Risk Modeling and Stochastic Programming

**Where does it occur?**

**Maximize CX**

**Subject to AX  b**

**X  0**

## **Uncertain objective function returns - C**

**variability in prices**

**variability in production quantities**

**variability in costs**

**variability in market sales**

## **Resource usages - A**

**variability in raw input quality**

**variability in working conditions**

**variability in intermediate product yields**

**variability in product requirements**

## **Resource endowments - b**

**variability in demand firm faces**

**variability in resources available**

**variability in working conditions**

Risk Modeling and Stochastic Programming

**Including Risk**

**When incorporating risk there are three big issues**

**1.    What is the nature of risk?**

**a.   What parameters of the model are**

**uncertain?**

1. **How do we describe their distribution?**
2. **When are risk outcomes revealed?**

**Does the producer receive information about**

**uncertain events and will make adaptive**

**decisions?**

1. **How do we model behavioral reaction to**

**risk?**

**Is expected profit maximization not the proper objective but rather some degree of aversion to the variation caused by risk?**

Risk Modeling and Stochastic Programming

Why Model Risk

Why not just solve for all values of risky parameters

# Curses of dimensionality and certainty

­Dimensionality: Number of possible plans

(5 possible values for 3 parameters 35 = 243)

Certainty: Each plan would be certain of data so we would have 243 different things we could do

–What would we do?

General Risk Modeling Aim

Generate Robust in the face of the Uncertainty

Not necessarily a best performer in any setting, but a good performer across many or most or the most likely spectrum

Risk Modeling and Stochastic Programming

Forms of assumed reaction to risk

Non Recourse or non adaptive decision making

* Decisions made now consequence felt later
* No decisions made between now and when

consequences felt

Example–Buy stock now make no decision for one year

## Recourse or adaptive decision making

Later time during model additional decisions made.

In this later decision period

Decision maker knows what happened between first decision and now.

Decision makers cannot revise prior actions but can adjust current decisions ie current decisions can be employed to make adjustments in the face of realized events -- phenomena called irreversibility and recourse

Example – Buy stock now make review decisions quarterly possibly selling and buying other stocks

Including Non Adaptive Risk

**Decision Maker reaction to risk**

**Expected Value Maximization**

****

**Conservative - Fat or thin coefficients**

****

**Where **

****

****

**E- V**

**Maximize E(income) - RAP \* Variance(income)**

# **Expected utility**

**Maximize Sum(p,Probability(p)\*U[Wealth(p)])**

**S.T. Wealth(p)=InitWealth + Income(p) for all p**

**Income(p)=C(p)\*X for all p**

# **Safety First based**

**Maximize Sum(p,Probability(p)\*Income(p))**

**S.T. Income(p)=C(p)\*X for all p**

**Income(p)safety for all p**

**Including Non Adaptive Risk**

**First Risk Model**

**Markowitz mean-variance portfolio choice formulation**

**Given Problem**

**Max sum(invest, moneyinvest(invest)\*avgreturn(invest))**

**s.t sum(invest,moneyinvest(invest)\*price(invest))  funds**

**Markowitz observed not all money in highest valued stock**

**Inconsistent with LP formulation**

**Why? Not a basic solution**

**Markowitz posed the hypothesis that average returns and the variance of returns were important**

Including Non Adaptive Risk

**EV Formulation – Statistical Background**

**Given a linear objective function**

****

**where X1, X2 are decision variables and c1 , c2 are uncertain parameters distributed with means and as well as variances s11 , s22, and covariance s12; then Z is distributed with mean**

****

**and variance **

**in terms of correlation**

**s12= ρ12 σ1σ2**

### **In matrix terms the mean and variance of Z are**

****

**where in the two by two case**



**Including Non Adaptive Risk**

**EV Formulation – Statistical Background**

## **Defining terms**

**sii is the variance of the objective function coefficient of Xi, which is calculated using the formula**

**sik =  (cik‑** **where cik is the kth observation on the objective value of Xi and N is the number of observations, assuming an equally likely probability (1/N) of occurrence.[[1]](#footnote-2)**

**sij for i ≠ j is the covariance of the objective function coefficients between ci and cj, calculated by the formula sij = ∑ (cik‑ )(cjk‑)/N. Note sij = sji.**

** is the mean value of the objective function coefficient ci, calculated by  cik/N. (Assuming an equally likely probability of occurrence.)**

**Including Non Adaptive Risk**

**E-V Model Commonly Used Formulation**

**Markowitz Formulation**

Min

s.t. E =K

or

**Freund Formulation**

****

**or Max E - RAP \* Variance**

**Why Use Later – reuse of RAP and Transferability**

**Commonly Used Freund Formulation**



**Where**

**E = expected value of risky c times choice of x**

**Var = sum over j of var (sjj) times the square of the x variables () minus sum over j and k where j of twice thecovariance(sjj) times xj times xk**

**= risk aversion parameter**

Risk Modeling and Stochastic Programming

EV Model – Example

Assume an investor wishes to develop a stock portfolio given the stock annual returns information shown in Table 14.1, 500 dollars to invest and prices of stock one $22.00, stock two $30.00, stock three $28.00 and stock four $26.00.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Table 14.1. Data for E-V Example -- Returns by Stock and Event** | | | | |
|  | ----Stock Returns by Stock and Event---- | | | |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Event1 | 7 | 6 | 8 | 5 |
| Event2 | 8 | 4 | 16 | 6 |
| Event3 | 4 | 8 | 14 | 6 |
| Event4 | 5 | 9 | ‑2 | 7 |
| Event5 | 6 | 7 | 13 | 6 |
| Event6 | 3 | 10 | 11 | 5 |
| Event7 | 2 | 12 | ‑2 | 6 |
| Event8 | 5 | 4 | 18 | 6 |
| Event9 | 4 | 7 | 12 | 5 |
| Event10 | 3 | 9 | ‑5 | 6 |
|  |  |  |  |  |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Price | 22 | 30 | 28 | 26 |

Risk Modeling and Stochastic Programming

EV Model – Example

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Table 14.2. Mean Returns and Variance Parameters for Stock Example | | | | |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Mean Returns | 4.70 | 7.60 | 8.30 | 5.80 |
|  |  |  |  |  |
| Variance-Covariance Matrix | |  |  |  |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Stock1 | 3.21 | -3.52 | 6.99 | 0.04 |
| Stock2 | -3.52 | 5.84 | -13.68 | 0.12 |
| Stock3 | 6.99 | -13.68 | 61.81 | -1.64 |
| Stock4 | 0.04 | 0.12 | -1.64 | 0.36 |

Risk Modeling and Stochastic Programming

EV Model – Example

**In turn the objective function is**

****

**or, in scalar notation**

****

**This objective function is maximized subject to a constraint on investable funds:**

****

**and non‑negativity conditions on the variables.**

GAMS Formulation

10 SETS STOCKS POTENTIAL INVESTMENTS / BUYSTOCK1\*BUYSTOCK4 /

11 EVENTS EQUALLY LIKELY RETURN STATES OF NATURE

12 /EVENT1\*EVENT10 / ;

14 ALIAS (STOCKS,STOCK);

16 PARAMETERS PRICES(STOCKS) PURCHASE PRICES OF THE STOCKS

17 / BUYSTOCK1 22, BUYSTOCK2 30

19 BUYSTOCK3 28, BUYSTOCK4 26 / ;

22 SCALAR FUNDS TOTAL INVESTABLE FUNDS / 500 / ;

24 TABLE RETURNS(EVENTS,STOCKS) RETURNS BY STATE OF NATURE EVENT

26 BUYSTOCK1 BUYSTOCK2 BUYSTOCK3 BUYSTOCK4

27 EVENT1 7 6 8 5

28 EVENT2 8 4 16 6

29 EVENT3 4 8 14 6

30 EVENT4 5 9 -2 7

31 EVENT5 6 7 13 6

32 EVENT6 3 10 11 5

33 EVENT7 2 12 -2 6

34 EVENT8 5 4 18 6

35 EVENT9 4 7 12 5

36 EVENT10 3 9 -5 6

38 PARAMETERS

39 MEAN (STOCKS) MEAN RETURNS TO X(STOCKS)

40 COVAR(STOCK,STOCKS) VARIANCE COVARIANCE MATRIX;

42 MEAN(STOCKS) = SUM(EVENTS , RETURNS(EVENTS,STOCKS) / CARD(EVENTS) );

43 COVAR(STOCK,STOCKS)

44 = SUM (EVENTS ,(RETURNS(EVENTS,STOCKS) - MEAN(STOCKS))

45 \*(RETURNS(EVENTS,STOCK)- MEAN(STOCK)))/CARD(EVENTS);

47 DISPLAY MEAN , COVAR ;

49 SCALAR RAP RISK AVERSION PARAMETER / 0.0 / ;

51 POSITIVE VARIABLES INVEST(STOCKS) MONEY INVESTED IN EACH STOCK

53 VARIABLE OBJ NUMBER TO BE MAXIMIZED ;

55 EQUATIONS OBJJ OBJECTIVE FUNCTION

56 INVESTAV INVESTMENT FUNDS AVAILABLE;

59 OBJJ..OBJ =E= SUM(STOCKS, MEAN(STOCKS) \* INVEST(STOCKS))

61 - RAP\*(SUM(STOCK, SUM(STOCKS,

62 INVEST(STOCK)\* COVAR(STOCK,STOCKS)\*invest(stocks));

64 INVESTAV.. SUM(STOCKS, PRICES(STOCKS) \* INVEST(STOCKS)) =L= FUNDS ;

66 MODEL EVPORTFOL /ALL/ ;

70 SCALAR VAR THE VARIANCE ;

75 SET RAPS RISK AVERSION PARAMETERS /R0\*R25/

77 PARAMETER RISKAVER(RAPS) RISK AVERSION COEFICIENT BY RISK AVERSION

78 /R0 0.00000, R1 0.00025, R2 0.00050, R3 0.00075,

79 R4 0.00100, R5 0.00150, R6 0.00200, R7 0.00300,

82 R20 5.00000, R21 10.0000, R22 15. , R23 20.

84 R24 40. , R25 80./

86 PARAMETER OUTPUT(\*,RAPS) RESULTS FROM MODEL RUNS WITH VARYING RAP

90 LOOP (RAPS,RAP=RISKAVER(RAPS);

91 SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;

92 VAR = SUM(STOCK, SUM(STOCKS,

93 INVEST.L(STOCK)\* COVAR(STOCK,STOCKS) \* INVEST.L(STOCKS)

94 OUTPUT("RAP",RAPS)=RAP;

95 OUTPUT(STOCKS,RAPS)=INVEST.L(STOCKS);

96 OUTPUT("OBJ",RAPS)=OBJ.L;

97 OUTPUT("MEAN",RAPS)=SUM(STOCKS, MEAN(STOCKS)\*invest.l(stock));

98 OUTPUT("VAR",RAPS) = VAR;

99 OUTPUT("STD",RAPS)=SQRT(VAR);

100 OUTPUT("SHADPRICE",RAPS)=INVESTAV.M;

101 OUTPUT("IDLE",RAPS)=FUNDS-INVESTAV.L); );

103 DISPLAY OUTPUT;

Risk Modeling and Stochastic Programming

EV Model – Example

**Table 14.4. E‑V Example Solutions for Alternative Risk Aversion Parameters**

**RAP or φ 0 0.00025 0.0005 0.00075 0.001**

BUYSTOCK2 1.263 5.324 7.355

BUYSTOCK3 17.857 17.857 16.504 12.152 9.977

OBJ 148.214 143.287 138.444 135.688 134.245

MEAN 148.214 148.214 146.581 141.331 138.705

VAR 19709.821 19709.821 16274.764 7523.441 4460.478

STD 140.392 140.392 127.573 86.738 66.787

SHADPRICE 0.296 0.277 0.261 0.260 0.260

**RAP or φ 0.0015 0.002 0.003 0.005 0.010**

BUYSTOCK2 9.386 10.401 11.416 12.229 12.838

BUYSTOCK3 7.801 6.713 5.625 4.755 4.102

OBJ 132.671 131.753 130.575 129.005 125.999

MEAN 136.080 134.767 133.454 132.404 131.617

VAR 2272.647 1506.907 959.949 679.907 561.764

STD 47.672 38.819 30.983 26.075 23.702

SHADPRICE 0.259 0.257 0.255 0.251 0.241

**RAP or φ 0.011 0.012 0.015 0.025 0.050**

BUYSTOCK1 1.273 4.372 4.405

BUYSTOCK2 12.893 12.960 12.420 11.070 8.188

BUYSTOCK3 4.043 3.972 3.550 2.561 1.753

BUYSTOCK4 4.168

OBJ 125.441 124.614 123.380 120.375 116.805

MEAN 131.545 131.459 129.839 125.939 121.656

VAR 554.929 547.587 430.560 222.576 97.026

STD 23.557 23.401 20.750 14.919 9.850

SHADPRICE 0.239 0.236 0.234 0.230 0.224

**RAP or φ 0.100 0.300 0.500 1.000 2.500**

BUYSTOCK1 4.105 3.905 3.865 3.835 1.777

BUYSTOCK2 6.488 5.354 5.128 4.958 2.289

BUYSTOCK3 1.340 1.064 1.009 0.968 0.446

BUYSTOCK4 6.829 8.602 8.957 9.223 4.296

OBJ 113.118 102.254 92.010 66.674 27.185

MEAN 119.327 117.774 117.463 117.230 54.370

VAR 62.086 51.734 50.905 50.556 10.874

STD 7.879 7.193 7.135 7.110 3.298

SHADPRICE 0.214 0.173 0.133 0.032 0

IDLE FUNDS 268.044

Risk Modeling and Stochastic Programming

EV Model – Example

Efficient Frontier:



Risk Modeling and Stochastic Programming

Characteristics of E‑V Model Optimal Solutions

**Properties of optimal E-V solutions may be examined via the Kuhn‑Tucker conditions. Given**

**the problem**

****

**Its Lagrangian function is**

****

**and the Kuhn‑Tucker conditions are**



Risk Modeling and Stochastic Programming

Unified Model

**Expected Income = sum (k,prob(k)\*Income(k))**

**Variance = sum (k,prob(k)\*(income(k)-Expected Income)\*\*2)**



Risk Modeling and Stochastic Programming

Unified Model- GAMS Formulation

10 SETS STOCKS POTENTIAL INVESTMENTS / BUYSTOCK1\*BUYSTOCK4 /

11 EVENTS EQUALLY LIKELY RETURN STATES OF NATURE

12 /EVENT1\*EVENT10 / ;

14 PARAMETERS PRICES(STOCKS) PURCHASE PRICES OF THE STOCKS

15 / BUYSTOCK1 22

16 BUYSTOCK2 30

17 BUYSTOCK3 28

18 BUYSTOCK4 26 / ;

19

20 SCALAR FUNDS TOTAL INVESTABLE FUNDS / 500 / ;

22 TABLE RETURNS(EVENTS,STOCKS) RETURNS BY STATE OF NATURE EVENT

23

24 BUYSTOCK1 BUYSTOCK2 BUYSTOCK3 BUYSTOCK4

25 EVENT1 7 6 8 5

26 EVENT2 8 4 16 6

27 EVENT3 4 8 14 6

28 EVENT4 5 9 -2 7

29 EVENT5 6 7 13 6

30 EVENT6 3 10 11 5

31 EVENT7 2 12 -2 6

32 EVENT8 5 4 18 6

33 EVENT9 4 7 12 5

34 EVENT10 3 9 -5 6

36

37 SCALAR RAP RISK AVERSION PARAMETER / 0.0 / ;

38

39 POSITIVE VARIABLES INVEST(STOCKS) MONEY INVESTED IN EACH STOCK

40 POSDEV(EVENTS) POSITIVE DEVIATIONS FROM MEAN INCOME

41 NEGDEV(EVENTS) NEGATIVE DEVIATIONS FROM MEAN INCOME

42

43 VARIABLES OBJ NUMBER TO BE MAXIMIZED

44 RETURN(EVENTS) RETURNS BY EVENT

45 MEAN MEAN RETURNS ;

46

47 EQUATIONS OBJJ OBJECTIVE FUNCTION

48 RETURNDEF(EVENTS) RETURNS DEFINITION

49 AVRET AVERAGE RETURNS

50 INVESTAV INVESTMENT FUNDS AVAILABLE

51 DEVIATION(EVENTS) DEVIATIONS FROM MEAN INCOME ;

53 OBJJ..

54 OBJ =E= MEAN

55 - RAP\*(SUM(EVENTS,(POSDEV(EVENTS)+NEGDEV(EVENTS))\*\*2)/CARD(EVENTS));

56

57 INVESTAV.. SUM(STOCKS, PRICES(STOCKS) \* INVEST(STOCKS)) =L= FUNDS ;

58

59 RETURNDEF(EVENTS)..SUM(STOCKS, RETURNS(EVENTS,STOCKS) \* INVEST(STOCKS))

60 - RETURN(EVENTS) =E= 0 ;

61

62 AVRET.. SUM(EVENTS,1/CARD(EVENTS)\*RETURN(EVENTS)) - MEAN=E= 0 ;

63

64 DEVIATION(EVENTS)..RETURN(EVENTS)-MEAN -POSDEV(EVENTS) + NEGDEV(EVENTS)

65 =E= 0 ;

67 MODEL EVPORTFOL /ALL/ ;

68

69 SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;

Risk Modeling and Stochastic Programming

Motad Model Development

Formally, the total absolute deviation of income from mean income under the kth state of nature (Dk) is



which can be rewritten as



Total absolute deviation (TAD) is the sum of Dk across the states of nature. Now introducing deviation variables to depict positive and negative deviations we get



The final MOTAD formulation is



Risk Modeling and Stochastic Programming

Motad Model Development

Model considering only negative deviations from the mean



the standard error of a normally distributed population can be estimated given sample size N, by multiplying mean absolute deviation (MAD), total absolute deviation (TAD), or total negative deviation (TND) by appropriate constraints. Thus,



This transformation is commonly used in MOTAD formulations such as:



Risk Modeling and Stochastic Programming

Motad Example

This example uses the same data as in the E-V Portfolio example.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Table 14.1. Data for E-V Example -- Returns by Stock and Event** | | | | |
|  | ----Stock Returns by Stock and Event---- | | | |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Event1 | 7 | 6 | 8 | 5 |
| Event2 | 8 | 4 | 16 | 6 |
| Event3 | 4 | 8 | 14 | 6 |
| Event4 | 5 | 9 | ‑2 | 7 |
| Event5 | 6 | 7 | 13 | 6 |
| Event6 | 3 | 10 | 11 | 5 |
| Event7 | 2 | 12 | ‑2 | 6 |
| Event8 | 5 | 4 | 18 | 6 |
| Event9 | 4 | 7 | 12 | 5 |
| Event10 | 3 | 9 | ‑5 | 6 |
|  |  |  |  |  |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Price | 22 | 30 | 28 | 26 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Table 14.5. Deviations from the Mean for Portfolio Example** | | | | |
|  | Stock1 | Stock2 | Stock3 | Stock4 |
| Event1 | 2.3 | ‑1.6 | ‑0.3 | ‑0.8 |
| Event2 | 3.3 | ‑3.6 | 7.7 | 0.2 |
| Event3 | ‑0.7 | 0.4 | 5.7 | 0.2 |
| Event4 | 0.3 | 1.4 | ‑10.3 | 1.2 |
| Event5 | 1.3 | ‑0.6 | 4.7 | 0.2 |
| Event6 | ‑1.7 | 2.4 | 2.7 | ‑0.8 |
| Event7 | ‑2.7 | 4.4 | ‑10.3 | 0.2 |
| Event8 | 0.3 | ‑3.6 | 9.7 | 0.2 |
| Event9 | ‑0.7 | ‑0.6 | 3.7 | ‑0.8 |
| Event10 | ‑1.7 | 1.4 | ‑13.3 | 0.2 |

Risk Modeling and Stochastic Programming

Motad Example

**Table 14.6. Example MOTAD Model Formulation**

|  |
| --- |
|  |



Risk Modeling and Stochastic Programming

Motad Example GAMS Formulation

10 SETS STOCKS POTENTIAL INVESTMENTS / BUYSTOCK1\*BUYSTOCK4 /

11 EVENTS EQUALLY LIKELY RETURN STATES OF NATURE

12 /EVENT1\*EVENT10 / ;

14 PARAMETERS PRICES(STOCKS) PURCHASE PRICES OF THE STOCKS

15 / BUYSTOCK1 22

16 BUYSTOCK2 30

17 BUYSTOCK3 28

18 BUYSTOCK4 26 / ;

20 SCALAR FUNDS TOTAL INVESTABLE FUNDS / 500 /

21 N SAMPLE SIZE

22 PI /3.141716/

23 TRAN TRANSFORMATION COEF MAD TO STD ERROR ;

25 N=CARD(EVENTS);

26 TRAN = ((PI \* N)/(2\*(N-1)))\*\*0.5 ;

28 TABLE RETURNS(EVENTS,STOCKS) RETURNS BY OBSERVATION

30 BUYSTOCK1 BUYSTOCK2 BUYSTOCK3 BUYSTOCK4

31 EVENT1 7 6 8 5

32 EVENT2 8 4 16 6

33 EVENT3 4 8 14 6

34 EVENT4 5 9 -2 7

35 EVENT5 6 7 13 6

36 EVENT6 3 10 11 5

37 EVENT7 2 12 -2 6

38 EVENT8 5 4 18 6

39 EVENT9 4 7 12 5

40 EVENT10 3 9 -5 6

42 PARAMETERS

43 MEAN (STOCKS) MEAN RETURNS TO X(STOCKS)

44 DEVS(EVENTS,STOCKS) DEVIATIONS FROM MEAN INCOME ;

46 MEAN(STOCKS) = SUM(EVENTS , RETURNS(EVENTS,STOCKS) / N );

48 DEVS(EVENTS,STOCKS) = RETURNS(EVENTS,STOCKS) - MEAN(STOCKS) ;

50 DISPLAY MEAN , DEVS ;

52 SCALAR RAP RISK AVERSION PARAMETER / 0.0 / ;

54 DISPLAY TRAN ;

56 POSITIVE VARIABLES INVEST(STOCKS) MONEY INVESTED IN EACH STOCK

57 DEVIATION(EVENTS) DEVIATION OF TOTAL INCOME BY EVENT

58 TRETURN TOTAL RETURNS

59 APPROXSTDE STANDARD ERROR AS APPROXIMATED

60 MAD MEAN ABSOLUTE DEVIATION

62 VARIABLE OBJ NUMBER TO BE MAXIMIZED ;

64 EQUATIONS OBJJ OBJECTIVE FUNCTION

65 INVESTAV INVESTMENT FUNDS AVAILABLE

66 DEVIATE(EVENTS) DEVIATION EQUATION FOR EVENTS

67 MADBALANCE MEAN ABSOLUTE DEVIATION DEFINITION

68 SEBALANCE STANDARD DEVIATION APPROXIMATION

71 OBJJ.. OBJ =E= SUM(STOCKS, MEAN(STOCKS) \* INVEST(STOCKS))

72 - RAP\*APPROXSTDE;

74 INVESTAV.. SUM(STOCKS, PRICES(STOCKS) \* INVEST(STOCKS)) =L= FUNDS ;

76 DEVIATE(EVENTS).. SUM(STOCKS, DEVS(EVENTS,STOCKS)\*INVEST(STOCKS))

77 + DEVIATION(EVENTS) =G= 0. ;

79 MADBALANCE.. 2\*SUM(EVENTS, DEVIATION(EVENTS))/N - MAD =E= 0. ;

81 SEBALANCE.. TRAN\*MAD - APPROXSTDE =E= 0. ;

83 MODEL MOTADPORTF /ALL/ ;

85 SOLVE MOTADPORTF USING LP MAXIMIZING OBJ ;

Risk Modeling and Stochastic Programming

Motad Example

###### Table 14.7. MOTAD Example Solutions for Alternative Risk Aversion Parameters

**RAP 0.050 0.100 0.110 0.120**

BUYSTOCK2 11.603

BUYSTOCK3 17.857 17.857 17.857 17.857 5.425

OBJ 148.214 140.146 132.078 130.464 129.390

MEAN 148.214 148.214 148.214 148.214 133.213

MAD 122.143 122.143 122.143 122.143 24.111

STDAPPROX 161.367 161.367 161.367 161.367 31.854

VAR 19709.821 19709.821 19709.821 19709.821 883.113

STD 140.392 140.392 140.392 140.392 29.717

SHADPRICE 0.296 0.280 0.264 0.261 0.259

**RAP 0.130 0.150 0.260 0.400 0.500**

BUYSTOCK1 2.663

BUYSTOCK2 11.603 11.603 11.916 12.379 10.985

BUYSTOCK3 5.425 5.425 5.090 4.594 3.995

OBJ 129.072 128.435 125.179 121.204 118.606

MEAN 133.213 133.213 132.809 132.210 129.161

MAD 24.111 24.111 22.212 20.827 15.979

STDAPPROX 31.854 31.854 29.345 27.515 21.110

VAR 883.113 883.113 771.228 643.507 455.983

STD 29.717 29.717 27.771 25.367 21.354

SHADPRICE 0.258 0.257 0.250 0.242 0.237

**RAP 0.750 1.000 1.250 1.500 1.750**

BUYSTOCK1 5.145 7.119 2.817 2.817 2.817

BUYSTOCK2 10.409 9.879 5.617 5.617 5.617

BUYSTOCK3 2.661 1.564 1.824 1.824 1.824

BUYSTOCK4 0.123 8.402 8.402 8.402

OBJ 114.168 111.009 108.372 106.086 103.801

MEAN 125.384 122.240 119.799 119.799 119.799

MAD 11.320 8.501 6.920 6.920 6.920

STDAPPROX 14.955 11.231 9.142 9.142 9.142

VAR 211.996 121.386 83.886 83.886 83.886

STD 14.560 11.018 9.159 9.159 9.159

SHADPRICE 0.228 0.222 0.217 0.212 0.208

**RAP 2.000 2.500 5.000 10.000 12.500**

BUYSTOCK1 2.817 2.817 2.858 2.858 2.858

BUYSTOCK2 5.617 5.617 4.178 4.178 4.178

BUYSTOCK3 1.824 1.824 1.242 1.242 1.242

BUYSTOCK4 8.402 8.402 10.654 10.654 10.654

OBJ 101.515 96.944 76.540 35.790 15.415

MEAN 119.799 119.799 117.289 117.289 117.289

MAD 6.920 6.920 6.169 6.169 6.169

STDAPPROX 9.142 9.142 8.150 8.150 8.150

VAR 83.886 83.886 57.695 57.695 57.695

STD 9.159 9.159 7.596 7.596 7.596

SHADPRICE 0.203 0.194 0.153 0.072 0.031

Risk Modeling and Stochastic Programming

Motad Example



Risk Modeling and Stochastic Programming

E-V – Nasty Assumptions

**Form of Distribution**

**Normality**

**Location and Scale**

**Form of Utility**

**Exponential**

**Taylor Series Approximation**

Risk Modeling and Stochastic Programming

Motad – Finding a Risk Aversion Parameter

The link between the E-standard error and E‑V risk aversion

parameters is as follows:

Consider the models



The first order conditions assuming X is nonzero are



For these two solutions to be identical in terms of X and , then



Risk Modeling and Stochastic Programming

Safety First

The Safety First model assumes that decision makers will choose plans to first assure a given safety level for income.



The overall problem then becomes



Risk Modeling and Stochastic Programming

Safety First – Example Formulation

**Table 14.8. Example Formulation of Safety First Problem**

|  |
| --- |
|  |



|  |
| --- |
| Solutions |

**Table 14.9. Safety First Example Solutions for Alternative Safety Levels**

|  |
| --- |
|  |

RUIN ‑100.000 ‑50.000 0.0 25.000 50.000

BUYSTOCK2 0.0 2.736 6.219 7.960 9.701

BUYSTOCK3 17.857 14.925 11.194 9.328 7.463

OBJ 148.214 144.677 140.174 137.923 135.672

MEAN 148.214 144.677 140.174 137.923 135.672

VAR 19709.821 12695.542 6066.388 3717.016 2011.116

STD 140.392 112.674 77.887 60.967 44.845

SHADPRICE 0.296 0.280 0.280 0.280 0.280

|  |
| --- |
|  |

Note: The abbreviations are the same as in the previous example solutions with RUIN giving the safety level.

Safety First – GAMS Formulation

12 SETS STOCKS POTENTIAL INVESTMENTS / BUYSTOCK1\*BUYSTOCK4 /

13 EVENTS EQUALLY LIKELY RETURN STATES OF NATURE/EVENT1\*EVENT10 / ;

16 PARAMETERS PRICES(STOCKS) PURCHASE PRICES OF THE STOCKS

17 / BUYSTOCK1 22, BUYSTOCK2 30

19 BUYSTOCK3 28, BUYSTOCK4 26 / ;

22 SCALAR FUNDS TOTAL INVESTABLE FUNDS / 500 / ;

24 TABLE RETURNS(EVENTS,STOCKS) RETURNS BY STATE OF NATURE EVENT

26 BUYSTOCK1 BUYSTOCK2 BUYSTOCK3 BUYSTOCK4

27 EVENT1 7 6 8 5

28 EVENT2 8 4 16 6

29 EVENT3 4 8 14 6

30 EVENT4 5 9 -2 7

31 EVENT5 6 7 13 6

32 EVENT6 3 10 11 5

33 EVENT7 2 12 -2 6

34 EVENT8 5 4 18 6

35 EVENT9 4 7 12 5

36 EVENT10 3 9 -5 6

39 SCALAR RUIN LEVEL OF SAFETY

40 N SAMPLE SIZE;

41 N = CARD(EVENTS) ;

43 PARAMETER STANDERROR(STOCKS) STANDARD ERROR OF STOCKS

44 AVRET(STOCKS) AVERAGE RETURN BY STOCK ;

45 AVRET(STOCKS) = SUM(EVENTS,RETURNS(EVENTS,STOCKS))/N;

46 STANDERROR(STOCKS) = SQRT(SUM(EVENTS,

47 (RETURNS(EVENTS,STOCKS)-AVRET(STOCKS))\*

48 (RETURNS(EVENTS,STOCKS)-AVRET(STOCKS))/(N-1)));

49 POSITIVE VARIABLES INVEST(STOCKS) MONEY INVESTED IN EACH STOCK

51 ART ARTIFICIAL VARIABLE

52 VARIABLES OBJ NUMBER TO BE MAXIMIZED

54 MEAN MEAN INCOME

55 EQUATIONS OBJJ OBJECTIVE FUNCTION

57 AVRETDEF AVERAGE RETURNS DEFINITION

58 INVESTAV INVESTMENT FUNDS AVAILABLE

59 SAFETY(EVENTS) SAFETY EQUATION ;

60 OBJJ.. OBJ =E= MEAN -99999\* ART;

62 AVRETDEF.. MEAN =E= SUM(STOCKS, AVRET(STOCKS) \* INVEST(STOCKS));

65 INVESTAV.. SUM(STOCKS, PRICES(STOCKS) \* INVEST(STOCKS)) =L=FUNDS ;

67 SAFETY(EVENTS).. SUM(STOCKS,RETURNS(EVENTS,STOCKS)\*INVEST(STOCKS))

68 + ART =G= RUIN;

70 MODEL SAFETYPORT /ALL/ ;

72 SET RUNS ALTERNATIVE RUIN LEVELS /R0\*R6/

74 PARAMETER RUINLEVEL(RUNS) SAFETY LEVELS

75 /R0 -100, R1 -50, R2 0, R3 25,

76 R4 50, R5 75 , R6 100/

78 PARAMETER OUTPUT(\*,RUNS) RESULTS FROM MODEL RUNS WITH VARYING RUIN LEVEL

79 RETURN(EVENTS) RETURN BY EVENTS

80 VAR VARIANCE;

84 LOOP (RUNS,RUIN=RUINLEVEL(RUNS);

86 SOLVE SAFETYPORT USING LP MAXIMIZING OBJ ;

88 RETURN(EVENTS) =SUM(STOCKS, RETURNS(EVENTS,STOCKS)\*INVEST.L(STOCKS));

90 VAR = SUM(EVENTS,(RETURN(EVENTS)-MEAN.L)\*(RETURN(EVENTS)-MEAN.L))/N;

92 OUTPUT("RUIN",RUNS)=RUIN;

93 OUTPUT(STOCKS,RUNS)=INVEST.L(STOCKS);

94 OUTPUT("OBJ",RUNS)=OBJ.L;

95 OUTPUT("MEAN",RUNS)=MEAN.L;

96 OUTPUT("VAR",RUNS) = VAR;

97 OUTPUT("STD",RUNS)=SQRT(VAR);

98 OUTPUT("SHADPRICE",RUNS)=INVESTAV.M;

99 OUTPUT("IDLE",RUNS)=FUNDS-INVESTAV.L);

Risk Modeling and Stochastic Programming

Target Motad

The Target MOTAD formulation incorporates a safety level of income while also allowing negative deviations from that safety level.



**Table 14.10. Example Formulation of Target MOTAD Problem**

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Risk Modeling and Stochastic Programming

Target Motad

**Table 14.11. Target MOTAD Example Solutions for Alternative Deviation Limits**

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| --- |
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**TARGETDEV 120.000 60.000 24.000 12.000 10.800**

BUYSTOCK2 0.0 0.0 7.081 10.193 10.516

BUYSTOCK3 17.857 17.857 10.270 6.936 6.590

OBJ 148.214 148.214 139.059 135.037 134.618

MEAN 148.214 148.214 139.059 135.037 134.618

VAR 19709.821 19709.821 4822.705 1646.270 1433.820

STD 140.392 140.392 69.446 40.574 37.866

SHADPRICE 0.296 0.296 0.286 0.295 0.295

**TARGETDEV 8.400 7.200 3.600**

BUYSTOCK1 0.0 0.0 3.459

BUYSTOCK2 11.259 11.782 11.405

BUYSTOCK3 5.794 5.234 2.919

OBJ 133.659 132.982 127.168

MEAN 133.659 132.982 127.168

VAR 1030.649 816.629 277.270

STD 32.104 28.577 16.651

SHADPRICE 0.298 0.298 0.815

|  |
| --- |
|  |

Note: The abbreviations are again the same with TARGETDEV giving the  value.

Risk Modeling and Stochastic Programming

DEMP Model

Direct Expected Maximizing Nonlinear Programming



where pk is the probability of the kth state of nature;

Wo is initial wealth;

Wk is the wealth under the kth state of nature; and

ckj is the return to one unit of the jth activity under the kth

state of nature.

Risk Modeling and Stochastic Programming

DEMP Model – Example

###### Table 14.12. Example Formulation of DEMP Problem

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Risk Modeling and Stochastic Programming

DEMP Model – Example

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| --- |
| **Table 14.13. DEMP Example Solutions for Alternative Utility Function Exponents** |

POWER 0.950 0.900 0.750 0.500 0.400

BUYSTOCK2 4.560 8.563 9.344

BUYSTOCK3 17.857 17.857 12.972 8.683 7.846

OBJ 186.473 140.169 60.363 15.282 8.848

MEAN 248.214 248.214 242.319 237.144 236.134

VAR 19709.821 19709.821 8903.295 3054.034 2309.233

STD 140.392 140.392 94.357 55.263 48.054

SHADPRICE 0.287 0.277 0.269 0.266 0.265

POWER 0.300 0.200 0.100 0.050 0.030

BUYSTOCK2 9.919 10.358 10.705 10.852 10.907

BUYSTOCK3 7.230 6.759 6.388 6.230 6.171

OBJ 5.127 2.972 1.724 1.313 1.177

MEAN 235.390 234.822 234.374 234.184 234.113

VAR 1843.171 1534.736 1320.345 1236.951 1207.076

STD 42.932 39.176 36.337 35.170 34.743

SHADPRICE 0.264 0.264 0.263 0.263 0.263

POWER 0.020 0.010 0.001 0.0001

BUYSTOCK2 10.934 10.960 10.960 10.960

BUYSTOCK3 6.143 6.115 6.115 6.115

OBJ 1.115 1.056 1.005 1.001

MEAN 234.079 234.045 234.045 234.045

VAR 1192.805 1178.961 1178.961 1178.961

STD 34.537 34.336 34.336 34.336

SHADPRICE 0.263 0.263 0.263 0

|  |
| --- |
|  |

DEMP Model – GAMS Formulation

12 SETS STOCKS POTENTIAL INVESTMENTS / BUYSTOCK1\*BUYSTOCK4 /

13 EVENTS EQUALLY LIKELY RETURN STATES OF NATURE

14 /EVENT1\*EVENT10 / ;

16 PARAMETERS PRICES(STOCKS) PURCHASE PRICES OF THE STOCKS

17 / BUYSTOCK1 22

18 BUYSTOCK2 30

19 BUYSTOCK3 28

20 BUYSTOCK4 26 / ;

22 SCALAR FUNDS TOTAL INVESTABLE FUNDS / 500 / ;

24 TABLE RETURNS(EVENTS,STOCKS) RETURNS BY STATE OF NATURE EVENT

26 BUYSTOCK1 BUYSTOCK2 BUYSTOCK3 BUYSTOCK4

27 EVENT1 7 6 8 5

28 EVENT2 8 4 16 6

29 EVENT3 4 8 14 6

30 EVENT4 5 9 -2 7

31 EVENT5 6 7 13 6

32 EVENT6 3 10 11 5

33 EVENT7 2 12 -2 6

34 EVENT8 5 4 18 6

35 EVENT9 4 7 12 5

36 EVENT10 3 9 -5 6

39 SCALAR INITWEALTH INITIAL WEALTH /100/

40 POWER EXPONENT IN UTILITY FUNCTION /0.5/;

42 POSITIVE VARIABLES INVEST(STOCKS) MONEY INVESTED IN EACH STOCK

44 VARIABLES OBJ NUMBER TO BE MAXIMIZED

45 WEALTH(EVENTS) RETURNS BY EVENT

46 MEAN MEAN RETURNS ;

48 EQUATIONS OBJJ OBJECTIVE FUNCTION

49 WEALTHDEF(EVENTS) WEALTHS DEFINITION

50 AVWEALTH AVERAGE WEALTH

51 INVESTAV INVESTMENT FUNDS AVAILABLE;

53 OBJJ.. OBJ =E= SUM(EVENTS,(WEALTH(EVENTS)\*\*POWER)/CARD(EVENTS));

55 INVESTAV.. SUM(STOCKS, PRICES(STOCKS) \* INVEST(STOCKS)) =L= FUNDS ;

57 WEALTHDEF(EVENTS).. WEALTH(EVENTS)

58 - SUM(STOCKS, RETURNS(EVENTS,STOCKS) \* INVEST(STOCKS)) =E=INITWEALTH ;

60 AVWEALTH.. MEAN =E= SUM(EVENTS,1/CARD(EVENTS)\*WEALTH(EVENTS));

63 MODEL EVPORTFOL /ALL/ ;

65 SET POWERS UTILITY FUNCTION POWER PARAMETERS /R0\*R13/

67 PARAMETER POWERSET(POWERS) RISK AVERSION COEF BY RISK AVERSION PARAMETER

68 /R0 0.95 , R1 0.9 , R2 0.75, R3 0.5 ,

69 R4 0.4 , R5 0.3 , R6 0.2 , R7 0.1 ,

70 R8 0.05 , R9 0.03 , R10 0.02, R11 0.01,

71 R12 0.001, R13 0.0001/

73 PARAMETER VAR VARIANCE OF RETURNS

75 PARAMETER OUTPUT(\*,POWERS) RESULTS FROM MODEL RUNS WITH VARYING EXPONENT

79 LOOP (POWERS,POWER=POWERSET(POWERS);

80 SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;

81 VAR = SUM(EVENTS,(WEALTH.L(EVENTS)-MEAN.L)\*(WEALTH.L(EVENTS) -MEAN.L))

82 /CARD(EVENTS);

83 OUTPUT("OBJ",POWERS)=OBJ.L;

84 OUTPUT("POWER",POWERS)=POWER;

85 OUTPUT(STOCKS,POWERS)=INVEST.L(STOCKS);

86 OUTPUT("MEAN",POWERS)=MEAN.L;

87 OUTPUT("VAR",POWERS) = VAR;

88 OUTPUT("STD",POWERS)=SQRT(VAR);

89 OUTPUT("SHADPRICE",POWERS)$(MEAN.L GT 0.001)=

90 INVESTAV.M/(power\*mean.l\*\*(power-1));

91 OUTPUT("IDLE",POWERS)=FUNDS-INVESTAV.L

92 );

Risk Modeling and Stochastic Programming

Chance Constrained Programming

The probability of a constraint being satisfied is greater than or equal to a prespecified value 



If the average value of the RHS is subtracted from both sides of the inequality and in turn both sides are divided by the standard deviation of the RHS then the constraint becomes



Those familiar with probability theory will note that the term



gives the number of standard errors that bi is away from the mean. Let Z denote this term. When a particular probability limit () is used, then the appropriate value of Z is Z and the constraint becomes



Assuming we discount for risk, then the constraint can be restated as



Risk Modeling and Stochastic Programming

Chance Constrained Programming

**Table 14.14. Chance Constrained Example Data**

|  |  |  |  |
| --- | --- | --- | --- |
| Event | Small Lathe | Large Lathe | Carver |
| 1 | 140 | 90 | 120 |
| 2 | 120 | 94 | 132 |
| 3 | 133 | 88 | 110 |
| 4 | 154 | 97 | 118 |
| 5 | 133 | 87 | 133 |
| 6 | 142 | 86 | 107 |
| 7 | 155 | 90 | 120 |
| 8 | 140 | 94 | 114 |
| 9 | 142 | 89 | 123 |
| 10 | 141 | 85 | 123 |
| Mean | 140 | 90 | 120 |
| Standard Error | 9.63 | 3.69 | 8.00 |

Then the resultant chance constrained formulation is

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | 67X1 | + | 66X2 | + | 66.3X3 | + | 80X4 | + | 78.5X5 | + | 78.4X6 |  |  |
| s.t | 0.8X1 | + | 1.3X2 | + | 0.2X3 | + | 1.2X4 | + | 1.7X5 | + | 0.5X6 |  | 140 - 9.63 Z |
|  | 0.5X1 | + | 0.2X2 | + | 1.3X3 | + | 0.7X4 | + | 0.3X5 | + | 1.5X6 |  | 90 - 3.69 Z |
|  | 0.4X1 | + | 0.4X2 | + | 0.4X3 | + | X4 | + | X5 | + | X6 |  | 120 - 8.00 Z |
|  | X1 | + | 1.05X2 | + | 1.1X3 | + | 0.8X4 | + | 0.82X5 | + | 0.84X6 |  | 125 |

Risk Modeling and Stochastic Programming

Chance Constrained Programming

**Table 14.15. Chance Constrained Example Solutions for Alternative Alpha Levels**

|  |
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|  |

**Z 0.00 1.280 1.654 2.330**

PROFIT 10417.291 9884.611 9728.969 9447.647

SMLLATHE 140.000 127.669 124.067 117.554

LRGLATHE 90.000 85.280 83.900 81.407

CARVER 120.000 109.760 106.768 101.360

LABOR 125.000 125.000 125.000 125.000

FUNCTNORM 62.233 78.102 82.739 91.120

FANCYNORM 73.020 51.495 45.205 33.837

FANCYMXLRG 5.180 6.788 7.258 8.108

|  |
| --- |
|  |

Note: Z is the risk aversion parameter.

Chance Constrained Programming

12 SET PROCESS TYPES OF PRODUCTION PROCESSES

13 /FUNCTNORM , FUNCTMXSML , FUNCTMXLRG

14 ,FANCYNORM , FANCYMXSML , FANCYMXLRG/

15 RESOURCE TYPES OF RESOURCES

16 /SMLLATHE,LRGLATHE,CARVER,LABOR/

17 EVENT STATES OF NATURE /S1\*S10/;

19 PARAMETER PRICE(PROCESS) PRODUCT PRICES BY PROCESS

20 /FUNCTNORM 82, FUNCTMXSML 82, FUNCTMXLRG 82

21 ,FANCYNORM 105, FANCYMXSML 105, FANCYMXLRG 105/

22 PRODCOST(PROCESS) COST BY PROCESS

23 /FUNCTNORM 15, FUNCTMXSML 16 , FUNCTMXLRG 15.7

24 ,FANCYNORM 25, FANCYMXSML 26.5, FANCYMXLRG 26.6/

26 TABLE RESORAVAIL(RESOURCE,EVENT) RESOURCE AVAILABLITY

28 S1 S2 S3 S4 S5 S6 S7 S8 S9 S10

29 SMLLATHE 140 120 133 154 133 142 155 140 142 141

30 LRGLATHE 90 94 88 97 87 86 90 94 89 85

31 CARVER 120 132 110 118 133 107 120 114 123 123

32 LABOR 125 125 125 125 125 125 125 125 125 125

34 TABLE RESOURUSE(RESOURCE,PROCESS) RESOURCE USAGE

36 FUNCTNORM FUNCTMXSML FUNCTMXLRG

37 SMLLATHE 0.80 1.30 0.20

38 LRGLATHE 0.50 0.20 1.30

39 CARVER 0.40 0.40 0.40

40 LABOR 1.00 1.05 1.10

41 + FANCYNORM FANCYMXSML FANCYMXLRG

42 SMLLATHE 1.20 1.70 0.50

43 LRGLATHE 0.70 0.30 1.50

44 CARVER 1.00 1.00 1.00

45 LABOR 0.80 0.82 0.84;

47 PARAMETER MEANAVAIL(RESOURCE) MEAN AMOUNT OF RESOURCE AVAILABILITY

48 STDERROR (RESOURCE) STANDARD ERROR OF RESOURCE AVAILABILITY;

50 SCALAR ZALPHA CHANCE CONSTRAINT DISCOUNT FACTOR/1.96/;

52 MEANAVAIL(RESOURCE) = SUM(EVENT,RESORAVAIL(RESOURCE,EVENT))/CARD(EVENT);

53 STDERROR(RESOURCE) = SQRT(SUM(EVENT,

54 SQR(RESORAVAIL(RESOURCE,EVENT)-MEANAVAIL(RESOURCE)))/CARD(EVENT));

58 POSITIVE VARIABLES PRODUCTION(PROCESS) ITEMS PRODUCED BY PROCESS;

61 VARIABLES PROFIT TOTALPROFIT;

64 EQUATIONS OBJT OBJECTIVE FUNCTION ( PROFIT )

66 AVAILABLE(RESOURCE) RESOURCES AVAILABLE ;

68 OBJT.. PROFIT =E=

69 SUM(PROCESS,(PRICE(PROCESS)-PRODCOST(PROCESS))

70 \* PRODUCTION(PROCESS)) ;

72 AVAILABLE(RESOURCE)..

73 SUM(PROCESS,RESOURUSE(RESOURCE,PROCESS)\*PRODUCTION(PROCESS))

74 =L= MEANAVAIL(RESOURCE)-ZALPHA\*STDERROR(RESOURCE);

76 MODEL CHANCE /ALL/;

78 SET ZALPH ALTERNATIVE Z VALUE IDENTIFIERS /Z1\*Z4/

80 PARAMETER OUTPUT(\*,ZALPH) RESULTS FROM MODEL RUNS WITH VARYING RAP

81 ZALPHS(ZALPH) ALTERNATIVE Z ALPHA VALUES

82 /Z1 0.0, Z2 1.28, Z3 1.654 , Z4 2.33/

86 LOOP (ZALPH,ZALPHA=ZALPHS(ZALPH);

87 SOLVE CHANCE USING LP MAXIMIZING PROFIT ;

88 OUTPUT(RESOURCE,ZALPH)=MEANAVAIL(RESOURCE)-ZALPHA\*STDERROR(RESOURCE);

89 OUTPUT("PROFIT",ZALPH)=PROFIT.L;

90 OUTPUT("RAP",ZALPH)=ZALPHA;

91 OUTPUT(PROCESS,ZALPH)=PRODUCTION.L(PROCESS));

93 DISPLAY OUTPUT;

Risk Modeling and Stochastic Programming

Technical Coefficient Risk

Merrill’s Approach

a constraint containing uncertain coefficients can be rewritten as



or, using standard deviation,



Wicks-Guise

given that the ith constraint contains uncertain aij's, the following constraints may be set up.



The general Wicks Guise formulation is



Wicks-Guise Example

**Table 14.16. Feed Nutrients by State of Nature for Wicks Guise Example**

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|  |

Nutrient State CORN SOYBEANS WHEAT

ENERGY S1 1.15 0.26 1.05

ENERGY S2 1.10 0.31 0.95

ENERGY S3 1.25 0.23 1.08

ENERGY S4 1.18 0.28 1.12

PROTEIN S1 0.23 1.12 0.51

PROTEIN S2 0.17 1.08 0.59

PROTEIN S3 0.25 1.01 0.46

PROTEIN S4 0.15 0.99 0.56

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**Table 14.17. Wicks Guise Example**

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Note: EnDev is the energy deviation

EnMAD is the energy mean absolute deviation

En is the energy standard deviation approximations

PrDev is the protein deviation

PrMAD is the protein mean absolute deviation

Pr is the protein standard deviation approximation

Risk Modeling and Stochastic Programming

Wicks-Guise Example Solution

**Table 14.18. Results From Example Wicks Guise Model Runs With Varying RAP**

|  |
| --- |
|  |

**RAP 0.250 0.500 0.750 1.000**

CORN 0.091 0.046 0.211 0.230 0.221

SOYBEANS 0.105 0.129 0.137

WHEAT 0.909 0.954 0.684 0.641 0.642

OBJ 0.039 0.040 0.040 0.040 0.041

AVGPROTEIN 0.500 0.515 0.515 0.521 0.529

STDPROTEIN 0.054 0.059 0.030 0.028 0.029

AVGENERGY 1.061 1.056 0.993 0.977 0.969

STDENERGY 0.072 0.072 0.061 0.059 0.058

SHADPROT 0.030 0.033 0.036 0.037 0.038

**RAP 1.250 1.500 2.000**

CORN 0.211 0.200 0.177

SOYBEANS 0.146 0.156 0.176

WHEAT 0.643 0.644 0.647

OBJ 0.041 0.041 0.042

AVGPROTEIN 0.536 0.545 0.563

STDPROTEIN 0.029 0.030 0.031

AVGENERGY 0.961 0.953 0.934

STDENERGY 0.057 0.056 0.055

SHADPROT 0.039 0.040 0.042

|  |
| --- |
|  |

Note: RAP gives the risk aversion parameter used

CORN gives the amount of corn used in the solution

SOYBEANS gives the amount of soybeans used in the solution

WHEAT gives the amount of wheat used in the solution

OBJ gives the objective function value

AVGPROTEIN gives the average amount of protein in the diet

STDPROTEIN gives the standard error of protein in the diet

AVGENERGY gives the average amount of energy in the diet

STDENERGY gives the standard error of energy in the diet

SHADPROT gives the shadow price on the protein requirement constraint

1. One could also use the divisor N-1 when working with a sample. [↑](#footnote-ref-2)