1. Suppose that you are hired as the manager of Sam's Hog and Crop Conglomerate. Sam owns two crop farms and a hog operation. Sam wishes to maximize overall profit from the operation and has a single labor force that is shared between the three operations. Pertinent data follow for each operation.

|  | Farm 1 |  | Farm 2 |
| :--- | :--- | :--- | :--- |
| Cost/hour of plowing regardless of crop | 2 |  | 2.1 |
| Acres plowed per hour of plowing regardless of crop | 5 | 5 |  |
| Labor hours required per hour of plowing regardless of <br> crop | 1 |  | 1.1 |
| Cost/hour of planting when corn is planted | 10 |  | 11 |
| Cost/hour of planting when soybeans are planted | 5 | 6 |  |
| Acres planted per hour of planting corn | 10 | 10 |  |
| Acres planted per hour of planting soybeans | 12 | 15 |  |
| Labor per hour of planting when corn is being planted | 1 |  | 1.1 |
| Labor per hour of planting when soybeans are planted | 1.2 |  | 1.5 |
| Cost/hour of harvesting for corn | 3 | 3 |  |
| Cost/hour of harvesting for soybeans | 3 | 3 |  |
| Acres harvested per hour of harvesting for corn | 2 | 2 |  |
| Acres harvested per hour of harvesting for soybeans | 5 | 5 |  |
| Labor hours per hour of harvesting corn | 2 | 2 |  |
| Labor per hour of harvesting soybeans | 2 | 2 |  |
| Acres available | 100 | 100 |  |

In addition, the following dates are relevant:

|  | Farm 1 |  | Farm 2 |
| :--- | :--- | :--- | :--- |
| Plowing | Sept. 1 - April 30 |  | Jan. 1 - April 30 |
| Planting Corn | April 1 - May 15 |  | April 16 - May 15 |
| Planting Soybeans | April 16 - April 30 |  | May 1 - May 30 |
| Harvesting Corn | Oct. 16 - Nov. 15 |  | Oct. 16 - Nov. 15 |
| Harvesting Soybeans | Oct. 1 - Nov.. 15 |  | Oct. 16 - Nov. 15 |

Data on yields in bushels are:

| CORN |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Harvest | Farm 1Planting Dates |  |  |  | Farm 2 Planting Dates |  |
| Dates | April 1-15 | April 16-30 | May 1-15 |  | April 16-30 | May 1-15 |
| Oct. 16-30 | 140 | 130 | 120 |  | 130 | 125 |
| Nov. 1-15 | 130 | 120 | 110 |  | 130 | 115 |


| SOYBEANS |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Harvest Dates | Farm 1Planting Dates |  |  | Farm 2 Planting Dates |  |  |
|  |  | April 16-30 |  |  | May 1-15 | May 16-30 |
| Oct. 1-15 | 42 |  |  | - | - |  |
| Oct. 16-30 | 45 |  |  | 42 | 45 |  |
| Nov. 1-15 | 38 |  |  | 38 | 37 |  |

Input usage in dollars:

|  |  | Corn |  |  | Soybeans |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | Farm 1 | Farm 2 |  | Farm 1 | Farm 2 |
| Fertilizer |  | 55 | 60 |  | 10 | 20 |
| Seed |  | 20 | 20 |  | 15 | 15 |
| Insecticide |  | 40 | 40 |  | 30 | 25 |

In addition, the farmer has discovered that on farm 2, by adding more fertilizer, that corn yield can be increased according to the function

Additional yield $=3 \mathrm{~F}-.05 \mathrm{~F}^{2}$
where F is the additional fertilizer in dollars.
Once these crops are grown, Sam either sells them to one of two elevators or feeds them to pigs. The cost of moving the grain to each location is

|  |  | Cost/bushel transported of corn or soybeans to |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | Elevator 1 |  | Elevator 2 |  |
|  | Pig Feeding Site |  |  |  |  |
| from Farm 1 | $\$ 0.05$ |  | $\$ 0.04$ |  | $\$ 0.01$ |
|  | Farm 2 | $\$ 0.06$ |  | $\$ 0.05$ |  |
|  |  | Current Prices |  |  | $\$ 0.02$ |
|  | Elevator 1 |  | Elevator 2 |  |  |
| Corn Price | $\$ 2.00$ |  | $\$ 2.05$ |  |  |
| Soybean Price | $\$ 5.00$ |  | $\$ 4.95$ |  |  |

Sam's pig operation operates as follows: Sam has hogs. Each hog gives birth to 12 pigs three months after it is retained instead of sold as a fed pig. Sam feeds these pigs for sale, slaughter or retention for future hogs. Sam's hogs can bear
pigs on Sept.1, Jan. 1, April 1 and July 1. Pigs are fed for 6 months and require 12.5 lbs. of protein per week and 900 calorie units. Hogs require twice as much.

The nutrient composition of corn and soybeans is:

|  |  | Corn |  | Soybeans |
| :--- | :--- | :--- | :--- | :--- |
| Protein | $3 \%$ |  | $20 \%$ |  |
| Calories |  | $3 / \mathrm{lb}$. |  | $1 / \mathrm{lb}$. |
| Weight/bushel | 60 lbs. |  | 56 lbs. |  |

Pig input usage other than feed involves 1 hour of labor per week. Hogs take 2 hours. Total labor endowments to be spread across all three operations are 50 hours per week.
Fed pigs weigh 180 lbs. at the end of the feeding period and sell for $\$ 0.50 / \mathrm{lb}$. If the pigs are slaughtered on the farm, the farmer receives 65 percent meat which sells for $\$ 0.80 / \mathrm{lb}$. and 35 percent offal which sells for $\$ 0.20 / \mathrm{lb}$. The farm can retain the pigs to become hogs up to the total hog capacity which is 14 in each of the 4 time periods. Hogs give only one litter and give birth 3 months after they have been converted from pig to hog status. They are kept 1 month after giving birth and are then sold for $\$ 100$.

Set up a LP profit maximizing model to tell:
a) How much of each crop to raise by farm;
b) How much of the crop to sell and how much to feed to pigs;
c) How many hogs to have bearing pigs in each of the 4 month periods, how many pigs to raise, and then how many pigs to sell, slaughter, or retain?
2. The Greenhouse, Inc. is a wholesale nursery which raises and sells petunias, marigolds, and geraniums. The nursery divides its year into 4 quarters. The length of time, space requirement, cash cost of production, and expected selling price for each plant are

|  | Time <br> (Quarters) | Space <br> $($ sq. ft.) | Cash <br> Cost | Selling <br> Price |
| :--- | :--- | :--- | :--- | :--- |
| Petunia | 2 | 1 | 2.25 | 3.00 |
| Marigold | 1 | 1 | 1.00 | 1.50 |
| Geranium | 4 | 1 | 4.00 | 6.50 |

Assume you can start the flowers in any quarter and that 10,000 square feet are available. Set up both an equilibrium and a 2 -year disequilibrium LP.
3. Develop a formulation to fit a linear function to the data

| Price of a <br> Good | Quantity of that <br> Good (own <br> quantity) |  | Quantity of a <br> Substitute Good |
| :--- | :--- | :--- | :--- |
| 3 | 10 | 12 |  |
| 4 | 9 | 13 |  |
| 2 | 15 | 9 |  |
| 1 | 22 | 6 |  |
| 7 | 3 | 15 |  |

where you wish to
a) Minimize Total Absolute Deviation
b) Minimize the largest Absolute Deviation
c) Impose the hypothesis that the own quantity slope is positive
d) Impose the restriction that the own quantity slope term is larger than the intercept
4. Better Beef Feeders Inc. wishes to manage its herd of cattle. Better Beef may buy feeders at $\$ 130$. Feeders gain weight according to the following production function:

$$
\mathrm{FWT}=300 \mathrm{G}^{.3} 1^{2} \mathrm{LAB}^{2} \mathrm{CF}^{2}
$$

Where G = grain in bushels
$\mathrm{L}=$ land in acres
LAB = labor in hours
$\mathrm{CF}=$ cut forage in hundredweight
FWT = finished weight of a feeder animal in pounds
In addition to weight of animal, the firm also receives animal wastes. The production function for waste is

$$
\mathrm{AW}=1.1 \mathrm{G}^{.6} \mathrm{CF}^{3} \mathrm{~L}^{.1}
$$

Where AW is waste production per animal in pounds. The price of feeders is $\$ .40 / \mathrm{lb}$. on the hoof. Waste is worth $\$ 1.00 / 100 \mathrm{lbs}$.
a) Formulate an LP problem for the feeding of one animal assuming land use of 2 or less acres, labor use of 1 or less hours, grain costs $\$ 2.00$ per bushel and cut forage $\$ .20 / \mathrm{cwt}$.
b) Assume the firm wishes to optimize over its whole herd and has 600 acres of land, 200 hours of labor and may buy grain and forage at the above prices. Formulate this LP.
c) Discuss the anticipated answers of these two formulations.
5. The Grow-It-Fast, Co. (known as GIF) makes fertilizer for home gardeners. Their two main products are the all-purpose fertilizer and the quick-green high nitrogen fertilizer. The all-purpose fertilizer is 10-10-10 which means it is $10 \%$ nitrogen,
$10 \%$ phosphorus, and $10 \%$ potassium. The quick-green fertilizer is 25-3-3.
GIF uses three primary ingredients to make the fertilizers. Data relevant to these ingredients are

| Input | Composition |  | Cost/ton |
| :--- | :--- | :--- | :--- |
| Ammonium nitrate | $33 \%$ nitrogen | $67 \%$ filler | $\$ 100$ |
| Phosphate | $100 \%$ phosphorus | $0 \%$ filler | $\$ 120$ |
| Potassium chloride | $50 \%$ potassium | $50 \%$ filler | $\$ 40$ |
| Filler |  | $100 \%$ filler | $\$ 3$ |

The processing plant is such that processing costs are not constant per unit. The processing function is

$$
C=10+5 Q+1 / 2 Q^{2}
$$

where Q is hundreds of bags of fertilizer produced. Assume that only 100 lb . bags are produced. The 10-10-10 fertilizer sells for $\$ 5$ per 100 lb . bag while the 25-3-3 fertilizer sells for $\$ 12$ per 100 lb . bag. The maximum capacity of the plant is 400 bags.

Formulate a LP model for this problem.

