**More on LP formulations**

**In these notes we turn to general rules for formulating problems including :**

**a. Variable/Constraint Identification**

**b. Data Development**

**c Homogeneity of units**

**d. Relaxation of Assumptions**

**Toward Proper Modeling**

**Formulating an applied LP problem**

Identify constraints, variables, relevant parameter values

**Constraints: AX ~ b**

·      Types: technical, institutional, subjective, etc.

·      # of constraints affects # of non-zero variables

·      carefully set up constraints – is this constraint necessary?

·      What restriction should it be <, >, or =?

·      When should it be relaxed?

**Variables Identification:**

·      Types: technical, accounting, convenience

**Objective Function:**

·      Maximization/Minimization

·      Determines optimal solution

Toward Proper Modeling

**Formulating an applied LP problem**

**Development of Model Structure**

Example

: A profit maximizing firm produces 4 crops

s.t. land and labor constraints. Crops are grown at different times of the year.

Crop1 is planted in the spring and harvested in the summer.

Crop2-3 are planted in the spring and harvested in the fall.

Crop4 is planted following crop1 and harvested in the fall.

1. Making a table laying out potential variables across the top and constraints /objective function down the side.

across the top and constraints /objective function down the side.

2. Entering profits for crops and resource coefficient and endowment.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Table 6.1. Initial Schematic for Example Farm Planning Problem** | | | | | |
|  | Crop 1 | Crop 2 | Crop 3 | Crop 4 | RHS |
| Objective | c1 | c2 | c3 | c4 |  |
| Land | 1 | 1 | 1 |  | < L |
| Labor |  |  |  |  |  |

**Toward Proper Modeling**

**Formulating an applied LP problem**

**Development of Model Structure**

1. Linking land use by crop4 to land use by crop1

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Table 6.2. Revised Schematic for Example Farm Planning Problem** | | | | | |
|  | Crop 1 | Crop 2 | Crop 3 | Crop 4 | RHS |
| Objective | c1 | c2 | c3 | c4 |  |
| Land | 1 | 1 | 1 |  | < L |
| Land After Crop 1 | -1 |  |  | 1 | < 0 |
| Labor |  |  |  |  |  |

**Development of Model Structure**

1. Developing time-specific labor constraints for spring, summer, and fall

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| .  **Table 6.3. Final Table for Example Farm Planning Problem** | | | | | |
|  | Crop 1 | Crop 2 | Crop 3 | Crop 4 | RHS |
| Objective | c1 | c2 | c3 | c4 |  |
| Land | 1 | 1 | 1 |  |  L |
| Land After Crop 1 | -1 |  |  | 1 |  0 |
| Labor –Spring | d1 | d3 | d5 |  | < sp |
| Labor – Summer | d2 |  |  | d7 |  su |
| Labor – Fall |  | d4 | d6 | d8 |  f |

Toward Proper Modeling

**Formulating an applied LP problem**

**Data Development**

**Good** solutions do not arise from **bad** data

**Key considerations:**

* Time frame

- objective function, technical coefficient (aij's) and RHS data must be mutually consistent i.e. annual basis vs. monthly basis

* Uncertainty

- how to incorporate data uncertainty

* Data sources

- vary by problem + judgments (statistical estimation or deductive process)

* Consistency

- homogeneity of units rules must hold

* Component specification

- objective, RHS, technical coefficients

**Toward Proper Modeling**



First solution Y=15, x=10 Second Solution X=10, Y=10

# Assume a mistake + 2Y should be -2Y

 

 

**Toward Proper Modeling**

**Homogeneity of Units**



***Rules***

1. All coefficients in a row have common numerators.

1. All coefficients in a column have common denominators.

**Toward Proper Modeling**

**Checking Structure**



Infeasible

Constraint i cannot be satisfied when

bi < 0 and aij > 0 for all j

dn < 0 and enj > 0 for all j

dn > 0 and enj < 0 for all j

gm > 0 and fmj < 0 for all j

Zero Variables

Constraint i will force all variables in it to equal zero when

bi = 0 and aij > 0 for all j implies all Xj = 0 with aij 0

dn = 0 and enj > 0 for all j implies all Xj = 0 with enj 0

dn = 0 and enj < 0 for all j implies all Xj = 0 with enj 0

gm = 0 and fmj < 0 for all j implies all Xj = 0 with fmj 0

Obviously Redundant

Constraint i will never be binding when

bi > 0 and aij < 0 for all j

gm < 0 and fmj > 0 for all j

Unbounded

Variable j will be unbounded when

cj > 0 and aij < 0, enj = 0 and fmj > 0 for all i, n and m

Always Zero

Variable j will be zero when

cj < 0 and aij > 0 or enj = 0 and fmj < 0 for all i, n and m

**Toward Proper Modeling**

**Improper Joint Products**

Alternative Formulations of Chicken Processing Problem:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Formulation 6.5(a) | | Chickens (lbs.) | | |  | Breast Quarter (lbs.) | | |  | Leg Quarter (lbs.) | | |  | Neck (lbs.) | | |  | Giblets (lbs.) | | |  | Maximize | | |
| Objective function ($) | |  | | | + | 1.00X1 | | | + | 0.80X2 | | | + | 0.20X3 | | | + | 0.70X4 | | |  |  | | |
| Balance (lbs.) | | -Y | | | + | 0.50X1 | | | + | 0.35X2 | | | + | 0.1X3 | | | + | 0.05X4 | | |  | 0 | | |
| Chickens Available (birds) | | | 1/3Y | |  |  | | |  |  | | |  |  | | |  |  | | |  | 1500 | | |
| Formulation 6.5(b) | | | Chickens (lbs.) | | |  | Breast Quarter (lbs.) | | |  | Leg Quarter (lbs.) | | |  | Neck (lbs.) | | |  | Giblets (lbs.) | | | Maximize | | |
| Objective Function ($) | | |  | | | + | 1.00X1 | | | + | 0.80X2 | | | + | 0.20X3 | | | + | 0.70X4 | | |  |  | |
| Breast Quarter | | | -0.50Y | | | + | X1 | | |  |  | | |  |  | | |  |  | | |  | 0 | |
| Leg Quarter | | | -0.35Y | | |  |  | | | + | X2 | | |  |  | | |  |  | | |  | 0 | |
| Neck | | | -0.10Y | | |  |  | | |  |  | | | + | X3 | | |  |  | | |  | 0 | |
| Giblets | | | -0.05Y | | |  |  | | |  |  | | |  |  | | | + | X4 | | |  | 0 | |
| Chickens | | | 1/3Y | | |  |  | | |  |  | | |  |  | | |  |  | | |  | 1500 | |

**Toward Proper Modeling**

**Improper Joint Product Alternatives**

Formulations for Processing Chickens with the Option of Deboning

Formulation 6.6(a)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Chicken |  | Breast Qtr. |  | Leg Qtr. |  | Neck |  | Giblet |  | Breast Q Meat |  | Leg Qtr. Meat |  | Neck Meat |  |  |
| Objective |  |  | 1.0X1 | + | 0.8X2 | + | 0.2X3 | + | 0.7X4 | + | 1.2M1 | + | 1.2M2 | + | 1.2M3 |  |  |
| Breast Qtr. | -0.5Y | + | X1 |  |  |  |  |  |  |  | 1 |  |  |  |  | < | 0 |
| Leg Qtr. | -0.35Y |  |  | + | X2 |  |  |  |  |  |  |  |  |  |  | < | 0 |
| Neck | -0.1Y |  |  |  |  | + | X3 |  |  |  |  |  |  |  |  | < | 0 |
| Giblets | -0.5Y |  |  |  |  |  |  | + | X4 |  |  |  |  |  |  | < | 0 |
| Chickens | 1/3Y |  |  |  |  |  |  |  |  |  |  |  |  |  |  | < | 1500 |
| BQ Meat | -(0.5)(0.75)Y |  |  |  |  |  |  |  |  | + | M1 |  |  |  |  | < | 0 |
| LQ Meat | -(0.35)(0.6)Y |  |  |  |  |  |  |  |  |  |  | + | M2 |  |  | < | 0 |
| N Meat | -(0.2)(0.1)Y |  |  |  |  |  |  |  |  |  |  |  |  | + | M3 | < | 0 |

Formulation 6.6(b)

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Chicken | Breast Qtr. | Leg Qtr. | Neck | Giblet | Breast Qtr. Meat | Leg Qtr. Meat | Neck Meat | Total Meat Sold |  |  |
| Objective |  | 1.0X1 | +0.8X2 | +0.2X3 | +0.7X4 |  |  |  | + 1.2M4 |  |  |
| BreastQtr | -0.5Y | + X1 |  |  |  | +M1 |  |  |  | < | 0 |
| Leg Qtr. | -0.35Y |  | + X2 |  |  |  | +M2 |  |  | < | 0 |
| Neck | -0.1Y |  |  | + X3 |  |  |  | +M3 |  | < | 0 |
| Giblets | -0.05Y |  |  |  | + X4 |  |  |  |  | < | 0 |
| Chickens | 1/3Y |  |  |  |  |  |  |  |  | < | 1500 |
| Meat |  |  |  |  |  | -0.75M1 | -0.6M2 | -0.2M3 | +M4 | < | 0 |

**Toward Proper Modeling**

**Formulations of the Chicken Assembly-Disassembly Problem**

Formulation 6.7(a)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Chicken | BQ | LQ | Neck | Giblet | BQMeat | LQMeat | NeckMeat | TotalMeat | MQSold |  |  |
| OBJ. |  | 1.0X1+ | 0.8X2 | + 0.2X3 + | 0.7X4 |  |  |  | + 1.2M4 | + 0.95Q |  |  |
| BQ | -0.5Y+ | X1 |  |  |  | + M1 |  |  |  | + 0.5Q |  | 0 |
| LQ | -0.35Y |  | + X2 |  |  |  | + M2 |  |  | + 0.5Q |  | 0 |
| Neck | -0.1Y |  |  | +X3 |  |  |  | + M3 |  |  |  | 0 |
| Giblets | -0.05Y |  |  |  | +X4 |  |  |  |  |  |  | 0 |
| Chickens | 1/3Y |  |  |  |  |  |  |  |  |  |  | 1500 |
| Meat |  |  |  |  | - | .75M1 | - 0.6M2 | - 0.2M3 | + M4 |  |  | 0 |

Formulation 6.7(b)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Chicken | BQ | | | LQ | | Neck | | | Giblet | | | BQ Meat | | | LQ Meat | | | Neck Meat | | | Total Meat | | | BQ in MQ | | LQinMQ | | | MQSold | |  |
| Obj |  |  | 1.0X1 | + | 0.8X2 | + | | 0.2X3 | + | | 0.7X4 |  | |  |  | |  |  | |  | + | | 1.2M4 |  | |  |  |  | + | | 0.95Q3 |  |  |
| BQ | -0.5Y |  | + X1 |  |  |  | |  |  | |  | + | | M1 |  | |  |  | |  |  | |  | + | | Q1 |  |  |  | |  | < | 0 |
| LQ | -0.35Y |  |  | + | X2 |  | |  |  | |  |  | |  | + | | M2 |  | |  |  | |  |  | |  | + | Q2 |  | |  | < | 0 |
| Neck | -0.1Y |  |  |  |  | + | | X3 |  | |  |  | |  |  | |  | + | | M3 |  | |  |  | |  |  |  |  | |  | < | 0 |
| Gib | -0.5Y |  |  |  |  |  | |  | + | | X4 |  | |  |  | |  |  | |  |  | |  |  | |  |  |  |  | |  | < | 0 |
| Chick | 1/3Y |  |  |  |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  |  |  | |  | < | 1500 |
| Meat |  |  |  |  |  |  | |  |  | |  | - | | 0.75M1 | - | | 0.6M2 | - | | 0.2M3 | + | | M4 |  | |  |  |  |  | |  | < | 0 |
| QtPac |  |  |  |  |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  | - | | Q1 | - | Q2 |  | | + Q3 | < | 0 |

**Toward Proper Modeling**

**Common Errors - Improper Substitution**

Formulation 6.8(a) – Leontief Isoquant problem – no substitution of labor.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Chair Production | Regular labor |  | Overtime labor |  | Chair Sale |  | |
| Objective |  | -10 |  | -15 |  | 220 |  |  |
| Chairs | -1 |  |  |  |  | 1 | ≤ | 0 |
| Regular labor | 7 | -1 |  |  |  |  | ≤ | 77 |
| Overtime Labor | 3 |  |  | -1 |  |  | ≤ | 27 |

Formulation 6.8(b) – Perfect substitution of labor.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Chair Production | Regular labor |  | Overtime labor |  | Chair Sale |  | |
| Objective |  | -10 |  | -15 |  | 220 |  |  |
| Chairs | -1 |  |  |  |  | 1 | ≤ | 0 |
| Labor | 10 | -1 |  | -1 |  |  | ≤ | 0 |
| Regular labor |  | 1 |  |  |  |  | ≤ | 77 |
| Overtime Labor |  |  |  | 1 |  |  | ≤ | 27 |

|  |
| --- |
| **Table 6.9 Example of GAMS Report Writing** |

‑‑‑‑ 53 PARAMETER MOVEMENT COMMODITY MOVEMENT

MIAMI HOUSTON MINEPLIS PORTLAND TOTAL

NEWYORK 30 35 15 80

CHICAGO 75 75

LOSANGLS 40 50 90

TOTAL 30 75 90 50 245

‑‑‑‑ 61 PARAMETER COSTS COMMODITY MOVEMENT COSTS BY ROUTE

MIAMI HOUSTON MINEPLIS PORTLAND TOTAL

NEWYORK 600 1400 525 2525

CHICAGO 1500 1500

LOSANGLS 1400 2000 3400

TOTAL 600 2800 2025 2000 7425

‑‑‑‑ 68 PARAMETER SUPPLYREP SUPPLY REPORT

AVAILABLE USED MARGVALUE

NEWYORK 100.00 80.00

CHICAGO 75.00 75.00 15.00

LOSANGLS 90.00 90.00 5.00

‑‑‑‑ 75 PARAMETER DEMANDREP DEMAND REPORT

REQUIRED RECEIVED MARGCOST

MIAMI 30.00 30.00 20.00

HOUSTON 75.00 75.00 40.00

MINEPLIS 90.00 90.00 35.00

PORTLAND 50.00 50.00 45.00

‑‑‑‑ 80 PARAMETER CMOVEMENT COSTS OF CHANGING COMMODITY MOVEMENT PATTERN

PATTERN

MIAMI HOUSTON MINEPLIS PORTLAND

NEWYORK 75.00

CHICAGO 45.00 35.00 40.00

LOSANGLS 75.00 40.00

**Transport GAMS Solution**

S O L V E S U M M A R Y

MODEL TRANSPORT OBJECTIVE TCOST

TYPE LP DIRECTION MINIMIZE

SOLVER OSL FROM LINE 49

\*\*\*\* SOLVER STATUS 1 NORMAL COMPLETION

\*\*\*\* MODEL STATUS 1 OPTIMAL

\*\*\*\* OBJECTIVE VALUE 7425.0000

LOWER LEVEL UPPER MARGINAL

‑‑‑‑ EQU TCOSTEQ . . . 1.000

‑‑‑‑ EQU SUPPLYEQ LIMIT ON SUPPLY AVAILABLE AT A PLANT

LOWER LEVEL UPPER MARGINAL

NEWYORK ‑INF 80.000 100.000 .

CHICAGO ‑INF 75.000 75.000 ‑15.000

LOSANGLS ‑INF 90.000 90.000 ‑5.000

‑‑‑‑ EQU DEMANDEQ MINIMUM REQUIREMENT AT A DEMAND MARKET

LOWER LEVEL UPPER MARGINAL

MIAMI 30.000 30.000 +INF 20.000

HOUSTON 75.000 75.000 +INF 40.000

MINEPLIS 90.000 90.000 +INF 35.000

PORTLAND 50.000 50.000 +INF 45.000

‑‑‑‑ VAR SHIPMENTS AMOUNT SHIPPED OVER A TRANSPORT ROUTE

LOWER LEVEL UPPER MARGINAL

NEWYORK .MIAMI . 30.000 +INF .

NEWYORK .HOUSTON . 35.000 +INF .

NEWYORK .MINEPLIS . 15.000 +INF .

NEWYORK .PORTLAND . . +INF 75.000

CHICAGO .MIAMI . . +INF 45.000

CHICAGO .HOUSTON . . +INF 35.000

CHICAGO .MINEPLIS . 75.000 +INF .

CHICAGO .PORTLAND . . +INF 40.000

LOSANGLS.MIAMI . . +INF 75.000

LOSANGLS.HOUSTON . 40.000 +INF .

LOSANGLS.MINEPLIS . . +INF 40.000

LOSANGLS.PORTLAND . 50.000 +INF .

LOWER LEVEL UPPER MARGINAL

‑‑‑‑ VAR TCOST ‑INF 7425.000 +INF .