Including Firm Adaptation to Risk

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Including Firm Adaptation to Risk

Now what about recourse (simplspr.gms)

What is recourse?

Make a Decision now for example investment in capital goods

Then make a decision later – but must adjust in face of prior decision cannot entirely undo it so we are stuck with earlier level of capital goods investment

Suppose we have the following decision

Today we can invest in a machine which costs $3

During the machine life we use it under differing price capacity and yield events that are uncertain

Two projected futures exist

At the time we use the machines we know the conditions

Two states of nature can occur

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Price | Yield  with  invest | Yield without invest | Unit that  can be produced | Probability |
| Son 1 | 4 | 1.2 | 1.1 | 2 | 0.3 |
| Son 2 | 6 | 1.9 | 0.9 | 2.2 | 0.7 |

Including Firm Adaptation to Risk

Now what about recourse (simplspr.gms)

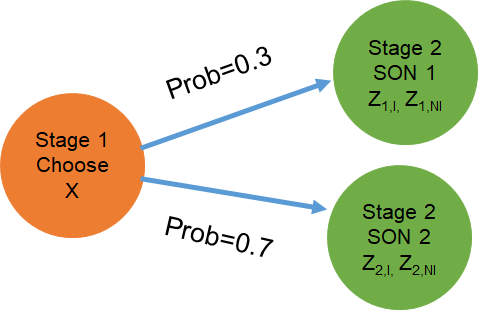
Problem will have 2 stages

Stage 1 Investment stage when we choose whether to buy machine for which we define a single variable X

Stage 2 Operation stage when we use machine and know prices, capacity and yield which results in variable to operate with (I) or without (NI) the investment under each state of nature (the 4 variables Z below)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | -3X | +0.3\*4( | 1.2 \* 1,I | +1.1 \* Z 1,NI) | +0.7\*6( | 1.9 \* Z 2,I | +0.9 \* Z 2,NI) |  |
| s.t. | -2X |  | + Z 1,I |  |  |  |  | ≤ 0 |
|  |  |  | + Z 1,I | + Z 1,NI |  |  |  | ≤ 2 |
|  | -2.2X |  |  |  |  | + Z 2,I |  | ≤ 0 |
|  |  |  |  |  |  | + Z 2,I | + Z 2,NI | ≤ 2.2 |

Objective maximizes expected income



Including Firm Adaptation to Risk

Now what about recourse (simplspr.gms)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | -3X | +0.3\*4( | 1.2 \* Z 1,I | +1.1 \* Z 1,NI) | +0.7\*6( | 1.9 \* Z 2,I | +0.9 \*Z 2,NI) |  |
|  |  |  |  |  |  |  |  |  |
| s.t. | -2X |  | + Z 1,I |  |  |  |  | ≤ 0 |
|  |  |  | + Z 1,I | + Z 1,NI |  |  |  | ≤ 2 |
|  | -2.2X |  |  |  |  | + Z 2,I |  | ≤ 0 |
|  |  |  |  |  |  | + Z 2,I | + Z 2,NI | ≤ 2.2 |

Note one decision variable (X) in first stage, 2 for each event at second stage (Z). Thus shows operation under 2 mutually exclusive second stages. ie at the same time we cannot have 2 prices, yields and capacities

When we solve we get

Solution obj=18.44 X=1 Z1,I=2 Z2,I=2.2

Note the X tells how to invest now, the Z’s tell how to use later

Including Firm Adaptation to Risk

Now what about recourse (simplspr.gms)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | -3X | +0.3\*4( | 1.2 \* Z 1,I | +1.1 \* Z 1,NI) | +0.7\*6( | 1.9 \* Z 2,I | +0.9 \* Z 2,NI) |  |
|  |  |  |  |  |  |  |  |  |
| s.t. | -2X |  | + Z 1,I |  |  |  |  | ≤ 0 |
|  |  |  | + Z 1,I | + Z 1,NI |  |  |  | ≤ 2 |
|  | -2.2X |  |  |  |  | + Z 2,I |  | ≤ 0 |
|  |  |  |  |  |  | + Z 2,I | + Z 2,NI | ≤ 2.2 |

SET STATE STATES OF NATURE /Son1 , Son2/

item /price,yieldwith,yieldwithout,capacity,PROBability/;

table data(item,STATE) Stochastic data

Son1 Son2

price 4 6

yieldwith 1.2 1.9

yieldwithout 1.1 0.9

capacity 2 2.2

PROBability 0.3 0.7 ;

set invest(item) /yieldwith,yieldwithout/;

POSITIVE VARIABLES

BuyMachine first stage variable

Use(state,invest) second stage variables;

VARIABLES

PROFIT TOTALPROFIT

EQUATIONS

OBJT OBJECTIVE FUNCTION ( PROFIT )

linkcapacity(state) New invest capacity AVAILABLE

totcapacity(state) Total Capacity AVAILABLE;

OBJT.. PROFIT =E= ‑3\*BuyMachine

+SUM(STATE,data("PROBability",STATE)\*data("price",STATE)

\*sum(invest,data(invest,state)\*Use(state,invest))) ;

linkcapacity(state).. ‑data("capacity",STATE)\*BuyMachine

+ Use(STATE,"yieldwith") =l= 0;

totcapacity(STATE).. sum(invest,Use(STATE,invest))=L=

data("capacity",STATE);

MODEL BASICSPR /ALL/;

Including Firm Adaptation to Risk -

Risk with Recourse (simplspr.gms)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | -3X | +0.3\*4( | 1.2 \* Z 1,I | +1.1 \* Z 1,NI) | +0.7\*6( | 1.9 \* Z 2,I | +0.9 \* Z 2,NI) |  |
|  |  |  |  |  |  |  |  |  |
| s.t. | -2X |  | + Z 1,I |  |  |  |  | ≤ 0 |
|  |  |  | + Z 1,I | + Z 1,NI |  |  |  | ≤ 2 |
|  | -2.2X |  |  |  |  | + Z 2,I |  | ≤ 0 |
|  |  |  |  |  |  | + Z 2,I | + Z 2,NI | ≤ 2.2 |

Solution

‑‑‑‑ EQU linkcapacity New invest capacity AVAILABLE

LOWER SLACK UPPER MARGINAL

Son1 ‑INF . . .

Son2 ‑INF . . 1.364

‑‑‑‑ EQU totcapacity Total Capacity AVAILABLE

LOWER SLACK UPPER MARGINAL

Son1 ‑INF . 2.000 1.440

Son2 ‑INF . 2.200 6.616

BuyMachine first stage variable

LOWER LEVEL UPPER MARGINAL

‑‑‑‑ VAR BuyMachine . 1.000 +INF .

‑‑‑‑ VAR Use second stage variables

LOWER LEVEL UPPER MARGINAL

Son1.yieldwith . 2.000 +INF .

Son1.yieldwithout . . +INF ‑0.120

Son2.yieldwith . 2.200 +INF .

Son2.yieldwithout . . +INF ‑2.836

LOWER LEVEL UPPER MARGINAL

‑‑‑‑ VAR PROFIT ‑INF 17.436 +INF .

Including Firm Adaptation to Risk



|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | E |  |  |  |  |  |  |  |  |
|  | -E | +0.3Y1 | +0.7Y2 |  |  |  |  |  | = 0 |
|  |  | -Y1 |  | -3X | +4\*(1.2 \* Z 1,I | +1.1 \*Z 1,NI) |  |  | = 0 |
| s.t. |  |  |  | -2X | + Z 1,I |  |  |  | ≤ 0 |
|  |  |  |  |  | + Z 1,I | + Z 1,NI |  |  | ≤ 2 |
|  |  |  | -Y2 | -3X |  |  | +6(1.9 \* Z 2,I | +0.9 \*Z 2,NI) | = 0 |
|  |  |  |  | -2.2X |  |  | + Z 2,I |  | ≤ 0 |
|  |  |  |  |  |  |  | + Z 2,I | + Z 2,NI | ≤ 2.2 |

Including Firm Adaptation to Risk -

Add Risk Aversion(spraver.gms)

Back to Unified model

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | E |  |  |  |  |  |  |  |  |  |
|  | E | +0.3Y1 | +0.7Y2 |  |  |  |  |  |  | = 0 |
|  |  | -Y1 |  | -3X | +4\*(1.2 \* Z 1,I | +1.1 \*Z 1,NI) |  |  |  | = 0 |
| s.t. |  |  |  | -2X | + Z 1,I |  |  |  |  | ≤ 0 |
|  |  |  |  |  | + Z 1,I | + Z 1,NI |  |  |  | ≤ 2 |
|  |  |  | -Y2 | -3X |  |  | +6(1.9 \* Z 2,I | +0.9 \*Z 2,NI) |  | = 0 |
|  |  |  |  | -2.2X |  |  | + Z 2,I |  |  | ≤ 0 |
|  |  |  |  |  |  |  | + Z 2,I | + Z 2,NI |  | ≤ 2.2 |

objt.. riskobj=e=avgincome ‑rap\*(sum(state, data("probability",state)

\*sqr(income(state)‑avgincome))\*\*0.5);

avgincomeacct.. sum(state,data("probability",state)\*income(state))=e=avgincome;

incomeacct(state).. income(state)=E= ‑3\*BuyMachine+

data("price",STATE)\*sum(invest,data(invest,state)\*Use(state,invest)) ;

linkcapacity(state).. ‑data("capacity",STATE)\*BuyMachine + Use(STATE,"yieldwith")

=l= 0;

totcapacity(STATE).. sum(invest,Use(STATE,invest))=L= data("capacity",STATE);

Including Firm Adaptation to Risk

Multiple Stages(sellspr.gms)

Suppose we can sell now, in 6 months and in one year and between now and then we get to observe the pricesIf we sell now we get certain price of $2, 6 months from now either 2.2 or 2.15, one year from now 2.01 or 2.44 with probability conditional on six month price

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Average  Ending  Net  Worth | Period 1 | | Period 2 | | | | Period 3 | | | | | | | |  |  |
|  |  | | State 1 | | State 2 | | Period 2  State 1 | | Period 2  State 1 | | Period 2  State 2 | | Period 2  State 2 | |  |  |
|  |  | |  | |  | | State A | | State B | | State A | | State B | |  |  |
|  | Sell | Keep | Sell | Keep | Sell | Keep | Sell | End  Worth | Sell | End  Worth | Sell | End  Worth | Sell | End  Worth |  |  |
| Objective | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | max |  |
| Starting Stock |  | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |  | < | 100 |
| Avg End Worth | 1 |  |  |  |  |  |  |  | ‑0.21 |  | ‑0.49 |  | ‑0.27 |  | ‑0.03 | = | 0 |
| Stock Kept pd 1 to 2 s1 |  |  | ‑1 | 1 | 1 |  |  |  |  |  |  |  |  |  |  | < | 0 |
| Stock Kept pd 1 to 2 s2 |  |  | ‑1 |  |  | 1 | 1 |  |  |  |  |  |  |  |  | < | 0 |
| Stock Kept pd 2 to 3 s1‑sA |  |  |  |  | ‑1 |  |  | 1 |  |  |  |  |  |  |  | < | 0 |
| Ending Worth s1‑sA |  | 2 |  | 2.2 |  |  |  | 2.01 | ‑1 |  |  |  |  |  |  | = | 0 |
| Stock Kept pd 2 to 3 s1‑sB |  |  |  |  | ‑1 |  |  |  |  | 1 |  |  |  |  |  | < | 0 |
| Ending Worth s1‑sB |  | 2 |  | 2.2 |  |  |  |  |  | 2.44 | ‑1 |  |  |  |  | = | 0 |
| Stock Kept pd 2 to 3 s2‑sA |  |  |  |  |  |  | ‑1 |  |  |  |  | 1 |  |  |  | < | 0 |
| Ending Worth s2‑sA |  | 2 |  |  |  | 2.15 |  |  |  |  |  | 2.01 | ‑1 |  |  | = | 0 |
| Stock Kept pd 2 to 3 s2‑sB |  |  |  |  |  |  | ‑1 |  |  |  |  |  |  | 1 |  | < | 0 |
| Ending Worth s2‑sB |  | 2 |  |  |  | 2.15 |  |  |  |  |  |  |  | 2.44 | ‑1 | = | 0 |

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Multiple Stages(sellspr.gms)

\* SECTION A SET DEFINITION

SET PERIODS TIME PERIODS /T1,T2,T3/

STATE2 STATES OF NATURE FOR PERIOD2 /S21,S22/

STATE3 STATES OF NATURE FOR PERIOD3 /S31,S32/ ;

\* SECTION B DATA DEFINITION

SCALAR INVENTORY STOCK ON HAND /100/

PRICE1 PRICE IN PERIOD1 /2.00/

PARAMETER PRICE2(STATE2) PRICE AT STATE NATURE 2 /S21 2.20 , S22 2.15/

PROB2(STATE2) PROBABILITY OF STATE OF NATURE PERIOD 2

/S21 .7 , S22 .3/

PRICE3(STATE3) PRICE AT STATE OF NATURE 3

/ S31 2.01 , S32 2.44 /

TABLE PROB3(STATE2,STATE3) PROBABILITY OF STATES IN PD 3

\* CONDITIONAL ON STATE RESULTING IN PD 2

S31 S32

S21 .3 .7

S22 .9 .1

POSITIVE VARIABLES

SELL1 SALES IN PERIOD 1

SELL2(STATE2) SALES IN PERIOD 2 BY STATE

SELL3(STATE2,STATE3) SALES IN PERIOD 3 BY STATE IN PD 2 & 3

KEEP1 STOCK KEPT ON HAND FROM PERIOD 1 TO 2

KEEP2(STATE2) STOCK KEPT ON HAND FROM PD 2 TO 3

VARIABLES

AVGWORTH TOTAL ENDING NET WORTH

ENDWORTH(STATE2,STATE3) INCOME BY STATE OF NATURE;

EQUATIONS

OBJT OBJECTIVE FUNCTION

BALANCE1 INITIAL STOCK AVAILABLE

KEPT12(STATE2) STOCK KEPT FROM PERIOD 1 INTO 2

KEPT23(STATE2,STATE3) STOCK KEPT FROM PERIOD 2 INTO 3

WORTHBAL(STATE2,STATE3) WORTH BALANCE BY STATE OF NATURE;

OBJT.. AVGWORTH =e=

SUM((STATE2,STATE3),

PROB2(STATE2)\*PROB3(STATE2,STATE3)\*ENDWORTH(STATE2,STATE3));

WORTHBAL(STATE2,STATE3)..

PRICE1\*SELL1 + PRICE2(STATE2)\*SELL2(STATE2)

+ PRICE3(STATE3)\*SELL3(STATE2,STATE3) =E= ENDWORTH(STATE2,STATE3);

BALANCE1.. SELL1 + KEEP1 =L= INVENTORY;

KEPT12(STATE2).. ‑KEEP1 + SELL2(STATE2) + KEEP2(STATE2) =L= 0;

KEPT23(STATE2,STATE3).. ‑KEEP2(STATE2) + SELL3(STATE2,STATE3) =L= 0;

MODEL STOCSTOCK /ALL/;

SOLVE STOCSTOCK USING LP MAXIMIZING AVGWORTH;

Including Firm Adaptation to Risk -

Multiple Stages(sellspr.gms) Solution

LOWER SLACK UPPER MARGINAL

‑‑‑‑ EQU BALANCE1 ‑INF . 100.000 2.263

‑‑‑‑ EQU KEPT12 STOCK KEPT FROM PERIOD 1 INTO 2

LOWER SLACK UPPER MARGINAL

S21 ‑INF . . 1.618

S22 ‑INF . . 0.645

‑‑‑‑ EQU KEPT23 STOCK KEPT FROM PERIOD 2 INTO 3

LOWER SLACK UPPER MARGINAL

S21.S31 ‑INF . . 0.422

S21.S32 ‑INF . . 1.196

S22.S31 ‑INF . . 0.543

S22.S32 ‑INF . . 0.073

‑‑‑‑ EQU WORTHBAL WORTH BALANCE BY STATE OF NATURE

LOWER SLACK UPPER MARGINAL

S21.S31 . . . ‑0.210

S21.S32 . . . ‑0.490

S22.S31 . . . ‑0.270

S22.S32 . . . ‑0.030

LOWER LEVEL UPPER MARGINAL

‑‑‑‑ VAR SELL1 . . +INF ‑0.263

SELL1 SALES IN PERIOD 1

‑‑‑‑ VAR SELL2 SALES IN PERIOD 2 BY STATE

LOWER LEVEL UPPER MARGINAL

S21 . . +INF ‑0.078

S22 . 100.000 +INF .

‑‑‑‑ VAR SELL3 SALES IN PERIOD 3 BY STATE IN PD 2 & 3

LOWER LEVEL UPPER MARGINAL

S21.S31 . 100.000 +INF .

S21.S32 . 100.000 +INF .

S22.S31 . . +INF .

S22.S32 . . +INF .

LOWER LEVEL UPPER MARGINAL

‑‑‑‑ VAR KEEP1 . 100.000 +INF .

KEEP1 STOCK KEPT ON HAND FROM PERIOD 1 TO 2

‑‑‑‑ VAR KEEP2 STOCK KEPT ON HAND FROM PD 2 TO 3

LOWER LEVEL UPPER MARGINAL

S21 . 100.000 +INF .

S22 . . +INF ‑0.029

LOWER LEVEL UPPER MARGINAL

‑‑‑‑ VAR AVGWORTH ‑INF 226.270 +INF .

AVGWORTH TOTAL ENDING NET WORTH

‑‑‑‑ VAR ENDWORTH INCOME BY STATE OF NATURE

LOWER LEVEL UPPER MARGINAL

S21.S31 ‑INF 201.000 +INF .

S21.S32 ‑INF 244.000 +INF .

S22.S31 ‑INF 215.000 +INF .

S22.S32 ‑INF 215.000 +INF .

Red and blue are adaptation



|  |  |  |
| --- | --- | --- |
| **Table 14.19. Data on Uncertain Parameters in First SPR Example** | | |
|  | Value Under | |
| Parameter | State of Nature 1 | State of Nature 2 |
| Probability | .6 | .4 |
| Corn Yield in bu | 100 | 105 |
| Wheat Yield in bu | 40 | 38 |
| Corn Harvest Rate hours per bu | .010 | .015 |
| Wheat Harvest Rate hours per bu | .030 | .034 |
| Corn Price per bu | 3.25 | 2.00 |
| Wheat Price per bu | 5.00 | 6.00 |
| Harvest Time hours | 122 | 143 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 14.20. Risk Free Formulation of First SPR Example** | | | | | | | |
|  | Grow Corn | Grow Wheat | Income | Harvest Corn | Harvest  Wheat | RHS | |
| Objective |  |  | 1 |  |  |  |  |
| Land | 1 | 1 |  |  |  |  | 100 |
| Corn Yield Balance | -yieldc |  |  | 1 |  |  | 0 |
| Wheat Yield Balance |  | -yieldw |  |  | 1 |  | 0 |
| Harvest Hours |  |  |  | +harvtimec | +harvtimew |  | harvavail |
| Income | -100 | -60 | -1 | +pricec | +pricew | = | 0 |

**Table 14.21. Formulation of First SPR Example**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | State 1 |  |  | State 2 |  |  |  |
|  |  | Grow Corn | Grow Wht. | Inc. s1 | Harv Corn s1 | Harv Wht  s1 | Inc. s2 | Harv Corn s2 | Harv Wht  s2 |  | RHS |
|  | Objective |  |  | .6 |  |  | .4 |  |  |  | max |
|  | Land | 1 | 1 |  |  |  |  |  |  |  | 100 |
| S t a t e 1 | Corn s1 | -100 |  |  | 1 |  |  |  |  |  | 0 |
| Wheat s1 |  | -40 |  |  | 1 |  |  |  |  | 0 |
| Harvest Hours s1 |  |  |  | .010 | .030 |  |  |  |  | 122 |
| Income s1 | -100 | -60 | -1 | 3.25 | 5.00 |  |  |  | = | 0 |
| S t a t e 2 | Corn s2 | -105 |  |  |  |  |  | 1 |  |  | 0 |
| Wheat s2 |  | -38 |  |  |  |  |  | 1 |  | 0 |
| Harvest Hours s2 |  |  |  |  |  |  | .015 | .034 |  | 143 |
| Income s2 | -100 | -60 |  |  |  | -1 | 2.00 | 6.00 | = | 0 |

|  |  |  |
| --- | --- | --- |
| **Table 14.22. Solution of First SPR Example** | | |
| Equation | Slack | Shadow Price |
| Objective | 16476 |  |
| Land | 0 | 24.28 |
| Corn s1 | 0 | -1.95 |
| Wheat s1 | 0 | 0.67 |
| Harvest Hours s1 | 11.75 | 0 |
| Income s1 | 0 | -0.6 |
| Corn s2 | 0 | -3.00 |
| Wheat s2 | 0 | 0.94 |
| Harvest Hours s2 | 0 | 98.23 |
| Income s2 | 0 | -0.4 |

|  |  |  |
| --- | --- | --- |
| Variable | Solution  Value | Marginal Cost |
| Grow Corn | 48.8 | 0 |
| Grow Wheat | 51.2 | 0 |
| Income S1 | 18144 | 0 |
| Harvest Corn s1 | 4876 | 0 |
| Harvest Wheat s1 | 2049 | 0 |
| Income S2 | 13972 | 0 |
| Harvest Corn s2 | 5120 | 0 |
| Harvest Wheat s2 | 1947 | 0 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 14.23. Second SPR Example Formulation (Partial Tableau)** | | | | | | | | | | | | | | | | | | | | |
|  | Corn | Soy | Wht | Avg Cost | Pos Prot  Dev s1 | Neg Prot  Dev s1 | Pos Eng  Dev s1 | Neg Eng  Dev s1 | Cost s1 | Pos  Cost  Dev  s1 | Negs  Cost  Dev  s1 | Pos Prot  Dev s2 | Neg Prot  Dev s2 | Pos Eng  Dev s2 | Neg Eng  Dev s2 | Cost s2 | Pos  Cost  Dev  s2 | Neg  Cost  Dev  s2 |  |  |
| Objective |  |  |  | 1 |  |  |  |  |  | + | + |  |  |  |  |  | + | + |  |  |
| Total Feed | 1 | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | = | 1 |
| Average Cost |  |  |  | 1 |  |  |  |  | -.25 |  |  |  |  |  |  | -.25 |  |  | = | 0 |
| Protein-s1 | 0.23 | 1.12 | 0.51 |  | -1 | 1 |  |  |  |  |  |  |  |  |  |  |  |  | = | 0.6 |
| Energy -s1 | 1.15 | 0.26 | 1.05 |  |  |  | -1 | 1 |  |  |  |  |  |  |  |  |  |  | = | 0.9 |
| Cost-s1 | 0.03 | 0.06 | 0.04 |  | 0.50 | 1.50 | 1.00 | 0.10 | -1 |  |  |  |  |  |  |  |  |  | = | 0 |
| Cost dev s1 |  |  |  | -1 |  |  |  |  | 1 | -1 | 1 |  |  |  |  |  |  |  | = | 0 |
| Protein-s2 | 0.17 | 1.08 | 0.59 |  |  |  |  |  |  |  |  | -1 | 1 |  |  |  |  |  | = | 0.6 |
| Energy -s2 | 1.10 | 0.31 | 0.95 |  |  |  |  |  |  |  |  |  |  | -1 | 1 |  |  |  | = | 0.9 |
| Cost-s2 | .03 | .06 | .04 |  |  |  |  |  |  |  |  | 0.50 | 1.50 | 1.00 | 0.10 | -1 |  |  | = | 0 |
| Cost dev s2 |  |  |  | -1 |  |  |  |  |  |  |  |  |  |  |  | 1 | -1 | 1 | = | 0 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Table 14.24. Second SPR Example Risk Neutral Solution** | | | | | |
|  | Slack | Shadow Price |  | Slack | Shadow Price |
| Objective | 0.067 |  | Corn Purchase | 0.283 | 0 |
| Total Feed | 0 | -0.14 | Soybean Purchase | 0.362 | 0 |
| Average Cost | 0.00 | 1. | Wheat Purchase | 0.355 | 0 |
| Protein-s1 | 0 | 0.125 | Average Cost | 0.067 | 0 |
| Energy -s1 | 0 | 0.025 | Pos Protein Dev s1 | 0.052 | 0 |
| Cost-s1 | 0 | 252.66 | Neg Protein Dev s1 | 0. | 0.50 |
| Cost dev s1 | 0 | 0.00 | Pos Energyn Dev s1 | 0.00 | 0 |
| Protein-s2 | 0 | 0.125 | Neg Energy Dev s1 | 0.108 | 0 |
| Energy -s2 | 0 | 0.025 | Cost - s1 | 0.081 | 0 |
| Cost-s2 | 0 | 0.25 | Pos Cost Dev - s1 | 0.014 | 0 |
| Cost dev s2 | 0 | 0 | Neg Cost Dev - s1 | 0.00 | 0 |
| Protein-s3 | 0 | -.366 | Pos Protein Dev s2 | 0.049 | 0 |
| Energy -s3 | 0 | 0.025 | Neg Protein Dev s2 | 0.000 | 0.50 |
| Cost-s3 | 0 | 0.25 | Pos Energyn Dev s2 | 0. | 0.275 |
| Cost dev s3 | 0 | 0 | Neg Energyn Dev s2 | 0.140 | 0 |
| Protein-s4 | 0 | .08 | Cost - s2 | 0.083 | 0 |
| Energy -s4 | 0 | .025 | Pos Cost Dev - s2 | .016 | 0 |
| Cost-s4 | 0 | 0.25 | Neg Cost Dev - s2 | 0.00 | 0 |
| Cost dev s4 | 0 | 0.00 | Pos Protein Dev s3 | 0. | 0.491 |
|  |  |  | Neg Protein Dev s3 | 0. | 0.009 |
|  |  |  | Pos Energy Dev s3 |  | 0.275 |
|  |  |  | Neg Energy Dev s3 | 0.080 | 0 |
|  |  |  | Cost - s3 | 0.052 | 0 |
|  |  |  | Pos Cost Dev - s3 | 0.00 | 0 |
|  |  |  | Neg Cost Dev - s3 | 0.014 | 0 |
|  |  |  | Pos Protein Dev s4 | 0. | 0.205 |
|  |  |  | Neg Protein Dev s4 | 0. | 0.295 |
|  |  |  | Pos Energyn Dev s4 | 0. | 0.275 |
|  |  |  | Neg Energy Dev s4 | 0.067 | 0 |
|  |  |  | Cost - s4 | 0.051 | 0 |
|  |  |  | Pos Cost Dev - s4 | 0. | 0 |
|  |  |  | Neg Cost Dev - s4 | 0.016 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 14.25. SPR Second Example Problem Soution Under Varying Risk Aversion** | | | | | | | |
| RAP | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.500 | 0.600 |
|  |  |  |  |  |  |  |  |
| Corn | 0.283 | 0.249 | 0.245 | 0.244 | 0.288 | 0.296 | 0.297 |
| Soybeans | 0.362 | 0.330 | 0.327 | 0.326 | 0.340 | 0.342 | 0.342 |
| Wheat | 0.355 | 0.422 | 0.428 | 0.430 | 0.372 | 0.363 | 0.361 |
| Avgcost | 0.067 | 0.067 | 0.067 | 0.067 | 0.071 | 0.071 | 0.071 |
| Cost s1 | 0.081 | 0.074 | 0.073 | 0.073 | 0.071 | 0.071 | 0.071 |
| Cost s2 | 0.083 | 0.080 | 0.080 | 0.080 | 0.074 | 0.073 | 0.073 |
| Cost s3 | 0.052 | 0.066 | 0.067 | 0.068 | 0.071 | 0.071 | 0.071 |
| Cost s4 | 0.051 | 0.048 | 0.048 | 0.048 | 0.067 | 0.070 | 0.071 |
| Std Error | 0.015 | 0.012 | 0.012 | 0.012 | 0.002 | 0.001 | 0.001 |

RAP is the risk aversion parameter.

**Table 14.26. Example Tableau for Third SPR Problem**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Average Ending Net Worth | Period 1 | | Period 2 | | | | Stage 3 | | | | | | | |  |  |
|  |  |  |  |  |  | |  | | Period 2 | | State 1 | | Period 2 | | State 2 | |  |  |
|  |  |  |  |  | State 1 | | State 2 | |  | |  | |  | |  | |  |  |
|  |  |  |  |  |  |  |  |  | Period 3 State A | | Period 3 State B | | Period 3 State A | | Period 3 State B | |  |  |
|  |  |  | Sell | Keep | Sell | Keep | Sell | Keep | Sell | End Worth | Sell | End Worth | Sell | End Worth | Sell | End Worth |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Objective | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | max |  |
|  | Starting Stock |  | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  | 100 |
|  | Avg End Worth | 1 |  |  |  |  |  |  |  | ‑0.42 |  | ‑0.28 |  | ‑0.21 |  | ‑0.09 | = | 0 |
|  | Stock Kept pd 1 to 2 s1 |  |  | ‑1 | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  | 0 |
|  | Stock Kept pd 1 to 2 s2 |  |  | ‑1 |  |  | 1 | 1 |  |  |  |  |  |  |  |  |  | 0 |
| P2  S1 | Stock Kept pd 2 to 3 s1‑sA |  |  |  |  | ‑1 |  |  | 1 |  |  |  |  |  |  |  |  | 0 |
|  | Ending Worth s1‑sA |  | 2.1412 |  | 2.332 |  |  |  | 2.18 | ‑1 |  |  |  |  |  |  | = | 0 |
|  | Stock Kept pd 2 to 3 s1‑sB |  |  |  |  | ‑1 |  |  |  |  | 1 |  |  |  |  |  |  | 0 |
|  | Ending Worth s1‑sB |  | 2.1008 |  | 2.288 |  |  |  |  |  | 2.44 | ‑1 |  |  |  |  | = | 0 |
| P2  S2 | Stock Kept pd 2 to 3 s2‑sA |  |  |  |  |  |  | ‑1 |  |  |  |  | 1 |  |  |  |  | 0 |
|  | Ending Worth s2‑sA |  | 2.1828 |  |  |  | 2.193 |  |  |  |  |  | 2.18 | ‑1 |  |  | = | 0 |
|  | Stock Kept pd 2 to 3 s2‑sB |  |  |  |  |  |  | ‑1 |  |  |  |  |  |  | 1 |  |  | 0 |
|  | Ending Worth s2‑sB |  | 2.1012 |  |  |  | 2.111 |  |  |  |  |  |  |  | 2.44 | ‑1 | = | 0 |

**Table 14.27. Solution for Third SPR Example**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Variable** | **Value** | **Reduced Cost** | **Variable** | **Slack** | **Shadow Price** |
| **Average Ending Net Worth** | **229.748** | **0** | **Objective** | **229.748** |  |
| **Sell In Period 1** | **0** | **-0.162** | **Starting Stock** | **0** | **2.297** |
| **Keep From Period 1 to 2** | **100** | **0** | **Avg End Worth** | **0** | **1** |
| **Sell In Period 2 Under State 1** | **100** | **0** | **Stock Kept pd 1 to 2 s1** | **0** | **1.62** |
| **Keep From Period 2 to 3 Under State 1** | **0** | **-0.021** | **Stock Kept pd 1 to 2 s1** | **0** | **0.677** |
| **Sell In Period 2 Under State 2** | **0** | **-0.027** | **Stock Kept pd 2 to 3 s1-s1** | **0** | **0.916** |
| **Keep From Period 2 to 3 Under State 2** | **100** | **0** | **Ending Worth s1-s1** | **0** | **-0.42** |
| **Sell in Period 3 Under State 1 -- State A** | **0** | **0** | **Stock Kept pd 2 to 3 s1-s2** | **0** | **0.683** |
| **Ending Worth Under State 1 -- State A** | **233.2** | **0** | **Ending Worth s1-s2** | **0** | **-0.28** |
| **Sell In Period 3 Under State 1 -- State B** | **0** | **0** | **Stock Kept pd 2 to 3 s2-s1** | **0** | **0.458** |
| **Ending Worth Under State 1 -- State B** | **228.8** | **0** | **Ending Worth s2-s1** | **0** | **-0.21** |
| **Sell In Period 3 Under State 2 -- State A** | **100** | **0** | **Stock Kept pd 2 to 3 s2-s2** | **0** | **0.22** |
| **Ending Worth Under State 2 -- State A** | **218** | **0** | **Ending Worth s2-s2** | **0** | **-0.09** |
| **Sell In Period 3 Under State 2 -- State B** | **100** | **0** |  |  |  |
| **Ending Worth Under State 2 -- State B** | **244** | **0** |  |  |  |



**Figure 14.3: Decision Tree for Sequential Programming Example**