that sequencing is modeled as illustrated earlier. This introduces the dynamic equilibrium assumption, in that plowing precedes planting, which precedes harvest, which precedes plowing, making the timing circular where the functions both precede themselves and follow themselves. The solution to this model is indicated in Table 8.9, and shows a continuous rotation with 200 acres of corn are planted after soybeans and 200 acres of soybeans are planted after corn.

8.2.3.3 Comments

Several aspects of the equilibrium model require discussion. First when a model reaches an equilibrium, the resources available must be in equilibrium. Thus, the same amount of resources must be available on a continuing basis. Similarly, resource use of a variable of a given age must be the same year after year. Such assumptions rule out the use of this model when examining cases with depletable resources.

A second assumption involves the word repeatability. Namely, the plan at any point in time must be repeatable. Given an alternative which lasts n periods the solution will have 1/n units of the alternative in each stage of its life. In the wheat strawberry 1/3 of the acreage is in each of the three years of the strawberries life. Further, in the corn-soybean example, no more than 1/2 the acreage will be devoted to corn following soybeans and an equal acreage must be devoted to soybeans followed by corn.

A final line of comments relates to discounting. These authors have not examined discounting at great length, but feel that discounting should not be included in this model since the return in every year is identical.

8.2.4 Equilibrium - Unknown Life

A version of the equilibrium model may be formulated in which the multi- period activities are of unknown life. The assumption is retained that the same thing is done each period. Thus, resource use in a typical period is summed across the possible ages that an activity may be kept. A general formulation of the model is

$$
\begin{array}{llrcll}
\text{Max} & \sum_j \sum_e & C_{je} X_{je} & & & \\
\text{s.t.} & \sum_j \sum_e & A_{ije} X_{je} & \leq & b_i & \text{for all } i \\
 & & -X_{j,e-1} + X_{je} & \leq & 0 & \text{for all } j \text{ and } e > 0 \\
 & & X_{je} & \geq & 0 & \text{for all } j \text{ and } e
\end{array}
$$

where: X_{je} is the quantity of the jth activity produced and kept until it is e periods old;

C_{je} is the per unit returns to X_{je};

A_{ije} is the per unit usage of resource i by X_{je};

b_i is the amount of resource i available;

The model maximizes profits subject to constraints on resource use and age sequencing. The age sequencing constraints state that the amount of enterprise j of age e must be less than or equal to the amount of that enterprise that was kept until age e-1.

8.2.4.1 Example

A tableau of the unknown life equilibrium model in the wheat, strawberry example context is given in Table 8.10. There we again have the structure where first year strawberries are required before second year strawberries can be grown. Similarly, two year old strawberries are required before three year old strawberries, etc. The solution, which is given in Table 8.11, indicates that the model chooses to terminate the life of strawberries at the end of three years and that it would cost $88 to continue them into the fourth year. The solution also is indicative of what terminal conditions would be needed had we set them appropriately in the equilibrium model, where the terminal value of zero year old strawberries is around $488 and the terminal value of one year old strawberries is $164 and the terminal value of two year old strawberries is $0. A slightly more compact formulation may be constructed in which the activities represent the activity kept for e years. This model is illustrated in Table 8.12.

8.2.5 Overall Comments on Dynamic Linear Programming

Dynamic LP may be used in many settings. The models, however, do involve numerous assumptions and require different computational and data resources. The disequilibrium models in particular become quite large when used and also require constructing "good" terminal conditions. Equilibrium models are more compact, yet use assumptions which may be unrealistic given a problem situation.

Further, both models suffer from a "curse of certainty"; clearly the optimal solution depends upon the assumed current and future conditions. However, the model can include uncertainty. Kaiser and Boeljhe (1980) included uncertainty in the disequilibrium model using the expected value variance and MOTAD formulations (although their MOTAD approach is wrong). Yaron and Horowitz (1972B) incorporated risk via a multi-stage discrete stochastic method.

Three types of models have been used in many research studies. In terms of equilibrium - known life models, Swanson and Fox (1954); and Peterson (1955) present early models. Throsby (1967) formalized the approach. In fact, the use of average statistical budgets in a single year model assumes equilibrium (e.g., the models in Heady and Srivistava [1975]). Brandao, McCarl and Schuh (1984); and El Nazer and McCarl (1986) present models which apply the equilibrium model.
Equilibrium - unknown life models are not as easily found, although El Nazer and McCarl (1986) and McCarl et al. (1977) provide examples.

In terms of disequilibrium models, early models with known life come from Swanson and Fox (1954) and Dean and deBeredictus (1964) while White et al. (1978); and Spreen et al. (1980) provide later examples. Unknown life type models are given in Loftsguard and Heady (1959); and Candler (1960) while Irwin (1968) provides a review article and Boussard (1971); Norton, Easter and Roe (1980); and Reid, Musser and Martin (1980) provide other examples and references.

8.3 Recursive Programming

Another LP formulation dealing with dynamic situations is known as recursive programming (Day, 1963). Basically, recursive programming models involve problems in which model coefficients are functionally dependent upon earlier model solutions and an exogenously specified time path. Following Day and Cigno, a recursive programming model consists of a constrained optimization model; and a data generator. The data generator given the solution in period t, prepares the input for period t+1 including defining a set of constraints which relates the feasible values of current variables to past values of variables and exogenous events. The model then is optimized for each time period with the data generator updating the model to the next time period.

While recursive programming does not have to be cast as a LP, the LP version will be presented. The general recursive LP sequentially maximizes an LP for each of a number of periods.

$$
\begin{array}{llcccccl}
\text{Max for period t} & Z_t = & \sum_j C_{jt} X_{jt} & + & \sum_k d_k Y_{kt} & & & \\
\text{s.t.} & & \sum_j A_{ijt} X_{jt} & + & \sum_k E_{ik} Y_{kt} & \leq & b_{it} & \text{for all i} \\
 & & X_{jt} & , & Y_{kt} & \geq & 0 & \text{for all j and k}
\end{array}
$$

Where: C_{jt} are objective function parameters functionally dependent upon the previous objective function parameters ($C_{j,t-1}$), lagged optimal decision variables (X_{jt-1}, Y_{kt-1}), and exogenous events;

X_{jt} are the values of the decision variables at time t;

A_{ijt} are the resource i usages by X_{jt} functionally dependent upon lagged values,

d_k are objective function coefficients which are stable over time;

Y_{kt} are the optimal values of the kth Y variable in time period t;

E_{ik} are the usages of resource i which do not change over time;

b_{it} are the resource i limits, functionally dependent upon lagged phenomena.

The recursive model is not solved over all time periods simultaneously, rather an optimal solution is constructed for each time period. There is no guarantee (or necessarily a desire) that these solutions are optimal over all periods. Rather, the solution represents an adaptive stream of decisions.

We have not specified the functional form of the lagged functions as they most often depend upon the problem at hand.

8.3.1 Example

A simple recursive programming example arises from a "cobweb" type model. Assume we have a group of producers who choose between two crops based on last year's price. Further assume that producers dampen their adjustment so that production varies only as much as 2 percent from last year's acreage. Production costs are \$135 per acre for Crop 1 and \$85 per acre for Crop 2. The yields are Crop 1 - 130 bu/acre; Crop 2 - 45 bu/acre. Assume the typical producers each have 600 acres. Suppose, last year's acreage mix was 50 percent Crop 1 and 50 percent Crop 2 (300 acres of each for the average producer). Lastly, assume the demand curve for Crop 1 in terms of this producers production price dependent form is $P_1 = 20 - .00045\ Q_1$ and the demand curve for Crop 2 is $P_2 = 10.9 - .00045\ Q_2$ where Q is the quantity produced. Such a situation leads to the following programming problem.

$$
\begin{array}{lllllll}
\text{Max} & Z_t = & (130\,P_{1t} - 135) & X_{1t} & + (45\,P_{2t} - 85) & X_{2t} & \\
 & & & X_{1t} & + & X_{2t} & \leq 600 \\
 & & & X_{1t} & & & \leq 1.02\,X_{1t-1} \\
 & & & X_{1t} & & & \geq .98\,X_{1t-1} \\
 & & & & & X_{2t} & \leq 1.02\,X_{2t-1} \\
 & & & & & X_{2t} & \leq .98\,X_{2t-1} \\
 & & & X_{1t'} & & X_{2t} & \geq 0
\end{array}
$$

where: $P_{lt+1} = 20 - 0.00045\ Q_{it}$

$Q_{lt} = 130\ X_{lt}$

$P_{2t+1} = 10.9 - 0.00045\ Q_{2t}$

$Q_{2t} = 45\ X_{2t}$

The recursive solution to this model over 6 periods is

Time Period	X_{1t}	X_{2t}	P_{1t+1}	P_{2t+1}	Z_t
0	300.000	300.000	2.450	4.825	----
1	306.000	294.000	2.099	4.703	94995.750
2	311.880	288.120	1.755	4.584	79491.454
3	306.118	293.882	2.092	4.701	64163.394
4	311.995	288.005	1.748	4.582	79182.851
5	306.235	293.765	2.085	4.699	63860.831
6	312.110	287.890	1.742	4.580	78874.189

8.3.2 Comments on Recursive Programming

Recursive programming models explicitly represent lagged adjustment. Several forms of lagged adjustment have been commonly used. A frequently used form involves "flexibility constraints" in which a variable is allowed to vary at most from its predecessor by a percentage (as done in the example). Day (1963) first used this form of constraint; Schaller and Dean did so later. Sahi and Craddock (1974, 1975) present information on estimation of these parameters.

Models involving lagged adjustment to price are also common. Day (1978) reviews the early literature involving the cobweb model and goes over a number of considerations involved in its use. The approach involved is similar to the example above where current price is a function of lagged production. Such an approach is common in recursive programming, and in fact has been used in other contexts (e.g., see the discussion of iterative methods in Judge and Wallace (1958) and Tramel and Seale (1959,1963)). The recursive programming model generally adjusts in one-year sequences. Multi-year activities must then be converted to single-year activities by annualizing costs and putting in expected future returns. The literature does not cover considerations where resource use of an activity is expected to change over time, but they are possible.

Finally, we come to usage of recursive programming. Actually, the vast majority of the usages are centered in and around the work of Day. A review of applications is given in Day and Cigno and several other references appear in Judge and Takayama (1973).

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Table 8.1. Disequilibrium Known Life Example

Rows	Year 1 Wheat	Year 1 Straw-berries	Year 2 Wheat	Year 2 Straw-berries	Year 3 Wheat	Year 3 Straw-berries	Year 4 Wheat	Year 4 Straw-berries	Year 5 Wheat	Year 5 Straw-berries	Terminal Condition Strawberries Age 0	Terminal Condition Strawberries Age 1		RHS
Objective	340	1230	340	1230	340	1230	340	550	340	-150	160	110		Max
Land Year 1	1	1											≤	640
Water Year 1	1	0.8											≤	967
Land Year 2		1	1	1									≤	650
Water Year 2		4.5	1	0.8									≤	975
Land Year 3		1		1	1	1							≤	700
Water Year 3		4.5		4.5	1	0.8							≤	1200
Land Year 4				1		1	1	1					≤	700
Water Year 4				4.5		4.5	1	0.8					≤	1200
Land Year 5						1		1	1	1			≤	700
Water Year 5						4.5		4.5	1	0.8			≤	1200
Strawberry 0										-1	1		≤	0
Strawberry 1								-1				1	≤	0

Table 8.2. Disequilibrium Known Life Example Model Solution

Objective = 1224296

Variables	Value	Reduced Cost	Equation	Slack	Shadow Price
Wheat Year 1	506.2	0	Land Year 1	0	340.0
Strawberries Year 1	133.8	0	Water Year 1	316.76	0
Wheat Year 2	539.9	0	Land Year 2	0	283.1
Strawberries Year 2	16.3	0	Water Year 2	0	56.9
Wheat Year 3	423.4	0	Land Year 3	0	336.9
Strawberries Year 3	126.6	0	Water Year 3	0	3.1
Wheat Year 4	557.1	0	Land Year 4	0	279.8
Strawberries Year 4	0	0	Water Year 4	0	60.2
Wheat Year 5	573.4	0	Land Year 5	0	340.0
Strawberries Year5	0	0	Water Year 5	57.05	0
Term Straw -0	0	0	Strawberry 0	0	-160
Term Straw -1	0	-8	Strawberry 1	0	-118

Table 8.3 Disequilibrium Unknown Life Sample Problem

	Year 1					Year 2					Year 3					Year 4					Year 5					Terminal Conditions			
		Strawberries					Strawberries					Strawberries					Strawberries					Strawberries				Strawberries			RHS
Rows	Wheat	0	1	2	3	Wheat	0	1	2	3	Wheat	0	1	2	3	Wheat	0	1	2	3	Wheat	0	1	2	3	0	1	2	
Objective	340	-150	700	680	300	340	-150	700	680	300	340	-150	700	680	300	340	-150	700	680	300	340	-150	700	680	300	160	110	20	Max
Land Year 1	1	1	1	1	1																								≤ 700
Water Year 1	1	0.8	4.5	4.5	5.7																								≤ 1200
Init Straw 0			1																										≤ 50
Init Straw 1				1																									≤ 10
Init Straw 2					1																								≤ 0
Straw 0-1 Year 1		-1						1																					≤ 0
Straw 1-2 Year 1			-1						1																				≤ 0
Straw 2-3 Year 1				-1						1																			≤ 0
Land Year 2						1	1	1	1	1																			≤ 700
Water Year 2						1	0.8	4.5	4.5	5.7																			≤ 1200
Straw 0-1 Year 2							-1						1																≤ 0
Straw 1-2 Year 2								-1						1															≤ 0
Straw 2-3 Year 2									-1						1														≤ 0
Land Year 3											1	1	1	1	1														≤ 700
Water Year 3											1	0.8	4.5	4.5	5.7														≤ 1200
Straw 0-1 Year 3												-1						1											≤ 0
Straw 1-2 Year 3													-1						1										≤ 0
Straw 2-3 Year 3														-1						1									≤ 0
Land Year 4																1	1	1	1	1									≤ 700
Water Year 4																1	0.8	4.5	4.5	5.7									≤ 1200
Straw 0-1 Year 4																	-1						1						≤ 0
Straw 1-2 Year 4																		-1						1					≤ 0
Straw 2-3 Year 4																			-1						1				≤ 0
Land Year 5																					1	1	1	1	1				≤ 700
Water Year 5																					1	0.8	4.5	4.5	5.7				≤ 1200
Term Straw 0																						-1				1			≤ 0
Term Straw 1																							-1				1		≤ 0
Term Straw 2																								-1				1	≤ 0

Table 8.4. Disequilibrium Unknown Life Example Model Solution

Objective = 1280757.0

Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
Wheat year 1	547.1	0	Land Year 1	0	340
Straw 0 year old year 1	92.9	0	Water Year 1	308.57	0
Straw 1 year old year 1	50.0	0	Init Straw 0	0	490
Straw 2 year old year 1	10.0	0	Init Straw 1	0	340
Straw 3 year old year 1	0	-40	Init Straw 2	0	0
Wheat year 2	557.1	0	Straw 0-1 Year 1	0	-490
Straw 0 year old year 2	0	-8	Straw 1-2 Year 1	0	-130
Straw 1 year old year 2	92.9	0	Straw 2-3 Year 1	10	0
Straw 2 year old year 2	50.0	0	Land Year 2	0	280
Straw 3 year old year 2	0	-322	Water Year 2	0	60
Wheat year 3	464.3	0	Straw 0-1 Year 2	0	-470
Straw 0 year old year 3	142.9	0	Straw 1-2 Year 2	0	-340
Straw 1 year old year 3	0	0	Straw 2-3 Year 2	50	0
Straw 2 year old year 3	92.9	0	Land Year 3	0	340
Straw 3 year old year 3	0	-40	Water Year 3	203.57	0
Wheat year 4	557.1	0	Straw 0-1 Year 3	0	-490
Straw 0 year old year 4	0	0	Straw 1-2 Year 3	0	-110
Straw 1 year old year 4	142.9	0	Straw 2-3 Year 3	92.857	0
Straw 2 year old year 4	0	0	Land Year 4	0	274
Straw 3 year old year 4	0	-349	Water Year 4	0	65.7
Wheat year 5	557.1	0	Straw 0-1 Year 4	0	-470
Straw 0 year old year 5	0	-330	Straw 1-2 Year 4	0	-360
Straw 1 year old year 5	0		Straw 2-3 Year 4		0
Straw 2 year old year 5	142.9	0	Land Year 5	0	340
Straw 3 year old year 5	0	-40	Water Year 5	0	0
Term Straw 0		0	Term Straw 0	0	-160
Term Straw 1		0	Term Straw 1	0	-110
Term Straw 2		0	Term Straw 2	142.86	-20

Table 8.5. Equilibrium Known Life Example Formulation

	Wheat	Strawberries		
Objective	340	1230		
Land	1	3	≤	700
Water	1	9.8	≤	1200

Table 8.6. Equilibrium Known Life Example Solution

Objective = 253441

Variables	Value	Reduced Cost	Equation	Slack	Shadow Price
Wheat	479	0	Land	0	309
Strawberries	74	0	Water	0	31

Table 8.7. Equilibrium Known Life Example Formulation with Average Activities

	Wheat	Strawberries		
Objective	340	410		
Land	1	1	≤	700
Water	1	3.27	≤	1200

Table 8.8 Rotation Example in Equilibrium Known Life Context

	Plow after corn					Plow after soybeans					Plant Corn								Plant Soybeans								RHS
											after corn				after soybeans				after corn				after soybeans				
Rows											pl2	pl2	pl3	pl3	pl2	pl2	pl3	pl3	pl2	pl2	pl3	pl3	pl2	pl2	pl3	pl3	
	pd1	pd2	pd3	pd4	pd5	pd1	pd2	pd3	pd4	pd5	hr4	hr5	hr4	hr5	hr4	hr5	hr4	hr5	hr4	hr5	hr4	hr5	hr4	hr5	hr4	hr5	
Objective											225	213	200	175	263	243	233	223	178	165	243	223	178	165	243	223	Max
Land	1	1	1	1	1	1	1	1	1	1																	≤ 400
Labor pd1	.4					.3																					≤ 80
Labor pd2		.4					.3				.2	.2			.2	.2			.2	.2			.2	.2			≤ 65
Labor pd3			.4					.3					.2	.2			.2	.2			.2	.2			.2	.2	≤ 75
Labor pd4				.4					.3		.4		.4		.4		.4		.3		.3		.3		.3		≤ 100
Labor pd5					.4					.3		.4		.4		.4		.4		.3		.3		.3		.3	≤ 80
plant after plow																											
corn pd2	-1	-1		-1	-1						1	1							1	1							≤ 0
corn pd3	-1	-1	-1	-1	-1						1	1	1	1					1	1	1	1					≤ 0
soybeans pd2						-1	-1		-1	-1					1	1							1	1			≤ 0
soybeans pd3						-1	-1	-1	-1	-1					1	1	1	1					1	1	1	1	≤ 0
plow after harvest																											
corn pd4				1							-1		-1		-1		-1										≤ 0
corn pd5	1	1	1	1	1						-1	-1	-1	-1	-1	-1	-1	-1									≤ 0
soyb pd4									1										-1		-1		-1		-1		≤ 0
soyb pd5						1	1	1	1	1									-1	-1	-1	-1	-1	-1	-1	-1	≤ 0

Table 8.9. Solution to Rotation Model

Obj = 100777

Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
Plow After Corn			Land	0	238
pd1	0	0	Labor pd1	80	0
pd2	87.5	0	Labor pd2	0	0
pd3	72.5	0	Labor pd3	16	0
pd4	0	-22	Labor pd4	0	56
pd5	0	0	Labor pd5	0	0
Plow After Soybeans			Plant After Plow		
pd1	0	0	corn pd2	128	0
pd2	0	0	corn pd3	0	238
pd3	0	0	soybeans pd2	0	0
pd4	0	-17	soybeans pd3	0	253
pd5	200	0	Plow After		
			corn pd4	200	0
Plant corn after corn					
pl2 hr4	0	-33	corn pd5	0	0
pl2 hr5	0	-26	soyb pd4	189	0
pl3 hr4	0	-58	soyb pd5	0	15
pl3 hr5	0	-63			
Plant Corn After Soybeans					
pl2 hr4	200	0			
pl2 hr5		-11			
pl3 hr4	0	-30			
pl3 hr5	0	-31			
Plant Soybeans After Corn					
pl2 hr4	0	-70			
pl2 hr5	0	-59			
pl3 hr4	189	0			
pl3 hr5	11	0			
Plant Soybeans After Soybeans					
pl2 hr4	0	-70			
pl2 hr5	0	-74			
Pl3 hr4	0	-5			
pl3 hr5	0	-15			

Table 8.10. Equilibrium Unknown Life Example Formulation

		Strawberries				RHS	
Rows	Wheat	1	2	3	4	MAX	
Objective	340	-150	700	680	300		
Land	1	1	1	1	1	≤	700
Water	1	0.8	4.5	4.5	5.7	≤	1200
Straw 1-2		-1	1			≤	0
Straw 2-3			-1	1		≤	0
Straw 3-4				-1	1	≤	0

Table 8.11. Equilibrium Unknown Life Example Solution

Objective 253441

Variables	Value	Reduced Cost	Equation	Slack	Shadow Price
Wheat	479	0	Land	0	309
Strawberries 1 year old	74	0	Water	0	31
Strawberries 2 year old	74	0	Straw 1-2	0	483
Strawberries 3 year old	74	0	Straw 2-3	0	232
Strawberries 4 year old	0	185	Straw 3-4	74	0

Table 8.12. Alternative Formulation of Equilibrium Unknown Life

		Keep Strawberries				RHS	
Rows	Wheat	1	2	3	4		
Objective	340	-150	550	1230	1530		
Land	1	1	2	3	4	≤	700
Water	1	0.8	5.3	9.8	15.5	≤	1200