Multi-Objective Programming

Lexicographic Utility

# The formulation is: Select X so that



and so that the goals are handled in the following priority order:



then



on through to



for the Rth and last goal.



Multi-Objective Programming Lexicographic Utility - Example

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| .**Table 11.1. Tableau for Lexicographic Example** | | | | | | | | | | | | | | | | | | | | | | |
|  |  | Original Decision Variables | | | | | | Profit | Idle Resources | | | | Goal Levels | | | | Goal Deviations | | | | RHS | |
|  |  | X1 | X2 | X3 | X4 | X5 | X6 |  | Sml Lathe | Lrg Lathe | Carver | Labor | Profit  9000 | Idle Labor | Idle Lathe | Profit 9500 | Prof  9000 | Idle Labor | Idle  Lathe | Profit  9500 |  | |
| Objective | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 12 |  |  | Min | |
| Original Problem Equations | Profit | 67 | 66 | 66.3 | 80 | 78.5 | 78.4 | -1 |  |  |  |  |  |  |  |  |  |  |  |  | = | 0 |
| Lexicographic Satisfaction | Profit 9000 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  | ≤ | 0 |
|  | Small Lathe | 0.8 | 1.3 | 0.2 | 1.2 | 1.7 | 0.5 |  | 1 |  |  |  |  |  |  |  |  |  |  |  | = | 140 |
|  | Large Lathe | 0.5 | 0.2 | 1.3 | 0.7 | 0.3 | 1.5 |  |  | 1 |  |  |  |  |  |  |  |  |  |  | = | 90 |
|  | Carver | 0.4 | 0.4 | 0.4 | 1.0 | 1.0 | 1.0 |  |  |  | 1 |  |  |  |  |  |  |  |  |  | = | 120 |
|  | Labor | 1.0 | 1.05 | 1.1 | 0.8 | 0.82 | 0.84 |  |  |  |  | 1 |  |  |  |  |  |  |  |  | = | 125 |
|  | Idle Labor |  |  |  |  |  |  |  |  |  |  | 1 |  | -1 |  |  |  |  |  |  | = | 0 |
|  | Idle Lathe |  |  |  |  |  |  |  | 1 | 1 |  |  |  |  | -1 |  |  |  |  |  | = | 0 |
|  | Profit 9500 |  |  |  |  |  |  | 1 |  |  |  |  |  |  |  | -1 |  |  |  |  | = | 0 |
|  | Idle Labor |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  | 1 |  |  | ≥ | 30 |
|  | Idle Lathe |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  | 1 |  | ≥ | 25 |
|  | Profit 9500 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  | 1 | ≥ | 9500 |
|  | Idle Labor |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  | ≤ | 99999999 |
|  | Idle Lathe |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  | ≤ | 99999999 |
|  | Profit 9500 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 | ≤ | 99999999 |
| Goal Satisfaction | Profit 9000 |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  | 1 |  |  |  | ≥ | 9000 |
| Goal Level Identity | Profit 9000 level |  |  |  |  |  |  | 1 |  |  |  |  | -1 |  |  |  |  |  |  |  | = | 0 |

Multi-Objective Programming

Lexicographic Utility - Example

|  |
| --- |
| Table 11.2. GAMS Formulation of Lexicographic Example |

5 SET GOALS GOALS IN THE ORDER THEY ARE TO BE MET

6 /PROFIT1,LABOR,LATHETIME,PROFIT2/

7 PROCESS TYPES OF PRODUCTION PROCESSES

8 /FUNCTNORM , FUNCTMXSML , FUNCTMXLRG

9 ,FANCYNORM , FANCYMXSML , FANCYMXLRG/

10 RESOURCE TYPES OF RESOURCES

11 /SMLLATHE,LRGLATHE,CARVER,LABOR/

13 ALIAS(GOALS,GOAL) ;

15 PARAMETER PRICE(PROCESS) PRODUCT PRICES BY PROCESS

16 /FUNCTNORM 82, FUNCTMXSML 82, FUNCTMXLRG 82

17 ,FANCYNORM 105, FANCYMXSML 105, FANCYMXLRG 105/

18 PRODCOST(PROCESS) COST BY PROCESS

19 /FUNCTNORM 15, FUNCTMXSML 16 , FUNCTMXLRG 15.7

20 ,FANCYNORM 25, FANCYMXSML 26.5, FANCYMXLRG 26.6/

21 RESORAVAIL(RESOURCE) RESOURCE AVAILABLITY

22 /SMLLATHE 140, LRGLATHE 90,

23 CARVER 120, LABOR 125/

24 TARGET(GOALS) GOAL TARGET LEVELS

25 /PROFIT1 9000,LABOR 30,LATHETIME 25

26 ,PROFIT2 9500/

27 DEV(GOALS) MAXIMUM DEVIATION BY GOAL

28 WEIGHTS(GOALS) WEIGHTS BY GOAL ;

30 DEV(GOALS)=999999;

31 WEIGHTS(GOALS)=0.00001;

33 TABLE RESOURUSE(RESOURCE,PROCESS) RESOURCE USAGE

35 FUNCTNORM FUNCTMXSML FUNCTMXLRG

36 SMLLATHE 0.80 1.30 0.20

37 LRGLATHE 0.50 0.20 1.30

38 CARVER 0.40 0.40 0.40

39 LABOR 1.00 1.05 1.10

40 + FANCYNORM FANCYMXSML FANCYMXLRG

41 SMLLATHE 1.20 1.70 0.50

42 LRGLATHE 0.70 0.30 1.50

43 CARVER 1.00 1.00 1.00

44 LABOR 0.80 0.82 0.84;

46 POSITIVE VARIABLES

47 PRODUCTION(PROCESS) ITEMS PRODUCED BY PROCESS

48 IDLE(RESOURCE) SLACK VARIABLES FOR RESOURCES

49 GOALLEVEL(GOALS) GOAL LEVELS

50 PROFIT TOTALPROFIT

51 SHORTFALL(GOALS) GOAL SHORTFALLS;

52 VARIABLES

53 GOALOBJ GOAL OBJECTIVE;

54 EQUATIONS

55 OBJT OBJECTIVE FUNCTION

56 PROFITACCT PROFIT ACCOUNTING

57 AVAILABLE(RESOURCE) RESOURCES AVAILABLE

58 IDLLABGOAL IDLE LABOR GOAL

59 PROFITGL1 PROFIT1 GOAL

60 PROFITGL2 PROFIT2 GOAL

61 LATHEGOAL IDLE LATHE GOAL

62 TARGS(GOALS) GOAL TARGETS

63 MAXSHORT(GOALS) SHORTFALL LIMITS;

64

|  |
| --- |
| **Table 11.2. GAMS Formulation of Lexicographic Example (Continued)** |

65 OBJT.. GOALOBJ =E= SUM(GOALS,WEIGHTS(GOALS)\*SHORTFALL(GOALS)) ;

66

67 PROFITACCT.. PROFIT =E=

68 SUM(PROCESS,(PRICE(PROCESS)‑PRODCOST(PROCESS))

69 \* PRODUCTION(PROCESS)) ;

70

71 AVAILABLE(RESOURCE)..

72 SUM(PROCESS,RESOURUSE(RESOURCE,PROCESS)\*PRODUCTION(PROCESS))

73 +IDLE(RESOURCE) =E= RESORAVAIL(RESOURCE);

74

75 IDLLABGOAL.. IDLE("LABOR") =E= GOALLEVEL("LABOR") ;

76 PROFITGL1.. PROFIT =E= GOALLEVEL("PROFIT1");

77 PROFITGL2.. PROFIT =E= GOALLEVEL("PROFIT2");

78 LATHEGOAL.. IDLE("LRGLATHE")+IDLE("SMLLATHE")

79 =E= GOALLEVEL("LATHETIME");

80 TARGS(GOALS).. GOALLEVEL(GOALS) + SHORTFALL(GOALS) =G= TARGET(GOALS) ;

81

82 MAXSHORT(GOALS).. SHORTFALL(GOALS) =L= DEV(GOALS);

83

84 MODEL RESALLOC /ALL/;

85 PARAMETER GOALDATA(GOAL,\*,\*)

86 LOOP(GOAL,

87 WEIGHTS(GOAL)=1.

88

89 SOLVE RESALLOC USING LP MINIMIZING GOALOBJ;

90 DEV(GOAL)=SHORTFALL.L(GOAL);

91 WEIGHTS(GOAL)=0.00001;

92 GOALDATA(GOAL,GOALS,"ATTAIN")=GOALLEVEL.L(GOALS) ;

93 GOALDATA(GOAL,GOALS,"SHORT")=SHORTFALL.L(GOALS) ;

94 GOALDATA(GOAL,PROCESS,"XLEVEL")=PRODUCTION.L(PROCESS);

95 );

96 DISPLAY GOALDATA;

|  |
| --- |
|  |

Multi-Objective Programming

Lexicographic Utility – Example Solution

|  |
| --- |
| **Lexicographic Utility**  **Table 11.3 Solution to Lexicographic Example** |

**Goal Being Solution Goal Goal Goal Production**

**Pursued Item Idle Attainment Level Shortfall Level**

PROFIT1 PROFIT1 GOAL 9500.000 9500 0

.LABOR GOAL 25.976 30 4.024

.LATHETIME GOAL 7.927 25 17.073

.PROFIT2 GOAL 9500.000 9500 0

.FUNCTNORM PROD 12.195

.FANCYNORM PROD 108.537

LABOR .PROFIT1 GOAL 9481.579 9500 0

.LABOR GOAL 30.000 30 0

.LATHETIME GOAL 4.359 25 20.641

.PROFIT2 GOAL 9481.579 9500 18.421

.FANCYNORM PROD 115.296

.FANCYMXLRG PROD 3.289

LATHETIME .PROFIT1 GOAL 9000.000 9500 0

.LABOR GOAL 30.000 30 0

.LATHETIME GOAL 20.663 25 4.337

.PROFIT2 GOAL 9000.000 9500 500.000

.FUNCTNORM PROD 15.152

.FANCYNORM PROD 99.811

PROFIT2 .PROFIT1 GOAL 9000.000 9500 0

.LABOR GOAL 30.000 30 0

.LATHETIME GOAL 20.663 25 4.337

.PROFIT2 GOAL 9000.000 9500 500.000

.FUNCTNORM PROD 15.152

.FANCYNORM PROD 99.811

Multi-Objective Programming Weighted Trade-Off

The second utility function type involves tradeoffs between various objectives

First Formulation (No Targets):



* This formulation does not take into account target levels
* The objective function, maximizes multi‑dimensional utility summed across all objectives

Multi-Objective Programming Weighted Trade-Off

The second weighted tradeoff formulation embodies goal target levels

The formulation is:



Multi-Objective Programming

Weighted Trade-Off / Example 1 No Targets:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| .**Table 11.4. Tableau for Weighted Tradeoff Example** | | | | | | | | | | | | | | | | | | | | |
|  |  | Original Decision Variables | | | | | | Profit | Idle Resources | | | | Goal Levels | | | Goal Deviations | | | RHS | |
|  |  | X1 | X2 | X3 | X4 | X5 | X6 |  | Lrg Lathe | Sml Lathe | Carver | Labor | Profit | Idle Labor | Idle Lathe | Profit | Idle Labor | Idle  Lathe |  | |
| Objective | |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 | .4 | .4 | Min | |
| Original Problem Equations | Profit | 67 | 66 | 66.3 | 80 | 78.5 | 78.4 | -1 |  |  |  |  |  |  |  |  |  |  | = | 0 |
|  | Small Lathe | 0.8 | 1.3 | 0.2 | 1.2 | 1.7 | 0.5 |  | 1 |  |  |  |  |  |  |  |  |  | = | 140 |
|  | Large Lathe | 0.5 | 0.2 | 1.3 | 0.7 | 0.3 | 1.5 |  |  | 1 |  |  |  |  |  |  |  |  | = | 90 |
|  | Carver | 0.4 | 0.4 | 0.4 | 1.0 | 1.0 | 1.0 |  |  |  | 1 |  |  |  |  |  |  |  | = | 120 |
|  | Labor | 1.0 | 1.05 | 1.1 | 0.8 | 0.82 | 0.84 |  |  |  |  | 1 |  |  |  |  |  |  | = | 125 |
| Goal Identity | Profit |  |  |  |  |  |  | 1 |  |  |  |  | -1 |  |  |  |  |  | = | 0 |
|  | Idle Labor |  |  |  |  |  |  |  |  |  |  | 1 |  | -1 |  |  |  |  | = | 0 |
|  | Idle Lathe |  |  |  |  |  |  |  | 1 | 1 |  |  |  |  | -1 |  |  |  | = | 0 |
| Goal Level | Profit 9000 |  |  |  |  |  |  |  |  |  |  |  | -1 |  |  | 10500 |  |  | = | 0 |
|  | Idle Labor |  |  |  |  |  |  |  |  |  |  |  |  | -1 |  |  | 125 |  | = | 0 |
|  | Idle Lathe |  |  |  |  |  |  |  |  |  |  |  |  |  | -1 |  |  | 230 | = | 0 |

Multi-Objective Programming

Weighted Trade-Off / Example 1

|  |
| --- |
| **Table 11.5. GAMS Setup for Weighted Objective Example** |

4

5 SET GOALS /PROFIT,LABOR,LATHETIME/

6 PROCESS TYPES OF PRODUCTION PROCESSES

7 /FUNCTNORM , FUNCTMXSML , FUNCTMXLRG

8 ,FANCYNORM , FANCYMXSML , FANCYMXLRG/

9 RESOURCE TYPES OF RESOURCES

10 /SMLLATHE,LRGLATHE,CARVER,LABOR/

11

12 PARAMETER PRICE(PROCESS) PRODUCT PRICES BY PROCESS

13 /FUNCTNORM 82, FUNCTMXSML 82, FUNCTMXLRG 82

14 ,FANCYNORM 105, FANCYMXSML 105, FANCYMXLRG 105/

15 PRODCOST(PROCESS) COST BY PROCESS

16 /FUNCTNORM 15, FUNCTMXSML 16 , FUNCTMXLRG 15.7

17 ,FANCYNORM 25, FANCYMXSML 26.5, FANCYMXLRG 26.6/

18 RESORAVAIL(RESOURCE) RESOURCE AVAILABLITY

19 /SMLLATHE 140, LRGLATHE 90,

20 CARVER 120, LABOR 125/

21 WEIGHTS(GOALS) WEIGHT FOR GOALS

22 /PROFIT 1,LABOR 0.4,LATHETIME 0.4/

23 MAGNITUDE(GOALS) MAGNITUDE FOR GOALS

24 /PROFIT 10500/;

25 MAGNITUDE("LATHETIME")=RESORAVAIL("SMLLATHE")+RESORAVAIL("LRGL ATHE");

26 MAGNITUDE("LABOR")=RESORAVAIL("LABOR");

27

28

29 TABLE RESOURUSE(RESOURCE,PROCESS) RESOURCE USAGE

30

31 FUNCTNORM FUNCTMXSML FUNCTMXLRG

32 SMLLATHE 0.80 1.30 0.20

33 LRGLATHE 0.50 0.20 1.30

34 CARVER 0.40 0.40 0.40

35 LABOR 1.00 1.05 1.10

36 + FANCYNORM FANCYMXSML FANCYMXLRG

37 SMLLATHE 1.20 1.70 0.50

38 LRGLATHE 0.70 0.30 1.50

39 CARVER 1.00 1.00 1.00

40 LABOR 0.80 0.82 0.84;

41

42 POSITIVE VARIABLES

43 PRODUCTION(PROCESS) ITEMS PRODUCED BY PROCESS

44 IDLE(RESOURCE) SLACK VARIABLES FOR RESOURCES

45 GOALLEVEL(GOALS) GOAL LEVELS

46 PROFIT TOTALPROFIT;

47 VARIABLES

48 GOALOBJ GOAL OBJECTIVE;

49 EQUATIONS

50 OBJT OBJECTIVE FUNCTION

51 PROFITACCT PROFIT ACCOUNTING

52 AVAILABLE(RESOURCE) RESOURCES AVAILABLE

53 IDLLABGOAL IDLE LABOR GOAL

54 PROFITGOAL PROFIT GOAL

55 LATHEGOAL IDLE LATHE GOAL;

56

57 OBJT.. GOALOBJ =E= SUM(GOALS,WEIGHTS(GOALS)\*GOALLEVEL(GOALS)) ;

58

59 PROFITACCT.. PROFIT =E=

60 SUM(PROCESS,(PRICE(PROCESS)‑PRODCOST(PROCESS))

61 \* PRODUCTION(PROCESS)) ;

62

63 AVAILABLE(RESOURCE)..

64 SUM(PROCESS,RESOURUSE(RESOURCE,PROCESS)\*PRODUCTION(PROCESS))

65 +IDLE(RESOURCE) =E= RESORAVAIL(RESOURCE);

66

67 IDLLABGOAL.. IDLE("LABOR") =E= GOALLEVEL("LABOR")\*MAGNITUDE("LABOR");

68 PROFITGOAL.. PROFIT =E= GOALLEVEL("PROFIT")\*MAGNITUDE("PROFIT");

69 LATHEGOAL.. IDLE("LRGLATHE")+IDLE("SMLLATHE")

70 =E= GOALLEVEL("LATHETIME")\*MAGNITUDE("LATHETIM E");

71 MODEL RESALLOC /ALL/;

72 SOLVE RESALLOC USING LP MAXIMIZING GOALOBJ;

Multi-Objective Programming Utility Trade-Off / Example 2 With Targets:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| .**Table 11.6. Tableau for Weighted Tradeoff with Targets Example** | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  |  | Original Decision Variables | | | | | | Profit | Idle Resources | | | | Goal Levels | | | | Goal Deviations | | | | | | | | RHS | |
|  |  | X1 | X2 | X3 | X4 | X5 | X6 |  | Small Lathe | Large Lathe | Carv | Labor | Profit  9000 | Idle Labor | Idle Lathe | Profit 9500 | Profit  9000 + | Profit 9000 - | Idle Labor + | Idle Labor - | Idle Lathe + | Idle  Lathe  - | Profit  9500  + | Profit 9500 - |  | |
| Objective | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 | -10 | .1 | -.4 | .1 | -.4 | 9 | -1 | Max | |
| Original Problem Equations | Profit | 67 | 66 | 66.3 | 80 | 78.5 | 78.4 | -1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | = | 0 |
|  | Small Lathe | 0.8 | 1.3 | 0.2 | 1.2 | 1.7 | 0.5 |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | = | 140 |
|  | Large Lathe | 0.5 | 0.2 | 1.3 | 0.7 | 0.3 | 1.5 |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | = | 90 |
|  | Carver | 0.4 | 0.4 | 0.4 | 1.0 | 1.0 | 1.0 |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  | = | 120 |
|  | Labor | 1.0 | 1.05 | 1.1 | 0.8 | 0.82 | 0.84 |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  | = | 125 |
| Goal Level Identity | Profit 9000 level1 |  |  |  |  |  |  | 1 |  |  |  |  | -1 |  |  |  |  |  |  |  |  |  |  |  | = | 0 |
|  | Idle Labor |  |  |  |  |  |  |  |  |  |  | 1 |  | -1 |  |  |  |  |  |  |  |  |  |  | = | 0 |
|  | Idle Lathe |  |  |  |  |  |  |  | 1 | 1 |  |  |  |  | -1 |  |  |  |  |  |  |  |  |  | = | 0 |
|  | Profit 9500 |  |  |  |  |  |  | 1 |  |  |  |  |  |  |  | -1 |  |  |  |  |  |  |  |  | = | 0 |
| Goal Satisfaction | Profit 9000 |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  | -9000 | 9000 |  |  |  |  |  |  | = | 9000 |
|  | Idle Labor |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  |  | -30 | 30 |  |  |  |  | = | 30 |
|  | Idle Lathe |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  |  |  | -25 | 25 |  |  | = | 25 |
|  | Profit 9500 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  |  |  |  | -9500 | 9500 | = | 9500 |

Multi-Objective Programming

Utility Trade-Off / Example 2

|  |
| --- |
| **Table 11.7. GAMS Setup for Weighted Objective with Target Example** |

4

5 SET GOALS GOALS IN THE ORDER THEY ARE TO BE MET

6 /PROFIT1,LABOR,LATHETIME,PROFIT2/

7 PROCESS TYPES OF PRODUCTION PROCESSES

8 /FUNCTNORM , FUNCTMXSML , FUNCTMXLRG

9 ,FANCYNORM , FANCYMXSML , FANCYMXLRG/

10 RESOURCE TYPES OF RESOURCES

11 /SMLLATHE,LRGLATHE,CARVER,LABOR/

12 DIR GOAL DEVIATION DIRECTION /MORETHAN, LESSTHAN/

13

14 ALIAS(GOALS,GOAL) ;

15

16 PARAMETER PRICE(PROCESS) PRODUCT PRICES BY PROCESS

17 /FUNCTNORM 82, FUNCTMXSML 82, FUNCTMXLRG 82

18 ,FANCYNORM 105, FANCYMXSML 105, FANCYMXLRG 105/

19 PRODCOST(PROCESS) COST BY PROCESS

20 /FUNCTNORM 15, FUNCTMXSML 16 , FUNCTMXLRG 15.7

21 ,FANCYNORM 25, FANCYMXSML 26.5, FANCYMXLRG 26.6/

22 RESORAVAIL(RESOURCE) RESOURCE AVAILABLITY

23 /SMLLATHE 140, LRGLATHE 90,

24 CARVER 120, LABOR 125/

25 TARGET(GOALS) GOAL TARGET LEVELS

26 /PROFIT1 9000,LABOR 30,LATHETIME 25

27 ,PROFIT2 9500/

28 MAGNITUDE(GOALS) MAGNITUDE FOR GOALS;

29 MAGNITUDE(GOALS)=TARGET(GOALS);

30

31 TABLE WEIGHTS(GOALS,dir) WEIGHTS BY GOAL

32

33 MORETHAN LESSTHAN

34 PROFIT1 1 ‑10

35 LABOR 0.1 ‑0.4

36 LATHETIME 0.1 ‑0.4

37 PROFIT2 .9 ‑1. ;

38

39 TABLE RESOURUSE(RESOURCE,PROCESS) RESOURCE USAGE

40

41 FUNCTNORM FUNCTMXSML FUNCTMXLRG

42 SMLLATHE 0.80 1.30 0.20

43 LRGLATHE 0.50 0.20 1.30

44 CARVER 0.40 0.40 0.40

45 LABOR 1.00 1.05 1.10

46 + FANCYNORM FANCYMXSML FANCYMXLRG

47 SMLLATHE 1.20 1.70 0.50

48 LRGLATHE 0.70 0.30 1.50

49 CARVER 1.00 1.00 1.00

50 LABOR 0.80 0.82 0.84;

51

52 POSITIVE VARIABLES

53 PRODUCTION(PROCESS) ITEMS PRODUCED BY PROCESS

54 IDLE(RESOURCE) SLACK VARIABLES FOR RESOURCES

55 GOALLEVEL(GOALS) GOAL LEVELS

56 PROFIT TOTALPROFIT

57 SHORTFALL(GOALS) GOAL SHORTFALLS

58 EXCESS(GOALS) GOAL EXCESSES;

59 VARIABLES

60 GOALOBJ GOAL OBJECTIVE;

|  |
| --- |
| **Table 11.7. GAMS Setup for Weighted Objective with Target Example (Continued)** |

61 EQUATIONS

62 OBJT OBJECTIVE FUNCTION

63 PROFITACCT PROFIT ACCOUNTING

64 AVAILABLE(RESOURCE) RESOURCES AVAILABLE

65 IDLLABGOAL IDLE LABOR GOAL

66 PROFITGL1 PROFIT1 GOAL

67 PROFITGL2 PROFIT2 GOA

68 LATHEGOAL IDLE LATHE GOAL

69 TARGS(GOALS) GOAL TARGETS ;

70

71 OBJT.. GOALOBJ =E= SUM(GOALS,WEIGHTS(GOALS,"LESSTHAN")\*SHORTFALL(GOALS)

72 +WEIGHTS(GOALS,"MORETHAN")\*EXCESS(GOALS)) ;

73

74 PROFITACCT.. PROFIT =E=

75 SUM(PROCESS,(PRICE(PROCESS)‑PRODCOST(PROCESS))

76 \* PRODUCTION(PROCESS)) ;

77

78 AVAILABLE(RESOURCE)..

79 SUM(PROCESS,RESOURUSE(RESOURCE,PROCESS)\*PRODUCTION(PROCESS))

80 +IDLE(RESOURCE) =E= RESORAVAIL(RESOURCE);

81

82 PROFITGL1.. PROFIT =E= GOALLEVEL("PROFIT1");

83 IDLLABGOAL.. IDLE("LABOR") =E= GOALLEVEL("LABOR");

84 LATHEGOAL.. IDLE("LRGLATHE")+IDLE("SMLLATHE")

85 =E= GOALLEVEL("LATHETIME");

86 PROFITGL2.. PROFIT =E= GOALLEVEL("PROFIT2");

87

88 TARGS(GOALS)..

89 GOALLEVEL(GOALS) + MAGNITUDE(GOALS)\*( SHORTFALL(GOALS) ‑EXCESS(GOALS))

90 =E= TARGET(GOALS) ;

91

92

93 MODEL RESALLOC /ALL/;

94

95 SOLVE RESALLOC USING LP MAXIMIZING GOALOBJ;

Multi-Objective Programming

Developing Utility Functions

Conceptually, multiobjective programming problems look attractive. However, assuming one knows the objectives, it is difficult to specify the utility structure. Here I address how to find the utility function for the other formulations.

The easiest system to use is the lexicographic system, where one has to establish goal targets and the preemptive order. Targets such as the minimum amount of debt service plus consumption or the desired length of a vacation can be used. However, one must be careful in using these targets in comparative static analysis, as the relative ability to satisfy the targets changes with alterations in the resource base. Also, one must ask whether tradeoffs are in order.

Multi-Objective Programming

Developing Utility Functions

Weights are more difficult but approaches are

* Take decision makers' past actions and then through a grid search over alternative weights, choose weights so as to minimize deviations of the model solution from observed actions.
* Use survey techniques. Here decision makers are asked questions about the relative importance of objectives and then through a scaling procedure a set of objective weights is obtained.
* Have decision makers pick best solution and use goal weights consistent with the decision maker's preference
* Assume based on prior studies

Finally, we must comment that there is no real way to abstractly set up a multiple objective model. The weights for the multiple objectives clearly require interaction with the decision maker.

Multi-Objective Programming

Shadow Prices



Rearranging, we obtain



So



or, in matrix terms,

C = WG

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Shadow Prices

U = CB B‑1

The CB terms within the multiobjective programming model are given by the multiplication of goal weights times the goal levels involved with the basic variables



where the superscript b on the g terms refer to the coefficients associated with the basic variable in the various objectives.

The shadow price term can be rewritten as

U = CB B‑1 = W GB B‑1

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Shadow Prices

Suppose we were to maximize the following multiple goal objective problem and we are willing to assume that the weights are each 1.



Including the weights the composite objective function is



Solving this problem our solution is X1=7.5 and X2=2.5. The basis matrix and its inverse are



Multi-Objective Programming

Shadow Prices

The composite shadow prices are



However, if we break this down we get

