**Dynamic Linear Programming**

**Disequilibrium- Known Life**

Primal Algebra:



Where



**Inspecting the resource equation**

for year t

Suppose Kj =3 and for we expand the sum over e

This is for year t

is the *e=0* case and gives resource use in year when activity is new (*e=0*) times the amount started in year t-*e* or t-*0* or t

is the e=1 case and gives resource use when activity is one year old (e=1) times the amount started last year in year t-1 (t-e)

is the e=2 case and gives resource use when activity is two years old (e=2) times the amount started two years ago in year t-2

So we add up resource use for new plus one year old plus two years old when we discontinue in start of year 3

**Dynamic Linear Programming**

**Disequilibrium- Known Life / Example**

The Strawberry Problem:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Year 0 | Year 1 | Year 2 |  |
| Cost/Acre | 150 | 280 | 300 |  |
| Yield/acre in tons | 0 | 7 | 7 |  |
| Water/acre in acre-feet | .8 | 4.5 | 4.5 |  |

The price per ton of strawberries is $140. The farm has 700 acres, 50 planted in 0 year old strawberries and 10 in 1 year old strawberries. Water available consists of 1200 acre ft. per year.

We need terminal conditions to value those items which are carried into the fifth and beyond years. Assume that the following values have been derived.

Product Terminal Value

New strawberries (0 years old the year after the model) $160/acre

1 year old strawberries $110/acre

**Dynamic Linear Programming**

**Disequilibrium- Known Life / Example**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 8.1. Disequilibrium Known Life Example** | | | | | | | | | | | | | | |
|  | Year 1 | | Year 2 | | Year 3 | | Year 4 | | Year 5 | | Terminal  Conditions | |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Rows | Wheat | Straw-berries | Wheat | Straw-berries | Wheat | Straw-berries | Wheat | Straw-berries | Wheat | Straw-berries | Strawberries | | RHS | |
| Aje 0 | Aje 1 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Objective | 340 | 1230 | 340 | 1230 | 340 | 1230 | 340 | 550 | 340 | ‑150 | 160 | 110 | Max | |
| Land Year 1 | 1 | 1 |  |  |  |  |  |  |  |  |  |  | ≤ | 640 |
| Water Year 1 | 1 | 0.8 |  |  |  |  |  |  |  |  |  |  | ≤ | 930 |
| Land Year 2 |  | 1 | 1 | 1 |  |  |  |  |  |  |  |  | ≤ | 690 |
| Water Year 2 |  | 4.5 | 1 | 0.8 |  |  |  |  |  |  |  |  | ≤ | 1155 |
| Land Year 3 |  | 1 |  | 1 | 1 | 1 |  |  |  |  |  |  | ≤ | 700 |
| Water Year 3 |  | 4.5 |  | 4.5 | 1 | 0.8 |  |  |  |  |  |  | ≤ | 1200 |
| Land Year 4 |  |  |  | 1 |  | 1 | 1 | 1 |  |  |  |  | ≤ | 700 |
| Water Year 4 |  |  |  | 4.5 |  | 4.5 | 1 | 0.8 |  |  |  |  | ≤ | 1200 |
| Land Year 5 |  |  |  |  |  | 1 |  | 1 | 1 | 1 |  |  | ≤ | 700 |
| Water Year 5 |  |  |  |  |  | 4.5 |  | 4.5 | 1 | 0.8 |  |  | ≤ | 1200 |
| Strawberry 0 |  |  |  |  |  |  |  |  |  | ‑1 | 1 |  | ≤ | 0 |
| Strawberry 1 |  |  |  |  |  |  |  | ‑1 |  |  |  | 1 | ≤ | 0 |

**Dynamic Linear Programming**

**Disequilibrium- Known Life / Example**

Solutions:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Objective = 1224296 | | | | | |
| Variables | Value | Reduced Cost | Equation | Slack | Shadow Price |
| Wheat Year 1 | 506.2 | 0 | Land Year 1 | 0 | 340.0 |
| Strawberries Year 1 | 133.8 | 0 | Water Year 1 | 316.76 | 0 |
| Wheat Year 2 | 539.9 | 0 | Land Year 2 | 0 | 283.1 |
| Strawberries Year 2 | 16.3 | 0 | Water Year 2 | 0 | 56.9 |
| Wheat Year 3 | 423.4 | 0 | Land Year 3 | 0 | 336.9 |
| Strawberries Year 3 | 126.6 | 0 | Water Year 3 | 0 | 3.1 |
| Wheat Year 4 | 557.1 | 0 | Land Year 4 | 0 | 279.8 |
| Strawberries Year 4 | 0 | 0 | Water Year 4 | 0 | 60.2 |
| Wheat Year 5 | 573.4 | 0 | Land Year 5 | 0 | 340.0 |
| Strawberries Year5 | 0 | 0 | Water Year 5 | 57.05 | 0 |
| Term Straw -0 | 0 | 0 | Strawberry 0 | 0 | -160 |
| Term Straw -1 | 0 | -8 | Strawberry 1 | 0 | -118 |

**Dynamic Linear Programming**

**Disequilibrium- Unknown Life**

##### Primal Algebra



**Dynamic Linear Programming**

**Disequilibrium- Unknown Life / Example**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Year 0 | Year 1 | Year 2 |  |
| Cost/Acre | 150 | 280 | 300 |  |
| Yield/acre in tons | 0 | 7 | 7 |  |
| Water/acre in acre-feet | .8 | 4.5 | 4.5 |  |

For this example we use the data above plus longer retention of strawberries. Assume they may be kept up to 4 years and that in the fourth year the planting costs $400 with the yield being 5, and water use being 5.7 acre feet. We will also assume that the terminal value of 3‑year old strawberries is $20/acre.

**Disequilibrium- Unknown Life / Example**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 8.3 Disequilibrium Unknown Life Sample Problem** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |  |
|  |  | Year 1 | | | | | Year 2 | | | | | Year 3 | | | | | Year 4 | | | | | Year 5 | | | | | Terminal Conditions | | |  |
|  | |  | Strawberries | | | |  | Strawberries | | | |  | Strawberries | | | |  | Strawberries | | | |  | Strawberries | | | | Strawberries | | | RHS |
| Rows | | Wheat | 0 | 1 | 2 | 3 | Wheat | 0 | 1 | 2 | 3 | Wheat | 0 | 1 | 2 | 3 | Wheat | 0 | 1 | 2 | 3 | Wheat | 0 | 1 | 2 | 3 | 0 | 1 | 2 |
|  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | Max |
| Objective | | 340 | ‑150 | 700 | 680 | 300 | 340 | ‑150 | 700 | 680 | 300 | 340 | ‑150 | 700 | 680 | 300 | 340 | ‑150 | 700 | 680 | 300 | 340 | ‑150 | 700 | 680 | 300 | 160 | 110 | 20 |
| Land Year 1 | | 1 | 1 | 1 | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 700 |
| Water Year 1 | | 1 | 0.8 | 4.5 | 4.5 | 5.7 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 1200 |
| Init Straw 0 | |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 50 |
| Init Straw 1 | |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 10 |
| Init Straw 2 | |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 0 |
| Straw 0-1 Year 1 | |  | ‑1 |  |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 0 |
| Straw 1-2 Year 1 | |  |  | ‑1 |  |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 0 |
| Straw 2-3 Year 1 | |  |  |  | ‑1 |  |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 0 |
| Land Year 2 | |  |  |  |  |  | 1 | 1 | 1 | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 700 |
| Water Year 2 | |  |  |  |  |  | 1 | 0.8 | 4.5 | 4.5 | 5.7 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 1200 |
| Straw 0-1 Year 2 | |  |  |  |  |  |  | ‑1 |  |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 0 |
| Straw 1-2 Year 2 | |  |  |  |  |  |  |  | ‑1 |  |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 0 |
| Straw 2-3 Year 2 | |  |  |  |  |  |  |  |  | ‑1 |  |  |  |  | 100 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 0 |
| Land Year 3 | |  |  |  |  |  |  |  |  |  |  | 1 | 1 | 1 | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 700 |
| Water Year 3 | |  |  |  |  |  |  |  |  |  |  | 1 | 0.8 | 4.5 | 4.5 | 5.7 |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 1200 |
| Straw 0-1 Year 3 | |  |  |  |  |  |  |  |  |  |  |  | ‑1 |  |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  | ≤ 0 |
| Straw 1-2 Year 3 | |  |  |  |  |  |  |  |  |  |  |  |  | ‑1 |  |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  | ≤ 0 |
| Straw 2-3 Year 3 | |  |  |  |  |  |  |  |  |  |  |  |  |  | ‑1 |  |  |  |  |  | 1 |  |  |  |  |  |  |  |  | ≤ 0 |
| Land Year 4 | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 | 1 | 1 | 1 | 1 |  |  |  |  |  |  |  |  | ≤ 700 |
| Water Year 4 | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 | 0.8 | 4.5 | 4.5 | 5.7 |  |  |  |  |  |  |  |  | ≤ 1200 |
| Straw 0-1 Year 4 | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ‑1 |  |  |  |  |  | 1 |  |  |  |  |  | ≤ 0 |
| Straw 1-2 Year 4 | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ‑1 |  |  |  |  |  | 1 |  |  |  |  | ≤ 0 |
| Straw 2-3 Year 4 | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ‑1 |  |  |  |  |  | 1 |  |  |  | ≤ 0 |
| Land Year 5 | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 | 1 | 1 | 1 | 1 |  |  |  | ≤ 700 |
| Water Year 5 | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 | 0.8 | 4.5 | 4.5 | 5.7 |  |  |  | ≤ 1200 |
| Term Straw 0 | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ‑1 |  |  |  | 1 |  |  | ≤ 0 |
| Term Straw 1 | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ‑1 |  |  |  | 1 |  | ≤ 0 |
| Term Straw 2 | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ‑1 |  |  |  | 1 | ≤ 0 |

**Dynamic Linear Programming**

**Disequilibrium- Unknown Life / Example**

Solutions:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Table 8.4. Disequilibrium Unknown Life Example Model Solution** | | | | | |
| Objective = 1280757.0 | | | | | |
| Variable | Value | Reduced Cost | Equation | Slack | Shadow Price |
| Wheat year 1 | 547.1 | 0 | Land Year 1 | 0 | 340 |
| Straw 0 year old year 1 | 92.9 | 0 | Water Year 1 | 308.57 | 0 |
| Straw 1 year old year 1 | 50.0 | 0 | Init Straw 0 | 0 | 490 |
| Straw 2 year old year 1 | 10.0 | 0 | Init Straw 1 | 0 | 340 |
| Straw 3 year old year 1 | 0 | -40 | Init Straw 2 | 0 | 0 |
| Wheat year 2 | 557.1 | 0 | Straw 0-1 Year 1 | 0 | -490 |
| Straw 0 year old year 2 | 0 | -8 | Straw 1-2 Year 1 | 0 | -130 |
| Straw 1 year old year 2 | 92.9 | 0 | Straw 2-3 Year 1 | 10 | 0 |
| Straw 2 year old year 2 | 50.0 | 0 | Land Year 2 | 0 | 280 |
| Straw 3 year old year 2 | 0 | -322 | Water Year 2 | 0 | 60 |
| Wheat year 3 | 464.3 | 0 | Straw 0-1 Year 2 | 0 | -470 |
| Straw 0 year old year 3 | 142.9 | 0 | Straw 1-2 Year 2 | 0 | -340 |
| Straw 1 year old year 3 | 0 | 0 | Straw 2-3 Year 2 | 50 | 0 |
| Straw 2 year old year 3 | 92.9 | 0 | Land Year 3 | 0 | 340 |
| Straw 3 year old year 3 | 0 | -40 | Water Year 3 | 203.57 | 0 |
| Wheat year 4 | 557.1 | 0 | Straw 0-1 Year 3 | 0 | -490 |
| Straw 0 year old year 4 | 0 | 0 | Straw 1-2 Year 3 | 0 | -110 |
| Straw 1 year old year 4 | 142.9 | 0 | Straw 2-3 Year 3 | 92.857 | 0 |
| Straw 2 year old year 4 | 0 | 0 | Land Year 4 | 0 | 274 |
| Straw 3 year old year 4 | 0 | -349 | Water Year 4 | 0 | 65.7 |
| Wheat year 5 | 557.1 | 0 | Straw 0-1 Year 4 | 0 | -470 |
| Straw 0 year old year 5 | 0 | -330 | Straw 1-2 Year 4 | 0 | -360 |
| Straw 1 year old year 5 | 0 | 0 | Straw 2-3 Year 4 | 0 | 0 |
| Straw 2 year old year 5 | 142.9 | 0 | Land Year 5 | 0 | 340 |
| Straw 3 year old year 5 | 0 | -40 | Water Year 5 | 0 | 0 |
| Term Straw 0 |  | 0 | Term Straw 0 | 0 | -160 |
| Term Straw 1 |  | 0 | Term Straw 1 | 0 | -110 |
| Term Straw 2 |  | 0 | Term Straw 2 | 142.86 | -20 |

**Dynamic Linear Programming**

**Equilibrium- Known Life**

Equilibrium models are developed as follows: assume we have a variable with life of 4 periods and resource use, yield,7 etc., equal to Ae, where e is the elapsed age of the activity (0-3). Let us (assuming we start with zero ini­tial activity) portray the resource use over several periods.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Begin Activity in Period | | | | | | |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Period 1 Resource Usage | A0 |  |  |  |  |  |  |
| Period 2 Resource Usage | A1 | A0 |  |  |  |  |  |
| Period 3 Resource Usage | A2 | A1 | A0 |  |  |  |  |
| Period 4 Resource Usage | A3 | A2 | A1 | A0 |  |  |  |
| Period 5 Resource Usage |  | A3 | A2 | A1 | A0 |  |  |
| Period 6 Resource Usage |  |  | A3 | A2 | A1 | A0 |  |
| Period 7 Resource Usage |  |  |  | A3 | A2 | A1 | A0 |

A3Xt-3+ A2Xt-2+ A1Xt-1+ A0Xt

Xt-3=Xt-2= Xt-1=Xt =X

A3X+ A2X+ A1X+ A0X

(A3+ A2+ A1+ A0)X

Algebraically



**Dynamic Linear Programming**

**Equilibrium- Known Life**

**Example 1a**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Table 8.5. Equilibrium Known Life Example Formulation** | | | | |
|  | Wheat | Strawberries |  |  |
| Objective | 340 | 1230 |  |  |
| Land | 1 | 3 | ≤ | 700 |
| Water | 1 | 9.8 | ≤ | 1200 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Table 8.6. Equilibrium Known Life Example Solution** | | | | | |
| Objective = 253441 | | | | | |
| Variables | Value | Reduced Cost | Equation | Slack | Shadow Price |
| Wheat | 479 | 0 | Land | 0 | 309 |
| Strawberries | 74 | 0 | Water | 0 | 31 |

**Dynamic Linear Programming**

**Equilibrium- Known Life**

**Example 1b**

The above model can also be re-expressed in terms of average resource use. This is done in Table 8.7, where an average of one acre of land is used every year generates an average of $410 and the usage of 3.27 acre feet of water. The solution for this model essentially identical to the solution for the previous model, but the strawberry variable equals 221. This indicates that the equilibrium solution averages 221 acres of strawberries. Thus, in the strawberry rotation, one-third of the 221 acres (or 74 as in the earlier model) would be first year, one-third second year and one-third third year.

|  |  |  |  |
| --- | --- | --- | --- |
| **Table 8.7. Equilibrium Known Life Example Formulation with Average Activities** | | | |
|  | Wheat | Strawberries |  |
| Objective | 340 | 410 |  |
| Land | 1 | 1 | ≤ 700 |
| Water | 1 | 3.27 | ≤ 1200 |

**Dynamic Linear Programming**

**Equilibrium- Known Life**

**Toward a crop rotation**

Suppose we can grow 2 crops

Suppose their yields depend on what proceeds them on the land

(ie corn after corn has a different yield structure than does corn after soybeans and soybeans after soybeans has different performance than does soybeans after corn)

Lets look at modeling land precedence first in a one year model

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | ------------- Year1 ------------ | | | | | | ------------- Year2 ------------ | | | | | |  |
|  |  | Plow after corn | Plow after Soy | Plant corn after corn | Plant corn after soy | Plant soy after corn | Plant soy after Soy | Plow after corn | Plow after Soy | Plant corn after corn | Plant corn after soy | Plant soy after corn | Plant soy after Soy |  |
| -Year 1 - | **Land after corn** | **1** |  |  |  |  |  |  |  |  |  |  |  | **≤ last yr corn** |
| **Land after soybeans** |  | **1** |  |  |  |  |  |  |  |  |  |  | **≤ last yr soy** |
| Land Plowed aft corn | -1 |  | 1 |  | 1 |  |  |  |  |  |  |  | ≤ 0 |
| Land Plowed aft soy |  | -1 |  | 1 |  | 1 |  |  |  |  |  |  | ≤ 0 |
| -Year 2 - | **Land after corn next yr** |  |  | **-1** | **-1** |  |  |  |  |  |  |  |  | ≤ 0 |
| **Land aft soybeans nxt yr** |  |  |  |  | **-1** | **-1** |  |  |  |  |  |  | ≤ 0 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |

**Dynamic Linear Programming**

**Equilibrium- Known Life**

**Toward a crop rotation**

Now Lets look at the land precedence in a two year model

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | ------------- Year1 ------------ | | | | | | ------------- Year2 ------------ | | | | | |  |
|  |  | Plow after corn | Plow after Soy | Plant corn after corn | Plant corn after soy | Plant soy after corn | Plant soy after Soy | Plow after corn | Plow after Soy | Plant corn after corn | Plant corn after soy | Plant soy after corn | Plant soy after Soy |  |
| -Year 1 - | Land after corn | 1 |  |  |  |  |  |  |  |  |  |  |  | ≤ last yr corn |
| Land after soybeans |  | 1 |  |  |  |  |  |  |  |  |  |  | ≤ last yr soy |
| Plowed aft corn | -1 |  | 1 |  | 1 |  |  |  |  |  |  |  | ≤ 0 |
| Plowed aft soy |  | -1 |  | 1 |  | 1 |  |  |  |  |  |  | ≤ 0 |
| -Year 2 - | **Land after corn** |  |  | **-1** | **-1** |  |  | **1** |  |  |  |  |  | **≤ 0** |
| **Land after soybeans** |  |  |  |  | **-1** | **-1** |  | **1** |  |  |  |  | **≤ 0** |
| Plowed aft corn |  |  |  |  |  |  | -1 |  | 1 |  | 1 |  | ≤ 0 |
| Plowed aft soy |  |  |  |  |  |  |  | -1 |  | 1 |  | 1 | ≤ 0 |

**Dynamic Linear Programming**

**Equilibrium- Known Life**

**Toward a crop rotation**

But what if we go equilibrium - Lets look at the land precedence

Since next years plowing equals this years

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | ------------- Year1 ------------ | | | | | | ------------- Year2 ------------ | | | | | |  |
|  |  | Plow after corn | Plow after Soy | Plant corn after corn | Plant corn after soy | Plant soy after corn | Plant soy after Soy | Plow after corn | Plow after Soy | Plant corn after corn | Plant corn after soy | Plant soy after corn | Plant soy after Soy |  |
| -Year 1 - | Land after corn | 1 |  | **-1** | **-1** |  |  |  |  |  |  |  |  | ≤ 0 |
| Land after soybeans |  | 1 |  |  | **-1** | **-1** |  |  |  |  |  |  | ≤ 0 |
| Plowed aft corn | -1 |  | 1 |  | 1 |  |  |  |  |  |  |  | ≤ 0 |
| Plowed aft soy |  | -1 |  | 1 |  | 1 |  |  |  |  |  |  | ≤ 0 |
| -Year 2 - | **Land after corn** |  |  | **-1** | **-1** |  |  | **1** |  |  |  |  |  | **≤ 0** |
| **Land after soybeans** |  |  |  |  | **-1** | **-1** |  | **1** |  |  |  |  | **≤ 0** |
| Plowed aft corn |  |  |  |  |  |  | -1 |  | 1 |  | 1 |  | ≤ 0 |
| Plowed aft soy |  |  |  |  |  |  |  | -1 |  | 1 |  | 1 | ≤ 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | -------- Typical Year ------- | | | | | |  |
|  | Plow after corn | Plow after Soy | Plant corn after corn | Plant corn after soy | Plant soy after corn | Plant soy after Soy |  |
| Land after corn | 1 |  | **-1** | **-1** |  |  | ≤ 0 |
| Land after soybeans |  | 1 |  |  | **-1** | **-1** | ≤ 0 |
| Plowed aft corn | -1 |  | 1 |  | 1 |  | ≤ 0 |
| Plowed aft soy |  | -1 |  | 1 |  | 1 | ≤ 0 |

**Dynamic Linear Programming**

**Equilibrium- Known Life**

**Example 2**

###### Technical Data

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Corn | Soybeans |
| Prices/Unit | | $2.50 | $6.50 |
| Production Cost/ Acre | | 100 | 50 |
| Labor Use in hrs/acre | | | |
| Plowing | | .4 | .3 |
| Planting | | .15 | .15 |
| Harvesting | | .35 | .17 |

Yields:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Corn after Corn Planting Period | | Corn After Soybeans  Planting Period | | Soybeans  (after Anything) Planting Period | |
|  |  | Pd2 | Pd3 | Pd2 | Pd3 | Pd2 | Pd3 |
| Harvest | Pd4 | 130 | 120 | 145 | 133 | 35 | 45 |
| Period | Pd5 | 125 | 110 | 137 | 129 | 33 | 42 |

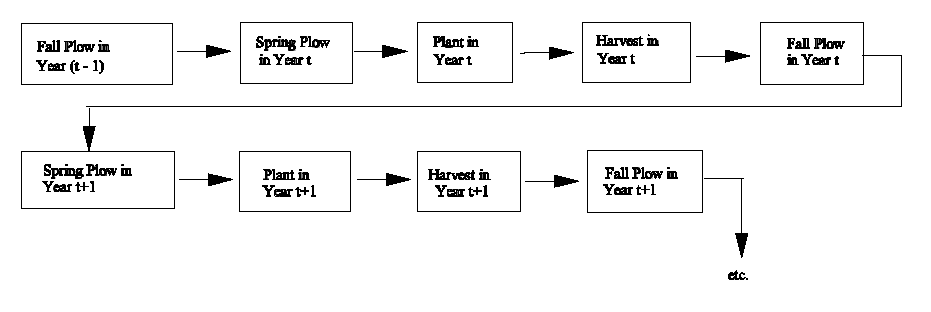
|  |  |
| --- | --- |
| Labor Availability (hrs) | |
| Period | Available Labor |
| Post harvest/Preplant (Pd1) | 80 |
| Plant (Pd2) | 65 |
| Plant (Pd3) | 75 |
| Harvest (Pd4) | 100 |
| Harvest (Pd5) | 80 |

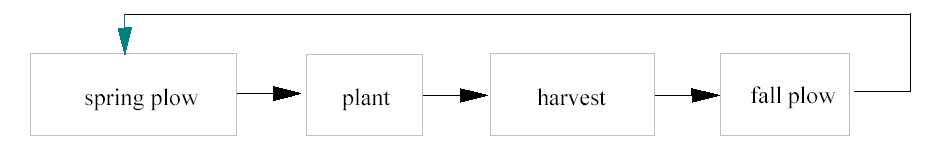
**Dynamic Linear Programming**

**Equilibrium- Known Life**

**Example 2**

The Dynamic Process





**Dynamic Linear Programming**

**Equilibrium- Known Life / Example 2**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 8.8 Rotation Example in Equilibrium Known Life Context** | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  |  | | | | |  | | | | | Plant Corn | | | | | | | | Plant Soybeans | | | | | | | | RHS |
|  | Plow after corn | | | | | Plow after soybeans | | | | | after corn | | | | after soybeans | | | | after corn | | | | after soybeans | | | |  |
| Rows |  | | | | |  | | | | | pl2 | pl2 | pl3 | pl3 | pl2 | pl2 | pl3 | pl3 | pl2 | pl2 | pl3 | pl3 | pl2 | pl2 | pl3 | pl3 |  |
|  | pd1 | pd2 | pd3 | pd4 | pd5 | pd1 | pd2 | pd3 | pd4 | pd5 | hr4 | hr5 | hr4 | hr5 | hr4 | hr5 | hr4 | hr5 | hr4 | hr5 | hr4 | hr5 | hr4 | hr5 | hr4 | hr5 |  |
| Objective |  |  |  |  |  |  |  |  |  |  | 225 | 213 | 200 | 175 | 263 | 243 | 233 | 223 | 178 | 165 | 243 | 223 | 178 | 165 | 243 | 223 | Max |
| Land | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 400 |
| Labor pd1 | .4 |  |  |  |  | .3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ 80 |
| Labor pd2 |  | .4 |  |  |  |  | .3 |  |  |  | .2 | .2 |  |  | .2 | .2 |  |  | .2 | .2 |  |  | .2 | .2 |  |  | ≤ 65 |
| Labor pd3 |  |  | .4 |  |  |  |  | .3 |  |  |  |  | .2 | .2 |  |  | .2 | .2 |  |  | .2 | .2 |  |  | .2 | .2 | ≤ 75 |
| Labor pd4 |  |  |  | .4 |  |  |  |  | .3 |  | .4 |  | .4 |  | .4 |  | .4 |  | .3 |  | .3 |  | .3 |  | .3 |  | ≤ 100 |
| Labor pd5 |  |  |  |  | .4 |  |  |  |  | .3 |  | .4 |  | .4 |  | .4 |  | .4 |  | .3 |  | .3 |  | .3 |  | .3 | ≤ 80 |
| plow after corn pd4 |  |  |  | 1 |  |  |  |  |  |  | ‑1 |  | ‑1 |  | ‑1 |  | ‑1 |  |  |  |  |  |  |  |  |  | ≤ 0 |
| plow after corn pd5 | 1 | 1 | 1 | 1 | 1 |  |  |  |  |  | ‑1 | ‑1 | ‑1 | ‑1 | ‑1 | ‑1 | ‑1 | ‑1 |  |  |  |  |  |  |  |  | ≤ 0 |
| plow after soyb pd4 |  |  |  |  |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  | ‑1 |  | ‑1 |  | ‑1 |  | ‑1 |  | ≤ 0 |
| plow after soyb pd5 |  |  |  |  |  | 1 | 1 | 1 | 1 | 1 |  |  |  |  |  |  |  |  | ‑1 | ‑1 | ‑1 | ‑1 | ‑1 | ‑1 | ‑1 | ‑1 | ≤ 0 |
| plnt aft plow corn pd2 | ‑1 | ‑1 |  | ‑1 | ‑1 |  |  |  |  |  | 1 | 1 |  |  |  |  |  |  | 1 | 1 |  |  |  |  |  |  | ≤ 0 |
| plnt aft plow corn pd3 | ‑1 | ‑1 | ‑1 | ‑1 | ‑1 |  |  |  |  |  | 1 | 1 | 1 | 1 |  |  |  |  | 1 | 1 | 1 | 1 |  |  |  |  | ≤ 0 |
| plnt aft plow soy pd2 |  |  |  |  |  | ‑1 | ‑1 |  | ‑1 | ‑1 |  |  |  |  | 1 | 1 |  |  |  |  |  |  | 1 | 1 |  |  | ≤ 0 |
| plnt aft plow soy pd3 |  |  |  |  |  | ‑1 | ‑1 | ‑1 | ‑1 | ‑1 |  |  |  |  | 1 | 1 | 1 | 1 |  |  |  |  | 1 | 1 | 1 | 1 | ≤ 0 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

**Hi**

**Dynamic Linear Programming**

**Equilibrium- Known Life / Example 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Table 8.9. Solution to Rotation Model** | | | | | |
| Objective= 100777 | | | |  |  |
| Variable | Value | Reduced Cost | Equation | Slack | Shadow  Price |
| Plow After Corn |  |  | Land | 0 | 238 |
| pd1 | 0 | 0 | Labor pd1 | 80 | 0 |
| pd2 | 87.5 | 0 | Labor pd2 | 0 | 0 |
| pd3 | 72.5 | 0 | Labor pd3 | 16 | 0 |
| pd4 | 0 | -22 | Labor pd4 | 0 | 56 |
| pd5 | 0 | 0 | Labor pd5 | 0 | 0 |
| Plow After Soybeans |  |  | Plant After Plow |  |  |
| pd1 | 0 | 0 | corn pd2 | 128 | 0 |
| pd2 | 0 | 0 | corn pd3 | 0 | 238 |
| pd3 | 0 | 0 | soybeans pd2 | 0 | 0 |
| pd4 | 0 | -17 | soybeans pd3 | 0 | 253 |
| pd5 | 200 | 0 | Plow After |  |  |
| Plant corn after corn |  |  | corn pd4 | 200 | 0 |
| pl2 hr4 | 0 | -33 | corn pd5 | 0 | 0 |
| pl2 hr5 | 0 | -26 | soyb pd4 | 189 | 0 |
| pl3 hr4 | 0 | -58 | soyb pd5 | 0 | 15 |
| pl3 hr5 | 0 | -63 |  |  |  |
| Plant Corn After Soybeans |  |  |  |  |  |
| pl2 hr4 | 200 | 0 |  |  |  |
| pl2 hr5 | 0 | -11 |  |  |  |
| pl3 hr4 | 0 | -30 |  |  |  |
| pl3 hr5 | 0 | -31 |  |  |  |
| Plant Soybeans After Corn |  |  |  |  |  |
| pl2 hr4 | 0 | -70 |  |  |  |
| pl2 hr5 | 0 | -59 |  |  |  |
| pl3 hr4 | 189 | 0 |  |  |  |
| pl3 hr5 | 11 | 0 |  |  |  |
| Plant Soybeans After Soybeans |  |  |  |  |  |
| pl2 hr4 | 0 | -70 |  |  |  |
| pl2 hr5 | 0 | -74 |  |  |  |
| pl3 hr4 | 0 | -5 |  |  |  |
| pl3 hr5 | 0 | -15 |  |  |  |

**Dynamic Linear Programming**

**Equilibrium- Unknown Life**



where: Xje is the quantity of the jth activity produced and kept until it is e periods old;

Cje is the per unit returns to Xje;

Aije is the per unit usage of resource i by Xje;

bi is the amount of resource i available;

Objective: The model maximizes profits subject to constraints on resource use and age sequencing.

Constraints: The age sequencing constraints state that the amount of enterprise j of age e must be less than or equal to the amount of that enterprise that was kept until age e‑1.

**Dynamic Linear Programming**

**Equilibrium- Unknown Life / Example**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 8.10. Equilibrium Unknown Life Example Formulation** | | | | | | | |
|  |  | Strawberries | | | | RHS | |
| Rows | Wheat | 1 | 2 | 3 | 4 | MAX | |
|  |  |  |  |  |  |  |  |
| Objective | 340 | ‑150 | 700 | 680 | 300 |  |  |
| Land | 1 | 1 | 1 | 1 | 1 | ≤ | 700 |
| Water | 1 | 0.8 | 4.5 | 4.5 | 5.7 | ≤ | 1200 |
| Straw 1-2 |  | ‑1 | 1 |  |  | ≤ | 0 |
| Straw 2-3 |  |  | ‑1 | 1 |  | ≤ | 0 |
| Straw 3-4 |  |  |  | ‑1 | 1 | ≤ | 0 |

**Dynamic Linear Programming**

**Equilibrium- Unknown Life / Example**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Table 8.11. Equilibrium Unknown Life Example Solution** | | | | | | |
| Objective = 253441 | | | | | | |
| Variables | Value | Reduced Cost |  | Equation | Slack | Shadow  Price |
| Wheat | 479 | 0 |  | Land | 0 | 309 |
| Strawberries 1 year old | 74 | 0 |  | Water | 0 | 31 |
| Strawberries 2 year old | 74 | 0 |  | Straw 1-2 | 0 | 483 |
| Strawberries 3 year old | 74 | 0 |  | Straw 2-3 | 0 | 232 |
| Strawberries 4 year old | 0 | 185 |  | Straw 3-4 | 74 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 8.12. Alternative Formulation of Equilibrium Unknown Life** | | | | | | | |
|  |  | Keep Strawberries | | | | RHS | |
| Rows | Wheat | 1 | 2 | 3 | 4 |
|  |  |  |  |  |  | MAX | |
| Objective | 340 | ‑150 | 550 | 1230 | 1530 |
| Land | 1 | 1 | 2 | 3 | 4 | ≤ | 700 |
| Water | 1 | 0.8 | 5.3 | 9.8 | 15.5 | ≤ | 1200 |

**Dynamic Linear Programming**

**Recursive Programming**



Where: Cjt are objective function parameters functionally dependent upon the previous objective function parameters (Cj,t‑1), lagged optimal decision variables (Xjt‑1, Ykt‑1), and exogenous events;

Xjt are the values of the decision variables at time t;

Aijt are the resource i usages by Xjt functionally dependent upon lagged values, dk are objective function coefficients which are stable over time;

Ykt are the optimal values of the kth Y variable in time period t; Eik are the usages of resource i which do not change over time;

bit are the resource i limits, functionally dependent upon lagged phenomena.

**Dynamic Linear Programming**

Recursive Programming / Example



where: Plt+1 = 20 - 0.00045 Qit

Qlt = 130 Xlt

P2t+1 = 10.9 - 0.00045 Q2t

Q2t = 45 X2t

**Dynamic Linear Programming**

**Recursive Programming / Example**

Solution

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time Period | X 1t | X2t | P1t+1 | P2t+1 | Zt |
| 0 | 300.000 | 300.000 | 2.450 | 4.825 | ---- |
| 1 | 306.000 | 294.000 | 2.099 | 4.703 | 94995.750 |
| 2 | 311.880 | 288.120 | 1.755 | 4.584 | 79491.454 |
| 3 | 306.118 | 293.882 | 2.092 | 4.701 | 64163.394 |
| 4 | 311.995 | 288.005 | 1.748 | 4.582 | 79182.851 |
| 5 | 306.235 | 293.765 | 2.085 | 4.699 | 63860.831 |
| 6 | 312.110 | 287.890 | 1.742 | 4.580 | 78874.189 |