Risk Modeling and Stochastic Programming

**Where does it occur?**

**Maximize CX**

**Subject to AX  b**

 **X  0**

## **Uncertain objective function returns - C**

 **variability in prices**

 **variability in production quantities**

 **variability in costs**

 **variability in market sales**

## **Resource usages - A**

 **variability in raw input quality**

 **variability in working conditions**

 **variability in intermediate product yields**

 **variability in product requirements**

## **Resource endowments - b**

 **variability in demand firm faces**

 **variability in resources available**

 **variability in working conditions**

Risk Modeling and Stochastic Programming

**Including Risk**

**When incorporating risk there are three big issues**

**1.    What is the nature of risk?**

 **a.   What parameters of the model are**

 **uncertain?**

1. **How do we describe their distribution?**
2. **When are risk outcomes revealed?**

**Does the producer receive information about**

 **uncertain events and will make adaptive**

**decisions?**

1. **How do we model behavioral reaction to**

 **risk?**

 **Is expected profit maximization not the proper objective but rather some degree of aversion to the variation caused by risk?**

Risk Modeling and Stochastic Programming

Why Model Risk

Why not just solve for all values of risky parameters

# Curses of dimensionality and certainty

­Dimensionality: Number of possible plans

(5 possible values for 3 parameters 35 = 243)

Certainty: Each plan would be certain of data so we would have 243 different things we could do

–What would we do?

General Risk Modeling Aim

 Generate Robust in the face of the Uncertainty

Not necessarily a best performer in any setting, but a good performer across many or most or the most likely spectrum

Risk Modeling and Stochastic Programming

Forms of assumed reaction to risk

Non Recourse or non adaptive decision making

* Decisions made now consequence felt later
* No decisions made between now and when

consequences felt

Example–Buy stock now make no decision for one year

## Recourse or adaptive decision making

Later time during model additional decisions made.

In this later decision period

Decision maker knows what happened between first decision and now.

Decision makers cannot revise prior actions but can adjust current decisions ie current decisions can be employed to make adjustments in the face of realized events -- phenomena called irreversibility and recourse

Example – Buy stock now make review decisions quarterly possibly selling and buying other stocks

Including Non Adaptive Risk

**Decision Maker reaction to risk**

**Expected Value Maximization**

****

**Conservative - Fat or thin coefficients**

****

**Where **

 ****

 ****

**E- V**

 **Maximize E(income) - RAP \* Variance(income)**

# **Expected utility**

 **Maximize Sum(p,Probability(p)\*U[Wealth(p)])**

 **S.T. Wealth(p)=InitWealth + Income(p) for all p**

 **Income(p)=C(p)\*X for all p**

# **Safety First based**

 **Maximize Sum(p,Probability(p)\*Income(p))**

 **S.T. Income(p)=C(p)\*X for all p**

 **Income(p)safety for all p**

**Including Non Adaptive Risk**

**First Risk Model**

**Markowitz mean-variance portfolio choice formulation**

**Given Problem**

**Max sum(invest, moneyinvest(invest)\*avgreturn(invest))**

**s.t sum(invest,moneyinvest(invest)\*price(invest))  funds**

**Markowitz observed not all money in highest valued stock**

 **Inconsistent with LP formulation**

 **Why? Not a basic solution**

**Markowitz posed the hypothesis that average returns and the variance of returns were important**

Including Non Adaptive Risk

**EV Formulation – Statistical Background**

**Given a linear objective function**

****

**where X1, X2 are decision variables and c1 , c2 are uncertain parameters distributed with means and as well as variances s11 , s22, and covariance s12; then Z is distributed with mean**

****

**and variance **

**in terms of correlation**

**s12= ρ12 σ1σ2**

### **In matrix terms the mean and variance of Z are**

****

**where in the two by two case**



**Including Non Adaptive Risk**

**EV Formulation – Statistical Background**

## **Defining terms**

**sii is the variance of the objective function coefficient of Xi, which is calculated using the formula**

 **sik =  (cik‑** **where cik is the kth observation on the objective value of Xi and N is the number of observations, assuming an equally likely probability (1/N) of occurrence.[[1]](#footnote-2)**

**sij for i ≠ j is the covariance of the objective function coefficients between ci and cj, calculated by the formula sij = ∑ (cik‑ )(cjk‑)/N. Note sij = sji.**

** is the mean value of the objective function coefficient ci, calculated by  cik/N. (Assuming an equally likely probability of occurrence.)**

**Including Non Adaptive Risk**

**E-V Model Commonly Used Formulation**

**Markowitz Formulation**

Min $σ\_{Z}^{2}$

s.t. E =K

or

**Freund Formulation**

****

**or Max E - RAP \* Variance**

**Why Use Later – reuse of RAP and Transferability**

**Commonly Used Freund Formulation**



**Where**

**E = expected value of risky c times choice of x**

**Var = sum over j of var (sjj) times the square of the x variables (**$x\_{j}^{2}$**) minus sum over j and k where j**$\ne k$ **of twice thecovariance(sjj) times xj times xk**

$∅$ **= risk aversion parameter**

Risk Modeling and Stochastic Programming

EV Model – Example

Assume an investor wishes to develop a stock portfolio given the stock annual returns information shown in Table 14.1, 500 dollars to invest and prices of stock one $22.00, stock two $30.00, stock three $28.00 and stock four $26.00.

|  |
| --- |
| **Table 14.1. Data for E-V Example -- Returns by Stock and Event** |
|  | ----Stock Returns by Stock and Event---- |
|  | Stock1 | Stock2 |  Stock3 |  Stock4 |
| Event1 | 7  | 6  | 8  | 5  |
| Event2 | 8  | 4  | 16  | 6  |
| Event3 | 4  | 8  | 14  | 6  |
| Event4 | 5  | 9  | ‑2  | 7  |
| Event5 | 6  | 7  | 13  | 6  |
| Event6 | 3  | 10  | 11  | 5  |
| Event7 | 2  | 12  | ‑2  | 6  |
| Event8 | 5  | 4  | 18  | 6  |
| Event9 | 4  | 7  | 12  | 5  |
| Event10 | 3  | 9  | ‑5  | 6  |
|  |  |  |  |  |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Price | 22 | 30 | 28 | 26 |

Risk Modeling and Stochastic Programming

EV Model – Example

|  |
| --- |
| Table 14.2. Mean Returns and Variance Parameters for Stock Example |
|  | Stock1 | Stock2 | Stock3 | Stock4  |
| Mean Returns  |  4.70 |  7.60 |  8.30 |  5.80 |
|  |  |  |  |  |
| Variance-Covariance Matrix |  |  |  |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Stock1 | 3.21 | -3.52 | 6.99 | 0.04 |
| Stock2 | -3.52 | 5.84 | -13.68 | 0.12 |
| Stock3 | 6.99 | -13.68 | 61.81 | -1.64 |
| Stock4 | 0.04 | 0.12 | -1.64 | 0.36 |

Risk Modeling and Stochastic Programming

EV Model – Example

**In turn the objective function is**

****

**or, in scalar notation**

****

**This objective function is maximized subject to a constraint on investable funds:**

****

**and non‑negativity conditions on the variables.**

GAMS Formulation

 10 SETS STOCKS POTENTIAL INVESTMENTS / BUYSTOCK1\*BUYSTOCK4 /

 11 EVENTS EQUALLY LIKELY RETURN STATES OF NATURE

 12 /EVENT1\*EVENT10 / ;

 14 ALIAS (STOCKS,STOCK);

 16 PARAMETERS PRICES(STOCKS) PURCHASE PRICES OF THE STOCKS

 17 / BUYSTOCK1 22, BUYSTOCK2 30

 19 BUYSTOCK3 28, BUYSTOCK4 26 / ;

 22 SCALAR FUNDS TOTAL INVESTABLE FUNDS / 500 / ;

 24 TABLE RETURNS(EVENTS,STOCKS) RETURNS BY STATE OF NATURE EVENT

 26 BUYSTOCK1 BUYSTOCK2 BUYSTOCK3 BUYSTOCK4

 27 EVENT1 7 6 8 5

 28 EVENT2 8 4 16 6

 29 EVENT3 4 8 14 6

 30 EVENT4 5 9 -2 7

 31 EVENT5 6 7 13 6

 32 EVENT6 3 10 11 5

 33 EVENT7 2 12 -2 6

 34 EVENT8 5 4 18 6

 35 EVENT9 4 7 12 5

 36 EVENT10 3 9 -5 6

 38 PARAMETERS

 39 MEAN (STOCKS) MEAN RETURNS TO X(STOCKS)

 40 COVAR(STOCK,STOCKS) VARIANCE COVARIANCE MATRIX;

 42 MEAN(STOCKS) = SUM(EVENTS , RETURNS(EVENTS,STOCKS) / CARD(EVENTS) );

 43 COVAR(STOCK,STOCKS)

 44 = SUM (EVENTS ,(RETURNS(EVENTS,STOCKS) - MEAN(STOCKS))

 45 \*(RETURNS(EVENTS,STOCK)- MEAN(STOCK)))/CARD(EVENTS);

 47 DISPLAY MEAN , COVAR ;

 49 SCALAR RAP RISK AVERSION PARAMETER / 0.0 / ;

 51 POSITIVE VARIABLES INVEST(STOCKS) MONEY INVESTED IN EACH STOCK

 53 VARIABLE OBJ NUMBER TO BE MAXIMIZED ;

 55 EQUATIONS OBJJ OBJECTIVE FUNCTION

 56 INVESTAV INVESTMENT FUNDS AVAILABLE;

 59 OBJJ..OBJ =E= SUM(STOCKS, MEAN(STOCKS) \* INVEST(STOCKS))

 61 - RAP\*(SUM(STOCK, SUM(STOCKS,

 62 INVEST(STOCK)\* COVAR(STOCK,STOCKS)\*invest(stocks));

 64 INVESTAV.. SUM(STOCKS, PRICES(STOCKS) \* INVEST(STOCKS)) =L= FUNDS ;

 66 MODEL EVPORTFOL /ALL/ ;

 70 SCALAR VAR THE VARIANCE ;

 75 SET RAPS RISK AVERSION PARAMETERS /R0\*R25/

 77 PARAMETER RISKAVER(RAPS) RISK AVERSION COEFICIENT BY RISK AVERSION

 78 /R0 0.00000, R1 0.00025, R2 0.00050, R3 0.00075,

 79 R4 0.00100, R5 0.00150, R6 0.00200, R7 0.00300,

 82 R20 5.00000, R21 10.0000, R22 15. , R23 20.

 84 R24 40. , R25 80./

 86 PARAMETER OUTPUT(\*,RAPS) RESULTS FROM MODEL RUNS WITH VARYING RAP

 90 LOOP (RAPS,RAP=RISKAVER(RAPS);

 91 SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;

 92 VAR = SUM(STOCK, SUM(STOCKS,

 93 INVEST.L(STOCK)\* COVAR(STOCK,STOCKS) \* INVEST.L(STOCKS)

 94 OUTPUT("RAP",RAPS)=RAP;

 95 OUTPUT(STOCKS,RAPS)=INVEST.L(STOCKS);

 96 OUTPUT("OBJ",RAPS)=OBJ.L;

 97 OUTPUT("MEAN",RAPS)=SUM(STOCKS, MEAN(STOCKS)\*invest.l(stock));

 98 OUTPUT("VAR",RAPS) = VAR;

 99 OUTPUT("STD",RAPS)=SQRT(VAR);

 100 OUTPUT("SHADPRICE",RAPS)=INVESTAV.M;

 101 OUTPUT("IDLE",RAPS)=FUNDS-INVESTAV.L); );

 103 DISPLAY OUTPUT;

Risk Modeling and Stochastic Programming

EV Model – Example

**Table 14.4. E‑V Example Solutions for Alternative Risk Aversion Parameters**

**RAP or φ 0 0.00025 0.0005 0.00075 0.001**

BUYSTOCK2 1.263 5.324 7.355

BUYSTOCK3 17.857 17.857 16.504 12.152 9.977

OBJ 148.214 143.287 138.444 135.688 134.245

MEAN 148.214 148.214 146.581 141.331 138.705

VAR 19709.821 19709.821 16274.764 7523.441 4460.478

STD 140.392 140.392 127.573 86.738 66.787

SHADPRICE 0.296 0.277 0.261 0.260 0.260

**RAP or φ 0.0015 0.002 0.003 0.005 0.010**

BUYSTOCK2 9.386 10.401 11.416 12.229 12.838

BUYSTOCK3 7.801 6.713 5.625 4.755 4.102

OBJ 132.671 131.753 130.575 129.005 125.999

MEAN 136.080 134.767 133.454 132.404 131.617

VAR 2272.647 1506.907 959.949 679.907 561.764

STD 47.672 38.819 30.983 26.075 23.702

SHADPRICE 0.259 0.257 0.255 0.251 0.241

**RAP or φ 0.011 0.012 0.015 0.025 0.050**

BUYSTOCK1 1.273 4.372 4.405

BUYSTOCK2 12.893 12.960 12.420 11.070 8.188

BUYSTOCK3 4.043 3.972 3.550 2.561 1.753

BUYSTOCK4 4.168

OBJ 125.441 124.614 123.380 120.375 116.805

MEAN 131.545 131.459 129.839 125.939 121.656

VAR 554.929 547.587 430.560 222.576 97.026

STD 23.557 23.401 20.750 14.919 9.850

SHADPRICE 0.239 0.236 0.234 0.230 0.224

**RAP or φ 0.100 0.300 0.500 1.000 2.500**

BUYSTOCK1 4.105 3.905 3.865 3.835 1.777

BUYSTOCK2 6.488 5.354 5.128 4.958 2.289

BUYSTOCK3 1.340 1.064 1.009 0.968 0.446

BUYSTOCK4 6.829 8.602 8.957 9.223 4.296

OBJ 113.118 102.254 92.010 66.674 27.185

MEAN 119.327 117.774 117.463 117.230 54.370

VAR 62.086 51.734 50.905 50.556 10.874

STD 7.879 7.193 7.135 7.110 3.298

SHADPRICE 0.214 0.173 0.133 0.032 0

IDLE FUNDS 268.044

Risk Modeling and Stochastic Programming

EV Model – Example

Efficient Frontier:



Risk Modeling and Stochastic Programming

Characteristics of E‑V Model Optimal Solutions

**Properties of optimal E-V solutions may be examined via the Kuhn‑Tucker conditions. Given**

**the problem**

****

**Its Lagrangian function is**

****

**and the Kuhn‑Tucker conditions are**



Risk Modeling and Stochastic Programming

 Unified Model

**Expected Income = sum (k,prob(k)\*Income(k))**

**Variance = sum (k,prob(k)\*(income(k)-Expected Income)\*\*2)**



Risk Modeling and Stochastic Programming

 Unified Model- GAMS Formulation

 10 SETS STOCKS POTENTIAL INVESTMENTS / BUYSTOCK1\*BUYSTOCK4 /

 11 EVENTS EQUALLY LIKELY RETURN STATES OF NATURE

 12 /EVENT1\*EVENT10 / ;

 14 PARAMETERS PRICES(STOCKS) PURCHASE PRICES OF THE STOCKS

 15 / BUYSTOCK1 22

 16 BUYSTOCK2 30

 17 BUYSTOCK3 28

 18 BUYSTOCK4 26 / ;

 19

 20 SCALAR FUNDS TOTAL INVESTABLE FUNDS / 500 / ;

 22 TABLE RETURNS(EVENTS,STOCKS) RETURNS BY STATE OF NATURE EVENT

 23

 24 BUYSTOCK1 BUYSTOCK2 BUYSTOCK3 BUYSTOCK4

 25 EVENT1 7 6 8 5

 26 EVENT2 8 4 16 6

 27 EVENT3 4 8 14 6

 28 EVENT4 5 9 -2 7

 29 EVENT5 6 7 13 6

 30 EVENT6 3 10 11 5

 31 EVENT7 2 12 -2 6

 32 EVENT8 5 4 18 6

 33 EVENT9 4 7 12 5

 34 EVENT10 3 9 -5 6

 36

 37 SCALAR RAP RISK AVERSION PARAMETER / 0.0 / ;

 38

 39 POSITIVE VARIABLES INVEST(STOCKS) MONEY INVESTED IN EACH STOCK

 40 POSDEV(EVENTS) POSITIVE DEVIATIONS FROM MEAN INCOME

 41 NEGDEV(EVENTS) NEGATIVE DEVIATIONS FROM MEAN INCOME

 42

 43 VARIABLES OBJ NUMBER TO BE MAXIMIZED

 44 RETURN(EVENTS) RETURNS BY EVENT

 45 MEAN MEAN RETURNS ;

 46

 47 EQUATIONS OBJJ OBJECTIVE FUNCTION

 48 RETURNDEF(EVENTS) RETURNS DEFINITION

 49 AVRET AVERAGE RETURNS

 50 INVESTAV INVESTMENT FUNDS AVAILABLE

 51 DEVIATION(EVENTS) DEVIATIONS FROM MEAN INCOME ;

 53 OBJJ..

 54 OBJ =E= MEAN

 55 - RAP\*(SUM(EVENTS,(POSDEV(EVENTS)+NEGDEV(EVENTS))\*\*2)/CARD(EVENTS));

 56

 57 INVESTAV.. SUM(STOCKS, PRICES(STOCKS) \* INVEST(STOCKS)) =L= FUNDS ;

 58

 59 RETURNDEF(EVENTS)..SUM(STOCKS, RETURNS(EVENTS,STOCKS) \* INVEST(STOCKS))

 60 - RETURN(EVENTS) =E= 0 ;

 61

 62 AVRET.. SUM(EVENTS,1/CARD(EVENTS)\*RETURN(EVENTS)) - MEAN=E= 0 ;

 63

 64 DEVIATION(EVENTS)..RETURN(EVENTS)-MEAN -POSDEV(EVENTS) + NEGDEV(EVENTS)

 65 =E= 0 ;

 67 MODEL EVPORTFOL /ALL/ ;

 68

 69 SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;

Risk Modeling and Stochastic Programming

 Motad Model Development

Formally, the total absolute deviation of income from mean income under the kth state of nature (Dk) is



which can be rewritten as



Total absolute deviation (TAD) is the sum of Dk across the states of nature. Now introducing deviation variables to depict positive and negative deviations we get



The final MOTAD formulation is



Risk Modeling and Stochastic Programming

 Motad Model Development

Model considering only negative deviations from the mean



the standard error of a normally distributed population can be estimated given sample size N, by multiplying mean absolute deviation (MAD), total absolute deviation (TAD), or total negative deviation (TND) by appropriate constraints. Thus,



This transformation is commonly used in MOTAD formulations such as:



Risk Modeling and Stochastic Programming

 Motad Example

This example uses the same data as in the E-V Portfolio example.

|  |
| --- |
| **Table 14.1. Data for E-V Example -- Returns by Stock and Event** |
|  | ----Stock Returns by Stock and Event---- |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Event1 | 7  | 6  | 8  | 5  |
| Event2 | 8  | 4  | 16  | 6  |
| Event3 | 4  | 8  | 14  | 6  |
| Event4 | 5  | 9  | ‑2  | 7  |
| Event5 | 6  | 7  | 13  | 6  |
| Event6 | 3  | 10  | 11  | 5  |
| Event7 | 2  | 12  | ‑2  | 6  |
| Event8 | 5  | 4  | 18  | 6  |
| Event9 | 4  | 7  | 12  | 5  |
| Event10 | 3  | 9  | ‑5  | 6  |
|  |  |  |  |  |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Price | 22 | 30 | 28 | 26 |

|  |
| --- |
| **Table 14.5. Deviations from the Mean for Portfolio Example** |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Event1 | 2.3  | ‑1.6  | ‑0.3  | ‑0.8  |
| Event2 | 3.3  | ‑3.6  | 7.7  | 0.2  |
| Event3 | ‑0.7  | 0.4  | 5.7  | 0.2  |
| Event4 | 0.3  | 1.4  | ‑10.3  | 1.2  |
| Event5 | 1.3  | ‑0.6  | 4.7  | 0.2  |
| Event6 | ‑1.7  | 2.4  | 2.7  | ‑0.8  |
| Event7 | ‑2.7  | 4.4  | ‑10.3  | 0.2  |
| Event8 | 0.3  | ‑3.6  | 9.7  | 0.2  |
| Event9 | ‑0.7  | ‑0.6  | 3.7  | ‑0.8  |
| Event10 | ‑1.7  | 1.4  | ‑13.3  | 0.2  |

Risk Modeling and Stochastic Programming

 Motad Example

**Table 14.6. Example MOTAD Model Formulation**

|  |
| --- |
|  |



Risk Modeling and Stochastic Programming

Motad Example GAMS Formulation

 10 SETS STOCKS POTENTIAL INVESTMENTS / BUYSTOCK1\*BUYSTOCK4 /

 11 EVENTS EQUALLY LIKELY RETURN STATES OF NATURE

 12 /EVENT1\*EVENT10 / ;

 14 PARAMETERS PRICES(STOCKS) PURCHASE PRICES OF THE STOCKS

 15 / BUYSTOCK1 22

 16 BUYSTOCK2 30

 17 BUYSTOCK3 28

 18 BUYSTOCK4 26 / ;

 20 SCALAR FUNDS TOTAL INVESTABLE FUNDS / 500 /

 21 N SAMPLE SIZE

 22 PI /3.141716/

 23 TRAN TRANSFORMATION COEF MAD TO STD ERROR ;

 25 N=CARD(EVENTS);

 26 TRAN = ((PI \* N)/(2\*(N-1)))\*\*0.5 ;

 28 TABLE RETURNS(EVENTS,STOCKS) RETURNS BY OBSERVATION

 30 BUYSTOCK1 BUYSTOCK2 BUYSTOCK3 BUYSTOCK4

 31 EVENT1 7 6 8 5

 32 EVENT2 8 4 16 6

 33 EVENT3 4 8 14 6

 34 EVENT4 5 9 -2 7

 35 EVENT5 6 7 13 6

 36 EVENT6 3 10 11 5

 37 EVENT7 2 12 -2 6

 38 EVENT8 5 4 18 6

 39 EVENT9 4 7 12 5

 40 EVENT10 3 9 -5 6

 42 PARAMETERS

 43 MEAN (STOCKS) MEAN RETURNS TO X(STOCKS)

 44 DEVS(EVENTS,STOCKS) DEVIATIONS FROM MEAN INCOME ;

 46 MEAN(STOCKS) = SUM(EVENTS , RETURNS(EVENTS,STOCKS) / N );

 48 DEVS(EVENTS,STOCKS) = RETURNS(EVENTS,STOCKS) - MEAN(STOCKS) ;

 50 DISPLAY MEAN , DEVS ;

 52 SCALAR RAP RISK AVERSION PARAMETER / 0.0 / ;

 54 DISPLAY TRAN ;

 56 POSITIVE VARIABLES INVEST(STOCKS) MONEY INVESTED IN EACH STOCK

 57 DEVIATION(EVENTS) DEVIATION OF TOTAL INCOME BY EVENT

 58 TRETURN TOTAL RETURNS

 59 APPROXSTDE STANDARD ERROR AS APPROXIMATED

 60 MAD MEAN ABSOLUTE DEVIATION

 62 VARIABLE OBJ NUMBER TO BE MAXIMIZED ;

 64 EQUATIONS OBJJ OBJECTIVE FUNCTION

 65 INVESTAV INVESTMENT FUNDS AVAILABLE

 66 DEVIATE(EVENTS) DEVIATION EQUATION FOR EVENTS

 67 MADBALANCE MEAN ABSOLUTE DEVIATION DEFINITION

 68 SEBALANCE STANDARD DEVIATION APPROXIMATION

 71 OBJJ.. OBJ =E= SUM(STOCKS, MEAN(STOCKS) \* INVEST(STOCKS))

 72 - RAP\*APPROXSTDE;

 74 INVESTAV.. SUM(STOCKS, PRICES(STOCKS) \* INVEST(STOCKS)) =L= FUNDS ;

 76 DEVIATE(EVENTS).. SUM(STOCKS, DEVS(EVENTS,STOCKS)\*INVEST(STOCKS))

 77 + DEVIATION(EVENTS) =G= 0. ;

 79 MADBALANCE.. 2\*SUM(EVENTS, DEVIATION(EVENTS))/N - MAD =E= 0. ;

 81 SEBALANCE.. TRAN\*MAD - APPROXSTDE =E= 0. ;

 83 MODEL MOTADPORTF /ALL/ ;

 85 SOLVE MOTADPORTF USING LP MAXIMIZING OBJ ;

Risk Modeling and Stochastic Programming

 Motad Example

###### Table 14.7. MOTAD Example Solutions for Alternative Risk Aversion Parameters

 **RAP 0.050 0.100 0.110 0.120**

 BUYSTOCK2 11.603

 BUYSTOCK3 17.857 17.857 17.857 17.857 5.425

 OBJ 148.214 140.146 132.078 130.464 129.390

 MEAN 148.214 148.214 148.214 148.214 133.213

 MAD 122.143 122.143 122.143 122.143 24.111

 STDAPPROX 161.367 161.367 161.367 161.367 31.854

 VAR 19709.821 19709.821 19709.821 19709.821 883.113

 STD 140.392 140.392 140.392 140.392 29.717

 SHADPRICE 0.296 0.280 0.264 0.261 0.259

 **RAP 0.130 0.150 0.260 0.400 0.500**

 BUYSTOCK1 2.663

 BUYSTOCK2 11.603 11.603 11.916 12.379 10.985

 BUYSTOCK3 5.425 5.425 5.090 4.594 3.995

 OBJ 129.072 128.435 125.179 121.204 118.606

 MEAN 133.213 133.213 132.809 132.210 129.161

 MAD 24.111 24.111 22.212 20.827 15.979

 STDAPPROX 31.854 31.854 29.345 27.515 21.110

 VAR 883.113 883.113 771.228 643.507 455.983

 STD 29.717 29.717 27.771 25.367 21.354

 SHADPRICE 0.258 0.257 0.250 0.242 0.237

 **RAP 0.750 1.000 1.250 1.500 1.750**

 BUYSTOCK1 5.145 7.119 2.817 2.817 2.817

 BUYSTOCK2 10.409 9.879 5.617 5.617 5.617

 BUYSTOCK3 2.661 1.564 1.824 1.824 1.824

 BUYSTOCK4 0.123 8.402 8.402 8.402

 OBJ 114.168 111.009 108.372 106.086 103.801

 MEAN 125.384 122.240 119.799 119.799 119.799

 MAD 11.320 8.501 6.920 6.920 6.920

 STDAPPROX 14.955 11.231 9.142 9.142 9.142

 VAR 211.996 121.386 83.886 83.886 83.886

 STD 14.560 11.018 9.159 9.159 9.159

 SHADPRICE 0.228 0.222 0.217 0.212 0.208

 **RAP 2.000 2.500 5.000 10.000 12.500**

 BUYSTOCK1 2.817 2.817 2.858 2.858 2.858

 BUYSTOCK2 5.617 5.617 4.178 4.178 4.178

 BUYSTOCK3 1.824 1.824 1.242 1.242 1.242

 BUYSTOCK4 8.402 8.402 10.654 10.654 10.654

 OBJ 101.515 96.944 76.540 35.790 15.415

 MEAN 119.799 119.799 117.289 117.289 117.289

 MAD 6.920 6.920 6.169 6.169 6.169

 STDAPPROX 9.142 9.142 8.150 8.150 8.150

 VAR 83.886 83.886 57.695 57.695 57.695

 STD 9.159 9.159 7.596 7.596 7.596

 SHADPRICE 0.203 0.194 0.153 0.072 0.031

Risk Modeling and Stochastic Programming

Motad Example



Risk Modeling and Stochastic Programming

E-V – Nasty Assumptions

**Form of Distribution**

 **Normality**

 **Location and Scale**

**Form of Utility**

 **Exponential**

 **Taylor Series Approximation**

Risk Modeling and Stochastic Programming

Motad – Finding a Risk Aversion Parameter

The link between the E-standard error and E‑V risk aversion

parameters is as follows:

Consider the models



The first order conditions assuming X is nonzero are



For these two solutions to be identical in terms of X and , then



Risk Modeling and Stochastic Programming

Safety First

The Safety First model assumes that decision makers will choose plans to first assure a given safety level for income.



 The overall problem then becomes



Risk Modeling and Stochastic Programming

Safety First – Example Formulation

**Table 14.8. Example Formulation of Safety First Problem**

|  |
| --- |
|  |



|  |
| --- |
| Solutions |

**Table 14.9. Safety First Example Solutions for Alternative Safety Levels**

|  |
| --- |
|  |

RUIN ‑100.000 ‑50.000 0.0 25.000 50.000

BUYSTOCK2 0.0 2.736 6.219 7.960 9.701

BUYSTOCK3 17.857 14.925 11.194 9.328 7.463

OBJ 148.214 144.677 140.174 137.923 135.672

MEAN 148.214 144.677 140.174 137.923 135.672

VAR 19709.821 12695.542 6066.388 3717.016 2011.116

STD 140.392 112.674 77.887 60.967 44.845

SHADPRICE 0.296 0.280 0.280 0.280 0.280

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Note: The abbreviations are the same as in the previous example solutions with RUIN giving the safety level.

Safety First – GAMS Formulation

 12 SETS STOCKS POTENTIAL INVESTMENTS / BUYSTOCK1\*BUYSTOCK4 /

 13 EVENTS EQUALLY LIKELY RETURN STATES OF NATURE/EVENT1\*EVENT10 / ;

 16 PARAMETERS PRICES(STOCKS) PURCHASE PRICES OF THE STOCKS

 17 / BUYSTOCK1 22, BUYSTOCK2 30

 19 BUYSTOCK3 28, BUYSTOCK4 26 / ;

 22 SCALAR FUNDS TOTAL INVESTABLE FUNDS / 500 / ;

 24 TABLE RETURNS(EVENTS,STOCKS) RETURNS BY STATE OF NATURE EVENT

 26 BUYSTOCK1 BUYSTOCK2 BUYSTOCK3 BUYSTOCK4

 27 EVENT1 7 6 8 5

 28 EVENT2 8 4 16 6

 29 EVENT3 4 8 14 6

 30 EVENT4 5 9 -2 7

 31 EVENT5 6 7 13 6

 32 EVENT6 3 10 11 5

 33 EVENT7 2 12 -2 6

 34 EVENT8 5 4 18 6

 35 EVENT9 4 7 12 5

 36 EVENT10 3 9 -5 6

 39 SCALAR RUIN LEVEL OF SAFETY

 40 N SAMPLE SIZE;

 41 N = CARD(EVENTS) ;

 43 PARAMETER STANDERROR(STOCKS) STANDARD ERROR OF STOCKS

 44 AVRET(STOCKS) AVERAGE RETURN BY STOCK ;

 45 AVRET(STOCKS) = SUM(EVENTS,RETURNS(EVENTS,STOCKS))/N;

 46 STANDERROR(STOCKS) = SQRT(SUM(EVENTS,

 47 (RETURNS(EVENTS,STOCKS)-AVRET(STOCKS))\*

 48 (RETURNS(EVENTS,STOCKS)-AVRET(STOCKS))/(N-1)));

 49 POSITIVE VARIABLES INVEST(STOCKS) MONEY INVESTED IN EACH STOCK

 51 ART ARTIFICIAL VARIABLE

 52 VARIABLES OBJ NUMBER TO BE MAXIMIZED

 54 MEAN MEAN INCOME

 55 EQUATIONS OBJJ OBJECTIVE FUNCTION

 57 AVRETDEF AVERAGE RETURNS DEFINITION

 58 INVESTAV INVESTMENT FUNDS AVAILABLE

 59 SAFETY(EVENTS) SAFETY EQUATION ;

 60 OBJJ.. OBJ =E= MEAN -99999\* ART;

 62 AVRETDEF.. MEAN =E= SUM(STOCKS, AVRET(STOCKS) \* INVEST(STOCKS));

 65 INVESTAV.. SUM(STOCKS, PRICES(STOCKS) \* INVEST(STOCKS)) =L=FUNDS ;

 67 SAFETY(EVENTS).. SUM(STOCKS,RETURNS(EVENTS,STOCKS)\*INVEST(STOCKS))

 68 + ART =G= RUIN;

 70 MODEL SAFETYPORT /ALL/ ;

 72 SET RUNS ALTERNATIVE RUIN LEVELS /R0\*R6/

 74 PARAMETER RUINLEVEL(RUNS) SAFETY LEVELS

 75 /R0 -100, R1 -50, R2 0, R3 25,

 76 R4 50, R5 75 , R6 100/

 78 PARAMETER OUTPUT(\*,RUNS) RESULTS FROM MODEL RUNS WITH VARYING RUIN LEVEL

 79 RETURN(EVENTS) RETURN BY EVENTS

 80 VAR VARIANCE;

 84 LOOP (RUNS,RUIN=RUINLEVEL(RUNS);

 86 SOLVE SAFETYPORT USING LP MAXIMIZING OBJ ;

 88 RETURN(EVENTS) =SUM(STOCKS, RETURNS(EVENTS,STOCKS)\*INVEST.L(STOCKS));

 90 VAR = SUM(EVENTS,(RETURN(EVENTS)-MEAN.L)\*(RETURN(EVENTS)-MEAN.L))/N;

 92 OUTPUT("RUIN",RUNS)=RUIN;

 93 OUTPUT(STOCKS,RUNS)=INVEST.L(STOCKS);

 94 OUTPUT("OBJ",RUNS)=OBJ.L;

 95 OUTPUT("MEAN",RUNS)=MEAN.L;

 96 OUTPUT("VAR",RUNS) = VAR;

 97 OUTPUT("STD",RUNS)=SQRT(VAR);

 98 OUTPUT("SHADPRICE",RUNS)=INVESTAV.M;

 99 OUTPUT("IDLE",RUNS)=FUNDS-INVESTAV.L);

Risk Modeling and Stochastic Programming

Target Motad

The Target MOTAD formulation incorporates a safety level of income while also allowing negative deviations from that safety level.



**Table 14.10. Example Formulation of Target MOTAD Problem**

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Risk Modeling and Stochastic Programming

Target Motad

**Table 14.11. Target MOTAD Example Solutions for Alternative Deviation Limits**

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 **TARGETDEV 120.000 60.000 24.000 12.000 10.800**

 BUYSTOCK2 0.0 0.0 7.081 10.193 10.516

 BUYSTOCK3 17.857 17.857 10.270 6.936 6.590

 OBJ 148.214 148.214 139.059 135.037 134.618

 MEAN 148.214 148.214 139.059 135.037 134.618

 VAR 19709.821 19709.821 4822.705 1646.270 1433.820

 STD 140.392 140.392 69.446 40.574 37.866

 SHADPRICE 0.296 0.296 0.286 0.295 0.295

 **TARGETDEV 8.400 7.200 3.600**

 BUYSTOCK1 0.0 0.0 3.459

 BUYSTOCK2 11.259 11.782 11.405

 BUYSTOCK3 5.794 5.234 2.919

 OBJ 133.659 132.982 127.168

 MEAN 133.659 132.982 127.168

 VAR 1030.649 816.629 277.270

 STD 32.104 28.577 16.651

 SHADPRICE 0.298 0.298 0.815

|  |
| --- |
|  |

Note: The abbreviations are again the same with TARGETDEV giving the  value.

Risk Modeling and Stochastic Programming

DEMP Model

Direct Expected Maximizing Nonlinear Programming



where pk is the probability of the kth state of nature;

Wo is initial wealth;

 Wk is the wealth under the kth state of nature; and

 ckj is the return to one unit of the jth activity under the kth

state of nature.

Risk Modeling and Stochastic Programming

DEMP Model – Example

###### Table 14.12. Example Formulation of DEMP Problem

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Risk Modeling and Stochastic Programming

DEMP Model – Example

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| **Table 14.13. DEMP Example Solutions for Alternative Utility Function Exponents** |

 POWER 0.950 0.900 0.750 0.500 0.400

 BUYSTOCK2 4.560 8.563 9.344

 BUYSTOCK3 17.857 17.857 12.972 8.683 7.846

 OBJ 186.473 140.169 60.363 15.282 8.848

 MEAN 248.214 248.214 242.319 237.144 236.134

 VAR 19709.821 19709.821 8903.295 3054.034 2309.233

 STD 140.392 140.392 94.357 55.263 48.054

 SHADPRICE 0.287 0.277 0.269 0.266 0.265

 POWER 0.300 0.200 0.100 0.050 0.030

 BUYSTOCK2 9.919 10.358 10.705 10.852 10.907

 BUYSTOCK3 7.230 6.759 6.388 6.230 6.171

 OBJ 5.127 2.972 1.724 1.313 1.177

 MEAN 235.390 234.822 234.374 234.184 234.113

 VAR 1843.171 1534.736 1320.345 1236.951 1207.076

 STD 42.932 39.176 36.337 35.170 34.743

 SHADPRICE 0.264 0.264 0.263 0.263 0.263

 POWER 0.020 0.010 0.001 0.0001

 BUYSTOCK2 10.934 10.960 10.960 10.960

 BUYSTOCK3 6.143 6.115 6.115 6.115

 OBJ 1.115 1.056 1.005 1.001

 MEAN 234.079 234.045 234.045 234.045

 VAR 1192.805 1178.961 1178.961 1178.961

 STD 34.537 34.336 34.336 34.336

 SHADPRICE 0.263 0.263 0.263 0

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|  |

DEMP Model – GAMS Formulation

 12 SETS STOCKS POTENTIAL INVESTMENTS / BUYSTOCK1\*BUYSTOCK4 /

 13 EVENTS EQUALLY LIKELY RETURN STATES OF NATURE

 14 /EVENT1\*EVENT10 / ;

 16 PARAMETERS PRICES(STOCKS) PURCHASE PRICES OF THE STOCKS

 17 / BUYSTOCK1 22

 18 BUYSTOCK2 30

 19 BUYSTOCK3 28

 20 BUYSTOCK4 26 / ;

 22 SCALAR FUNDS TOTAL INVESTABLE FUNDS / 500 / ;

 24 TABLE RETURNS(EVENTS,STOCKS) RETURNS BY STATE OF NATURE EVENT

 26 BUYSTOCK1 BUYSTOCK2 BUYSTOCK3 BUYSTOCK4

 27 EVENT1 7 6 8 5

 28 EVENT2 8 4 16 6

 29 EVENT3 4 8 14 6

 30 EVENT4 5 9 -2 7

 31 EVENT5 6 7 13 6

 32 EVENT6 3 10 11 5

 33 EVENT7 2 12 -2 6

 34 EVENT8 5 4 18 6

 35 EVENT9 4 7 12 5

 36 EVENT10 3 9 -5 6

 39 SCALAR INITWEALTH INITIAL WEALTH /100/

 40 POWER EXPONENT IN UTILITY FUNCTION /0.5/;

 42 POSITIVE VARIABLES INVEST(STOCKS) MONEY INVESTED IN EACH STOCK

 44 VARIABLES OBJ NUMBER TO BE MAXIMIZED

 45 WEALTH(EVENTS) RETURNS BY EVENT

 46 MEAN MEAN RETURNS ;

 48 EQUATIONS OBJJ OBJECTIVE FUNCTION

 49 WEALTHDEF(EVENTS) WEALTHS DEFINITION

 50 AVWEALTH AVERAGE WEALTH

 51 INVESTAV INVESTMENT FUNDS AVAILABLE;

 53 OBJJ.. OBJ =E= SUM(EVENTS,(WEALTH(EVENTS)\*\*POWER)/CARD(EVENTS));

 55 INVESTAV.. SUM(STOCKS, PRICES(STOCKS) \* INVEST(STOCKS)) =L= FUNDS ;

 57 WEALTHDEF(EVENTS).. WEALTH(EVENTS)

 58 - SUM(STOCKS, RETURNS(EVENTS,STOCKS) \* INVEST(STOCKS)) =E=INITWEALTH ;

 60 AVWEALTH.. MEAN =E= SUM(EVENTS,1/CARD(EVENTS)\*WEALTH(EVENTS));

 63 MODEL EVPORTFOL /ALL/ ;

 65 SET POWERS UTILITY FUNCTION POWER PARAMETERS /R0\*R13/

 67 PARAMETER POWERSET(POWERS) RISK AVERSION COEF BY RISK AVERSION PARAMETER

 68 /R0 0.95 , R1 0.9 , R2 0.75, R3 0.5 ,

 69 R4 0.4 , R5 0.3 , R6 0.2 , R7 0.1 ,

 70 R8 0.05 , R9 0.03 , R10 0.02, R11 0.01,

 71 R12 0.001, R13 0.0001/

 73 PARAMETER VAR VARIANCE OF RETURNS

 75 PARAMETER OUTPUT(\*,POWERS) RESULTS FROM MODEL RUNS WITH VARYING EXPONENT

 79 LOOP (POWERS,POWER=POWERSET(POWERS);

 80 SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;

 81 VAR = SUM(EVENTS,(WEALTH.L(EVENTS)-MEAN.L)\*(WEALTH.L(EVENTS) -MEAN.L))

 82 /CARD(EVENTS);

 83 OUTPUT("OBJ",POWERS)=OBJ.L;

 84 OUTPUT("POWER",POWERS)=POWER;

 85 OUTPUT(STOCKS,POWERS)=INVEST.L(STOCKS);

 86 OUTPUT("MEAN",POWERS)=MEAN.L;

 87 OUTPUT("VAR",POWERS) = VAR;

 88 OUTPUT("STD",POWERS)=SQRT(VAR);

 89 OUTPUT("SHADPRICE",POWERS)$(MEAN.L GT 0.001)=

 90 INVESTAV.M/(power\*mean.l\*\*(power-1));

 91 OUTPUT("IDLE",POWERS)=FUNDS-INVESTAV.L

 92 );

Risk Modeling and Stochastic Programming

Chance Constrained Programming

The probability of a constraint being satisfied is greater than or equal to a prespecified value 



If the average value of the RHS is subtracted from both sides of the inequality and in turn both sides are divided by the standard deviation of the RHS then the constraint becomes



Those familiar with probability theory will note that the term



gives the number of standard errors that bi is away from the mean. Let Z denote this term. When a particular probability limit () is used, then the appropriate value of Z is Z and the constraint becomes



Assuming we discount for risk, then the constraint can be restated as



Risk Modeling and Stochastic Programming

Chance Constrained Programming

**Table 14.14. Chance Constrained Example Data**

|  |  |  |  |
| --- | --- | --- | --- |
| Event | Small Lathe | Large Lathe | Carver |
| 1 | 140 | 90 | 120 |
| 2 | 120 | 94 | 132 |
| 3 | 133 | 88 | 110 |
| 4 | 154 | 97 | 118 |
| 5 | 133 | 87 | 133 |
| 6 | 142 | 86 | 107 |
| 7 | 155 | 90 | 120 |
| 8 | 140 | 94 | 114 |
| 9 | 142 | 89 | 123 |
| 10 | 141 | 85 | 123 |
| Mean | 140 | 90 | 120 |
| Standard Error | 9.63 | 3.69 | 8.00 |

Then the resultant chance constrained formulation is

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | 67X1 | + | 66X2 | + | 66.3X3 | + | 80X4 | + | 78.5X5 | + | 78.4X6 |  |  |
| s.t | 0.8X1 | + | 1.3X2 | + | 0.2X3 | + | 1.2X4 | + | 1.7X5 | + | 0.5X6 |  | 140 - 9.63 Z |
|  | 0.5X1 | + | 0.2X2 | + | 1.3X3 | + | 0.7X4 | + | 0.3X5 | + | 1.5X6 |  | 90 - 3.69 Z |
|  | 0.4X1 | + | 0.4X2 | + | 0.4X3 | + | X4 | + | X5 | + | X6 |  | 120 - 8.00 Z |
|  | X1 | + | 1.05X2 | + | 1.1X3 | + | 0.8X4 | + | 0.82X5 | + | 0.84X6 |  |  125 |

Risk Modeling and Stochastic Programming

Chance Constrained Programming

**Table 14.15. Chance Constrained Example Solutions for Alternative Alpha Levels**

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**Z 0.00 1.280 1.654 2.330**

PROFIT 10417.291 9884.611 9728.969 9447.647

SMLLATHE 140.000 127.669 124.067 117.554

LRGLATHE 90.000 85.280 83.900 81.407

CARVER 120.000 109.760 106.768 101.360

LABOR 125.000 125.000 125.000 125.000

FUNCTNORM 62.233 78.102 82.739 91.120

FANCYNORM 73.020 51.495 45.205 33.837

FANCYMXLRG 5.180 6.788 7.258 8.108

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Note: Z is the risk aversion parameter.

Chance Constrained Programming

 12 SET PROCESS TYPES OF PRODUCTION PROCESSES

 13 /FUNCTNORM , FUNCTMXSML , FUNCTMXLRG

 14 ,FANCYNORM , FANCYMXSML , FANCYMXLRG/

 15 RESOURCE TYPES OF RESOURCES

 16 /SMLLATHE,LRGLATHE,CARVER,LABOR/

 17 EVENT STATES OF NATURE /S1\*S10/;

 19 PARAMETER PRICE(PROCESS) PRODUCT PRICES BY PROCESS

 20 /FUNCTNORM 82, FUNCTMXSML 82, FUNCTMXLRG 82

 21 ,FANCYNORM 105, FANCYMXSML 105, FANCYMXLRG 105/

 22 PRODCOST(PROCESS) COST BY PROCESS

 23 /FUNCTNORM 15, FUNCTMXSML 16 , FUNCTMXLRG 15.7

 24 ,FANCYNORM 25, FANCYMXSML 26.5, FANCYMXLRG 26.6/

 26 TABLE RESORAVAIL(RESOURCE,EVENT) RESOURCE AVAILABLITY

 28 S1 S2 S3 S4 S5 S6 S7 S8 S9 S10

 29 SMLLATHE 140 120 133 154 133 142 155 140 142 141

 30 LRGLATHE 90 94 88 97 87 86 90 94 89 85

 31 CARVER 120 132 110 118 133 107 120 114 123 123

 32 LABOR 125 125 125 125 125 125 125 125 125 125

 34 TABLE RESOURUSE(RESOURCE,PROCESS) RESOURCE USAGE

 36 FUNCTNORM FUNCTMXSML FUNCTMXLRG

 37 SMLLATHE 0.80 1.30 0.20

 38 LRGLATHE 0.50 0.20 1.30

 39 CARVER 0.40 0.40 0.40

 40 LABOR 1.00 1.05 1.10

 41 + FANCYNORM FANCYMXSML FANCYMXLRG

 42 SMLLATHE 1.20 1.70 0.50

 43 LRGLATHE 0.70 0.30 1.50

 44 CARVER 1.00 1.00 1.00

 45 LABOR 0.80 0.82 0.84;

 47 PARAMETER MEANAVAIL(RESOURCE) MEAN AMOUNT OF RESOURCE AVAILABILITY

 48 STDERROR (RESOURCE) STANDARD ERROR OF RESOURCE AVAILABILITY;

 50 SCALAR ZALPHA CHANCE CONSTRAINT DISCOUNT FACTOR/1.96/;

 52 MEANAVAIL(RESOURCE) = SUM(EVENT,RESORAVAIL(RESOURCE,EVENT))/CARD(EVENT);

 53 STDERROR(RESOURCE) = SQRT(SUM(EVENT,

 54 SQR(RESORAVAIL(RESOURCE,EVENT)-MEANAVAIL(RESOURCE)))/CARD(EVENT));

 58 POSITIVE VARIABLES PRODUCTION(PROCESS) ITEMS PRODUCED BY PROCESS;

 61 VARIABLES PROFIT TOTALPROFIT;

 64 EQUATIONS OBJT OBJECTIVE FUNCTION ( PROFIT )

 66 AVAILABLE(RESOURCE) RESOURCES AVAILABLE ;

 68 OBJT.. PROFIT =E=

 69 SUM(PROCESS,(PRICE(PROCESS)-PRODCOST(PROCESS))

 70 \* PRODUCTION(PROCESS)) ;

 72 AVAILABLE(RESOURCE)..

 73 SUM(PROCESS,RESOURUSE(RESOURCE,PROCESS)\*PRODUCTION(PROCESS))

 74 =L= MEANAVAIL(RESOURCE)-ZALPHA\*STDERROR(RESOURCE);

 76 MODEL CHANCE /ALL/;

 78 SET ZALPH ALTERNATIVE Z VALUE IDENTIFIERS /Z1\*Z4/

 80 PARAMETER OUTPUT(\*,ZALPH) RESULTS FROM MODEL RUNS WITH VARYING RAP

 81 ZALPHS(ZALPH) ALTERNATIVE Z ALPHA VALUES

 82 /Z1 0.0, Z2 1.28, Z3 1.654 , Z4 2.33/

 86 LOOP (ZALPH,ZALPHA=ZALPHS(ZALPH);

 87 SOLVE CHANCE USING LP MAXIMIZING PROFIT ;

 88 OUTPUT(RESOURCE,ZALPH)=MEANAVAIL(RESOURCE)-ZALPHA\*STDERROR(RESOURCE);

 89 OUTPUT("PROFIT",ZALPH)=PROFIT.L;

 90 OUTPUT("RAP",ZALPH)=ZALPHA;

 91 OUTPUT(PROCESS,ZALPH)=PRODUCTION.L(PROCESS));

 93 DISPLAY OUTPUT;

Risk Modeling and Stochastic Programming

Technical Coefficient Risk

Merrill’s Approach

a constraint containing uncertain coefficients can be rewritten as



or, using standard deviation,



Wicks-Guise

given that the ith constraint contains uncertain aij's, the following constraints may be set up.



The general Wicks Guise formulation is



Wicks-Guise Example

**Table 14.16. Feed Nutrients by State of Nature for Wicks Guise Example**

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|  |

 Nutrient State CORN SOYBEANS WHEAT

ENERGY S1 1.15 0.26 1.05

ENERGY S2 1.10 0.31 0.95

ENERGY S3 1.25 0.23 1.08

ENERGY S4 1.18 0.28 1.12

PROTEIN S1 0.23 1.12 0.51

PROTEIN S2 0.17 1.08 0.59

PROTEIN S3 0.25 1.01 0.46

PROTEIN S4 0.15 0.99 0.56

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**Table 14.17. Wicks Guise Example**

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Note: EnDev is the energy deviation

 EnMAD is the energy mean absolute deviation

 En is the energy standard deviation approximations

 PrDev is the protein deviation

 PrMAD is the protein mean absolute deviation

 Pr is the protein standard deviation approximation

Risk Modeling and Stochastic Programming

Wicks-Guise Example Solution

**Table 14.18. Results From Example Wicks Guise Model Runs With Varying RAP**

|  |
| --- |
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 **RAP 0.250 0.500 0.750 1.000**

 CORN 0.091 0.046 0.211 0.230 0.221

 SOYBEANS 0.105 0.129 0.137

 WHEAT 0.909 0.954 0.684 0.641 0.642

 OBJ 0.039 0.040 0.040 0.040 0.041

 AVGPROTEIN 0.500 0.515 0.515 0.521 0.529

 STDPROTEIN 0.054 0.059 0.030 0.028 0.029

 AVGENERGY 1.061 1.056 0.993 0.977 0.969

 STDENERGY 0.072 0.072 0.061 0.059 0.058

 SHADPROT 0.030 0.033 0.036 0.037 0.038

 **RAP 1.250 1.500 2.000**

 CORN 0.211 0.200 0.177

 SOYBEANS 0.146 0.156 0.176

 WHEAT 0.643 0.644 0.647

 OBJ 0.041 0.041 0.042

 AVGPROTEIN 0.536 0.545 0.563

 STDPROTEIN 0.029 0.030 0.031

 AVGENERGY 0.961 0.953 0.934

 STDENERGY 0.057 0.056 0.055

 SHADPROT 0.039 0.040 0.042

|  |
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Note: RAP gives the risk aversion parameter used

 CORN gives the amount of corn used in the solution

 SOYBEANS gives the amount of soybeans used in the solution

 WHEAT gives the amount of wheat used in the solution

 OBJ gives the objective function value

 AVGPROTEIN gives the average amount of protein in the diet

 STDPROTEIN gives the standard error of protein in the diet

 AVGENERGY gives the average amount of energy in the diet

 STDENERGY gives the standard error of energy in the diet

 SHADPROT gives the shadow price on the protein requirement constraint

1. One could also use the divisor N-1 when working with a sample. [↑](#footnote-ref-2)