MIDTERM EXAM
AGEC 641

March 22, 1994

Answer 6 of the first 7 questions. Take the other question home along with question #8 and turn them in on Wednesday by 5:00 p.m.

1. Suppose we have a storage model of the following form:

$$
\begin{array}{lccccccc}
\text{Max} & c_1 g_1 & + & c_2 g_2 & - & h_{12} s_{12} & & \\
 & g_1 & & & + & s_{12} & \leq & I_1 \\
 & & & g_2 & - & s_{12} & \leq & I_2 \\
 & g_1, & & g_2, & & s_{12} & \geq & 0
\end{array}
$$

where c_i is the price of grain in period i

g_i is the quantity of grain in period i

I_i is the net addition of grain sold in period i

h_{12} is the holding cost from period 1 to 2.

a. Explain the function of the s_{12} variable.

b. What are the dual implications of s_{12}.

c. Explain the interrelationship of the shadow prices on the two constraints and the optimal value of s_{12}.

2. Assuming that a firm knows a function g(F) where F is an input and wishes to choose the optimum level of F. Also, suppose you are willing to approximate this at F levels of 1, 4, and 9. Set up an LP of:

$$\text{Max} \quad 1.4g(F) - 2F$$

$$\text{where} \quad g(F) = 5F^{0.5}$$

3. a. Derive (do **not** state) conditions in a linear programming maximization model which will identify the way the objective function changes when a new variable is bought into the basis.

b. State conditions for selecting a variable to enter the basis which indicates the objective function will probably increase. State whether under those conditions the objective function will always increase.

4. Suppose someone told you linear programs always deal with problems which maximize profits subject to resource constraints assuming Leontief isoquants. Do you agree? Cite examples if not.

5. Explain the relationship between linear programming right hand side ranging and a factor demand curve.

6. What implications do the homogeneity of units tests discussed in class have for the coefficients in the first row and column of the following:

$$
\begin{array}{rrcrcl}
\text{Max} & c_1x_1 & + & c_2x_2 & & \\
 & a_{11}x_1 & + & a_{12}x_2 & \leq & b_1 \\
 & a_{21}x_1 & + & a_{22}x_2 & \leq & b_2 \\
 & x_1 & & & \leq & b_3 \\
 & x_1, & & x_2 & \geq & 0
\end{array}
$$

7. Assume a firm feeds hogs and raises feed. Assume hogs may be started in each of the four quarters and are kept 3 quarters. Suppose the feed production LP is as follows

$$\begin{aligned} \text{Max}\quad & 3X_1 - 4Y_1 - 70Z \\ & X_1 - 100Y_1 \le 0 \\ & 3Y_1 - Z \le 0 \\ & Y_1 \le 500 \end{aligned}$$

with X_1 being the amount of feed available for sale in quarter 3; Y_1 the amount of feed raised; and Z the amount of inputs bought. Furthermore, the constraint $Y_1 \le 500$ depicts land availability.

You need to add hogs to this model. Suppose hogs need 10 units of feed in the first quarter after they are bought, 12 units of feed in the second quarter after they are bought, 15 units of feed in the third quarter after they are bought and are then sold. Suppose the hogs use .02 units of land in each quarter they are present and that the crops use that resource in equal amounts in quarters 2 and 3. Suppose hogs are bought for $20 and sold for $200, and that the firm can't have more than 75 in any quarter. Suppose it costs $0.10 to store feed one quarter. Set up an equilibrium model to determine how many hogs to buy in each quarter.

8. Pete's Petroleum Products (PPP) is trying to establish a long term plan for its refinery operations.

PPP makes 6 products from 3 kinds of crude oil. The breakout percentages are as follows:

Production of Products When Refining	**Crude Oil Source**		
	Middle East	West Texas	Mid Continent
		(Proportional Yield)	
Gasoline	.04	.18	.24
Naphtha	.14	.20	.22
Distillate	.09	.07	.09
Gas Oil	.18	.09	.11
Residuum	.54	.44	.32
Misc.	.01	.02	.02
Cost of refining 55 gallon barrel	$ 2	$2.5	$ 3

PPP now has three plants: one in Texas, one in Ohio and one in South Carolina. The crude oil can be shipped in as follows:

Crude Oil Shipping Cost per 55 Gallon Barrel

	Source of Crude Oil		
Destination Plant	Middle East	West Texas	Mid Continent
Texas	$ 3	$ 1	$ 4
Ohio	$ 8	$ 4	$ 2
South Carolina	$ 2	$ 6	$ 4

In turn, the products are sold at each of the plants for the following prices:

Price/Gallon

Product	Texas	Ohio	South Carolina
Gasoline	$ .63	$ .73	$ .69
Naphtha	.37	.33	.40
Distillate	.45	.50	.51
Gas Oil	.30	.40	.39
Residuum	.10	.11	.12
Misc. [a/]	-.01	-.02	-.01
Maximum sale in barrels	1,000,000	2,000,000	1,000,000

[a/] This is a cost of disposal

PPP may buy the crude oils at the following prices:

West Texas	$17 per 55 gallon barrel
Mid Continental	19 per 55 gallon barrel
Middle East	15 per 55 gallon barrel

West Texas supplies yield no more than 1,000,000 barrels this year while Mid Continental are limited to 200,000 and Middle East are unlimited.

PPP is also considering building some ships and will accept a non-integer answer. The ships can be of the following capacities (in 55 gallon barrels), costs per voyage and annual cost:

Ship Capacity

Per Voyage	Cost/Voyage[b/]	Time/Voyage	Annual Cost
100,000	100,000	1 month	1,000,000
500,000	400,000	1 month	5,500,000
1,000,000	800,000	1 month	10,000,000

b/ These are voyages to South Carolina. Direct voyages to Texas cost 20% more and take 20% longer. Voyages to Ohio are not possible. Ships have 12 months available. If the ships are bought then no other transport costs other than voyage and annual costs are incurred.

Further, suppose that transhipping Middle East oil to Ohio costs $3.00 from Texas or South Carolina.

Finally, PPP has a contract where for each gallon of gasoline sold at the above price in Ohio at least 1/2 gallon of Gas Oil must be sold. Gasoline available exceeding this limit may be sold at 2/3 price to local companies.

Formulate a profit maximizing LP assuming partial ships can be bought.