Basic LP formulations

Linear programming formulations are typically composed of a number of standard problem types.

In these notes we review **four basic problems examining their:**

a.              Basic Structure

b.             Formulation

c.              Example application

d.             Answer interpretation

**The problems examined are the:**

1. [Resource allocation problem](#_Resource_Allocation_Problem)
2. [Transportation problem](#_Transportation_Problem)
3. [Feed mix problem](#_Feeding_Problem)
4. [Joint products problem](#_Joint_Products_Problem)

### Resource Allocation Problem

Basic Concept

The classical LP problem involves the allocation of an endowment of scarce resources among a number of competing products so as to maximize profits.

Objective: Maximize Profits

Indexes: Let the competing products index be j

Let the scarce resources index be i

Variables: Let us define our primary decision variable Xj, as the number of units of the jth product made

Restrictions: Non negative production

Resource usage across all production possibilities is less than or equal to the resource endowment

**Resource Allocation Problem**

**Algebraic Setup:**



**where**

* cj is the profit contribution per unit of the jth product
* aij depicts the number of units of the ith resource used when producing one unit of the jth product
* bi depicts the endowment of the ith resource

**Resource Allocation Problem**

**Example: E-Z Chair Makers**

**Objective**: determine the number of two types of chairs to produce that will maximize profits.

**Chair Types**: Functional and Fancy

**Resources**: Large &, Small Lathe, Chair Bottom Carver, and Labor

**Profit Contributions:** (revenue – material cost - cost increase due to lathe shifts)

Functional $82 - $15 = $67

Fancy $105 - $25 = $80

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Resource Requirements When Using The Normal Pattern**  **Wh the Normal Pattern** | | | | | | | | |
|  | | Hours of Use per Chair Type | | | | | | |
|  | | Functional | | | | Fancy | | |
| Small Lathe | | 0.8 | | | | 1.2 | | |
| Large Lathe | | 0.5 | | | | 0.7 | | |
| Chair Bottom Carver | | 0.4 | | | | 1.0 | | |
| Labor | | 1.0 | | | | 0.8 | | |
| **Resource Requirements and Increased Costs for Alternative Methods of Production in Hours of Use per Chair and Dollars** | | | | | | | |
|  | Maximum Use of Small Lathe | | |  | Maximum Use of Large Lathe | | |
|  | Functional | | Fancy |  | Functional | | Fancy |
| Small Lathe | 1.30 | | 1.70 |  | 0.20 | | 0.50 |
| Large Lathe | 0.20 | | 0.30 |  | 1.30 | | 1.50 |
| Chair Bottom Carver | 0.40 | | 1.00 |  | 0.40 | | 1.00 |
| Labor | 1.05 | | 0.82 |  | 1.10 | | 0.84 |
| Cost Increase | $1.00 | | $1.50 |  | $0.70 | | $1.60 |

**Resource Allocation Problem**

Example: E-Z Chair Makers

Algebraic Setup:



Empirical Setup:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | 67X1 | + | 66X2 | + | 66.3X3 | + | 80X4 | + | 78.5X5 | + | 78.4X6 |  |  |
| s.t. | 0.8X1 | + | 1.3X2 | + | 0.2X3 | + | 1.2X4 | + | 1.7X5 | + | 0.5X6 | ≤ | 140 |
|  | 0.5X1 | + | 0.2X2 | + | 1.3X3 | + | 0.7X4 | + | 0.3X5 | + | 1.5X6 | ≤ | 90 |
|  | 0.4X1 | + | 0.4X2 | + | 0.4X3 | + | X4 | + | X5 | + | X6 | ≤ | 120 |
|  | X1 | + | 1.05X2 | + | 1.1X3 | + | 0.8X4 | + | 0.82X5 | + | 0.84X6 | ≤ | 125 |
|  | X1 | , | X2 | , | X3 | , | X4 | , | X5 | , | X6 | ≥ | 0 |

Where:

X1 = the # of functional chairs made with the normal pattern;

X2 = the # of functional chairs made with maximum use of the small lathe;

X3 = the # of functional chairs made with maximum use of the large lathe;

X4 = the # of fancy chairs made with the normal pattern;

X5 = the # of fancy chairs made with maximum use of the small lathe;

X6 = the # of fancy chairs made with maximum use of the large lathe.

**Resource Allocation Problem**

**GAMS Formulation:**

1 SET PROCESS TYPES OF PRODUCTION PROCESSES

2 /FUNCTNORM , FUNCTMXSML , FUNCTMXLRG

3 ,FANCYNORM , FANCYMXSML , FANCYMXLRG/

4 RESOURCE TYPES OF RESOURCES

5 /SMLLATHE,LRGLATHE,CARVER,LABOR/ ;

6

7 PARAMETER PRICE(PROCESS) PRODUCT PRICES BY PROCESS

8 /FUNCTNORM 82, FUNCTMXSML 82, FUNCTMXLRG 82

9 ,FANCYNORM 105, FANCYMXSML 105, FANCYMXLRG 105/

10 PRODCOST(PROCESS) COST BY PROCESS

11 /FUNCTNORM 15, FUNCTMXSML 16 , FUNCTMXLRG 15.7

12 ,FANCYNORM 25, FANCYMXSML 26.5,FANCYMXLRG 26.6/

13 RESORAVAIL(RESOURCE) RESOURCE AVAILABLITY

14 /SMLLATHE 140, LRGLATHE 90,

15 CARVER 120, LABOR 125/;

16

17 TABLE RESOURUSE(RESOURCE,PROCESS) RESOURCE USAGE

18

19 FUNCTNORM FUNCTMXSML FUNCTMXLRG

20 SMLLATHE 0.80 1.30 0.20

21 LRGLATHE 0.50 0.20 1.30

22 CARVER 0.40 0.40 0.40

23 LABOR 1.00 1.05 1.10

24 + FANCYNORM FANCYMXSML FANCYMXLRG

25 SMLLATHE 1.20 1.70 0.50

26 LRGLATHE 0.70 0.30 1.50

27 CARVER 1.00 1.00 1.00

28 LABOR 0.80 0.82 0.84;

29

30 POSITIVE VARIABLES

31 PRODUCTION(PROCESS) ITEMS PRODUCED BY PROCESS;

32 VARIABLES

33 PROFIT TOTALPROFIT;

34 EQUATIONS

35 OBJT OBJECTIVE FUNCTION ( PROFIT )

36 AVAILABLE(RESOURCE) RESOURCES AVAILABLE ;

37 OBJT.. PROFIT =E=

38 SUM(PROCESS,(PRICE(PROCESS)‑PRODCOST(PROCESS))

39 \* PRODUCTION(PROCESS)) ;

40 AVAILABLE(RESOURCE)..

41 SUM(PROCESS,RESOURUSE(RESOURCE,PROCESS)

42 \*PRODUCTION(PROCESS)) =L= RESORAVAIL(RESOURCE);

43 MODEL RESALLOC /ALL/;

44 SOLVE RESALLOC USING LP MAXIMIZING PROFIT;

**Resource Allocation Problem**

**Primal and Solution:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | 67X1 | + | 66X2 | + | 66.3X3 | + | 80X4 | + | 78.5X5 | + | 78.4X6 |  |  |
| s.t. | 0.8X1 | + | 1.3X2 | + | 0.2X3 | + | 1.2X4 | + | 1.7X5 | + | 0.5X6 | ≤ | 140 |
|  | 0.5X1 | + | 0.2X2 | + | 1.3X3 | + | 0.7X4 | + | 0.3X5 | + | 1.5X6 | ≤ | 90 |
|  | 0.4X1 | + | 0.4X2 | + | 0.4X3 | + | X4 | + | X5 | + | X6 | ≤ | 120 |
|  | X1 | + | 1.05X2 | + | 1.1X3 | + | 0.8X4 | + | 0.82X5 | + | 0.84X6 | ≤ | 125 |
|  | X1 | , | X2 | , | X3 | , | X4 | , | X5 | , | X6 | ≥ | 0 |

Recall:

Shadow Price represents marginal values of the resources

Reduced Cost represents marginal costs of forcing non-basic variable into the solution

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Table 5.5 Optimal Solution to the E-Z Chair Makers Problem** | | | | | |
| obj = 10417.29 | |  |  |  |  |
| Variables | Value | Reduced Cost | Const  raint | Level | Shadow Price |
| X1 | 62.23 | 0 | 1 | 140 | 33.33 |
| X2 | 0 | -11.30 | 2 | 90 | 25.79 |
| X3 | 0 | -4.08 | 3 | 103.09 | 0 |
| X4 | 73.02 | 0 | 4 | 125 | 27.44 |
| X5 | 0 | -8.40 |  |  |  |
| X6 | 5.18 | 0 |  |  |  |

**Resource Allocation Problem**

Simple GAMS Formulation

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | 67X1 | + | 66X2 | + | 66.3X3 | + | 80X4 | + | 78.5X5 | + | 78.4X6 |  |  |
| s.t. | 0.8X1 | + | 1.3X2 | + | 0.2X3 | + | 1.2X4 | + | 1.7X5 | + | 0.5X6 | ≤ | 140 |
|  | 0.5X1 | + | 0.2X2 | + | 1.3X3 | + | 0.7X4 | + | 0.3X5 | + | 1.5X6 | ≤ | 90 |
|  | 0.4X1 | + | 0.4X2 | + | 0.4X3 | + | X4 | + | X5 | + | X6 | ≤ | 120 |
|  | X1 | + | 1.05X2 | + | 1.1X3 | + | 0.8X4 | + | 0.82X5 | + | 0.84X6 | ≤ | 125 |
|  | X1 | , | X2 | , | X3 | , | X4 | , | X5 | , | X6 | ≥ | 0 |

1 POSITIVE VARIABLES X1, X2, X3, X4, X5, X6;

2 VARIABLE PROFIT;

3 EQUATIONS OBJFUNC, CONSTR1, CONSTR2, CONSTR3, CONSTR4;

4 OBJFUNC.. 67\*X1 + 66\*X2 + 66.3\*X3 + 80\*X4 + 78.5\*X5 + 78.4\*X6 =E= PROFIT;

5 CONSTR1.. 0.8\*X1 + 1.3\*X2 + 0.2\*X3 + 1.2\*X4 + 1.7\*X5 + 0.5\*X6 =L= 140;

6 CONSTR2.. 0.5\*X1 + 0.2\*X2 + 1.3\*X3 + 0.7\*X4 + 0.3\*X5 + 1.5\*X6 =L= 90;

7 CONSTR3.. 0.4\*X1 + 0.4\*X2 + 0.4\*X3 + X4 + X5 + X6 =L= 120;

8 CONSTR4.. X1 + 1.05\*X2 + 1.1\*X3 + 0.8\*X4 + 0.82\*X5 + 0.84\*X6 =L= 125;

9 MODEL EXAMPLE1 /ALL/;

10 SOLVE EXAMPLE1 USING LP MAXIMIZING PROFIT;

**Resource Allocation Problem**

**Primal and Dual Algebra:**

# Primal



## Dual



Resource Allocation Problem

**Primal and Dual Empirical:**

**Primal**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | 67X1 | + | 66X2 | + | 66.3X3 | + | 80X4 | + | 78.5X5 | + | 78.4X6 |  |  |
| s.t. | 0.8X1 | + | 1.3X2 | + | 0.2X3 | + | 1.2X4 | + | 1.7X5 | + | 0.5X6 | ≤ | 140 |
|  | 0.5X1 | + | 0.2X2 | + | 1.3X3 | + | 0.7X4 | + | 0.3X5 | + | 1.5X6 | ≤ | 90 |
|  | 0.4X1 | + | 0.4X2 | + | 0.4X3 | + | X4 | + | X5 | + | X6 | ≤ | 120 |
|  | X1 | + | 1.05X2 | + | 1.1X3 | + | 0.8X4 | + | 0.82X5 | + | 0.84X6 | ≤ | 125 |
|  | X1 | , | X2 | , | X3 | , | X4 | , | X5 | , | X6 | ≥ | 0 |

**Dual**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Min | 140U1 | + | 90U2 | + | 120U3 | + | 125U4 |  |  |
| s.t. | 0.8U1 | + | 0.5U2 | + | 0.4U3 | + | U4 | ≥ | 67 |
|  | 1.3U1 | + | 0.2U2 | + | 0.4U3 | + | 1.05U4 | ≥ | 66 |
|  | 0.2U1 | + | 1.3U2 | + | 0.4U3 | + | 1.1U4 | ≥ | 66.3 |
|  | 1.2U1 | + | 0.7U2 | + | U3 | + | 0.8U4 | ≥ | 80 |
|  | 1.7U1 | + | 0.3U2 | + | U3 | + | 0.82U4 | ≥ | 78.5 |
|  | 0.5U1 | + | 1.5U2 | + | U3 | + | 0.84U4 | ≥ | 78.4 |
|  | U1 | , | U2 | , | U3 | , | U4 | ≥ | 0 |

**Resource Allocation Problem**

**Dual:**



**Dual Solution:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| obj = 10417.29 | |  |  |  |  |
| Variables | Value | Reduced Cost | Row | Level | Shadow Price |
| U1 | 33.33 | 0 | 1 | 67.00 | 62.23 |
| U2 | 25.79 | 0 | 2 | 77.30 | 0 |
| U3 | 0 | 16.91 | 3 | 70.38 | 0 |
| U4 | 27.44 | 0 | 4 | 80.00 | 73.02 |
|  |  |  | 5 | 86.90 | 0 |
|  |  |  | 6 | 78.40 | 5.18 |

### Resource Allocation

**Primal and Dual Solution Comparison:**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 5.5 Optimal Solution to the E-Z Chair Makers Problem** | | | | | | | | | |
| obj = 10417.29 | | |  | |  |  | |  | |
| Variables | Value | | Reduced Cost | | Constraint | Level | | Shadow Price | |
| X1 | 62.23 | | 0 | | 1 | 140 | | 33.33 | |
| X2 | 0 | | -11.30 | | 2 | 90 | | 25.79 | |
| X3 | 0 | | -4.08 | | 3 | 103.09 | | 0 | |
| X4 | 73.02 | | 0 | | 4 | 125 | | 27.44 | |
| X5 | 0 | | -8.40 | |  |  | |  | |
| X6 | 5.18 | | 0 | |  |  | |  | |
| **Table 5.5a Optimal Solution to the Dual E-Z Chair Makers Problem** | | | | | | | | | | | |
| obj = 10417.29 | | | |  | | |  | |  | |  |
| Variables | | Value | | Reduced Cost | | | Constraint | | Level | | Shadow Price |
| U1 | | 33.33 | | 0 | | | 1 | | 67.00 | | 62.23 |
| U2 | | 25.79 | | 0 | | | 2 | | 77.30 | | 0 |
| U3 | | 0 | | 16.91 | | | 3 | | 70.38 | | 0 |
| U4 | | 27.44 | | 0 | | | 4 | | 80.00 | | 73.02 |
|  | |  | |  | | | 5 | | 86.90 | | 0 |
|  | |  | |  | | | 6 | | 78.40 | | 5.18 |

### Resource Allocation

**Evaluate Dual Constraints:**

**For X1 and X2**

0.8U1 + 0.5U2 + 0.4U3 + U4 > 67

1.3U1 + 0.2U2 + 0.4U3 + 1.05U4 > 66

Dual Solution

U1 33.33

U2 25.79

U3 0

U4 27.44

Evaluation

X1

0.8(33.33) + 0.5(25.79) +0.4(0) +27.44 > 67

26.6664 + 12.895 + 0 +27.44 > 67

67 > 67

Reduced cost=0

X2

1.3U1 + 0.2U2 + 0.4U3 + 1.05U4 > 66

1.3(33.33)+ 0.2(25.79) +0.4(0) +1.05(27.44) > 66

43.33 + 5.16 +0 + 28.812 > 66

77.3 > 66

66+11.3 > 66

Reduced cost=11.3

Table 5.3. First GAMS Formulation of Resource Allocation Example 1 SET PROCESS TYPES OF PRODUCTION PROCESSES

2 /FUNCTNORM , FUNCTMXSML , FUNCTMXLRG

3 ,FANCYNORM , FANCYMXSML ,FANCYMXLRG/

4 RESOURCE TYPES OF RESOURCES

5 /SMLLATHE,LRGLATHE,CARVER,LABOR/ ;

7 PARAMETER PRICE(PROCESS) PRODUCT PRICES BY PROCESS

8 /FUNCTNORM 82, FUNCTMXSML 82, FUNCTMXLRG 82

9 ,FANCYNORM 105, FANCYMXSML 105, FANCYMXLRG 105/

10 PRODCOST(PROCESS) COST BY PROCESS

11 /FUNCTNORM 15, FUNCTMXSML 16, FUNCTMXLRG 15.7

12 ,FANCYNORM 25, FANCYMXSML 26.5,FANCYMXLRG 26.6/

13 RESORAVAIL(RESOURCE) RESOURCE AVAILABLITY

14 /SMLLATHE 140, LRGLATHE 90,

15 CARVER 120, LABOR 125/;

16

17 TABLE RESOURUSE(RESOURCE,PROCESS) RESOURCE USAGE

18

19 FUNCTNORM FUNCTMXSML FUNCTMXLRG

20 SMLLATHE 0.80 1.30 0.20

21 LRGLATHE 0.50 0.20 1.30

22 CARVER 0.40 0.40 0.40

23 LABOR 1.00 1.05 1.10

24 + FANCYNORM FANCYMXSML FANCYMXLRG

25 SMLLATHE 1.20 1.70 0.50

26 LRGLATHE 0.70 0.30 1.50

27 CARVER 1.00 1.00 1.00

28 LABOR 0.80 0.82 0.84;

30 POSITIVE VARIABLES

31 PRODUCTION(PROCESS) ITEMS PRODUCED BY PROCESS;

32 VARIABLES

33 PROFIT TOTALPROFIT;

34 EQUATIONS

35 OBJT OBJECTIVE FUNCTION ( PROFIT )

36 AVAILABLE(RESOURCE) RESOURCES AVAILABLE ;

37

38 OBJT.. PROFIT =E=

39 SUM(PROCESS,(PRICE(PROCESS)‑PRODCOST(PROCESS))

40 \* PRODUCTION(PROCESS)) ;

42 AVAILABLE(RESOURCE)..

43 SUM(PROCESS,RESOURUSE(RESOURCE,PROCESS)\*PRODUCTION(PROCESS))

44 =L= RESORAVAIL(RESOURCE);

46 MODEL RESALLOC /ALL/;

47 SOLVE RESALLOC USING LP MAXIMIZING PROFIT;

|  |
| --- |
|  |

|  |
| --- |
| Table 5.4. Second GAMS Formulation of Resource Allocation Example |

1 SET CHAIRS TYPES OF CHAIRS /FUNCTIONAL,FANCY/

2 PROCESS TYPES OF PRODUCTION PROCESSES

3 /NORMAL , MXSMLLATHE , MXLRGLATHE/

4 RESOURCE TYPES OF RESOURCES

5 /SMLLATHE,LRGLATHE,CARVER,LABOR/ ;

6

7 PARAMETER PRICE(CHAIRS) PRODUCT PRICES BY PROCESS

8 /FUNCTIONAL 82, FANCY 105/

9 COST(CHAIRS) BASE COST BY CHAIR

10 /FUNCTIONAL 15, FANCY 25/

11 EXTRACOST(CHAIRS,PROCESS) EXTRA COST BY PROCESS

12 / FUNCTIONAL.MXSMLLATHE 1.0 ,

13 FUNCTIONAL.MXLRGLATHE 0.7

14 ,FANCY. MXSMLLATHE 1.5,

15 FANCY. MXLRGLATHE 1.6/

16 RESORAVAIL(RESOURCE) RESOURCE AVAILABLITY

17 /SMLLATHE 140, LRGLATHE 90,

18 CARVER 120, LABOR 125/;

20 TABLE RESOURUSE(RESOURCE,CHAIRS,PROCESS) RESOURCE USAGE

21 FUNCTIONAL.NORMAL FUNCTIONAL.MXSMLLATHEFUNCTIONAL.MXLRGLATHE

22 SMLLATHE 0.80 1.30 0.20

23 LRGLATHE 0.50 0.20 1.30

24 CARVER 0.40 0.40 0.40

25 LABOR 1.00 1.05 1.10

26 + FANCY.NORMAL FANCY.MXSMLLATHE FANCY.MXLRGLATHE

27 SMLLATHE 1.20 1.70 0.50

28 LRGLATHE 0.70 0.30 1.50

29 CARVER 1.00 1.00 1.00

30 LABOR 0.80 0.82 0.84 ;

31

32 POSITIVE VARIABLES

33 PRODUCTION(CHAIRS,PROCESS) ITEMS PRODUCED BY PROCESS;

34 VARIABLES

35 PROFIT TOTAL PROFIT;

36 EQUATIONS

37 OBJT OBJECTIVE FUNCTION ( PROFIT )

38 AVAILABLE(RESOURCE) RESOURCES AVAILABLE ;

39

40 OBJT.. PROFIT =E= SUM((CHAIRS,PROCESS),

41 PRICE(CHAIRS)‑COST(CHAIRS)‑EXTRACOST(CHAIRS,PROCESS))

42 \* PRODUCTION(CHAIRS,PROCESS)) ;

44 AVAILABLE(RESOURCE)..

45 SUM((CHAIRS,PROCESS),

46 RESOURUSE(RESOURCE,CHAIRS,PROCESS)\*PRODUCTION(CHAIRS,PROCESS))

47 =L= RESORAVAIL(RESOURCE);

48

49 MODEL RESALLOC /ALL/;

50 SOLVE RESALLOC USING LP MAXIMIZING PROFIT;

|  |
| --- |
|  |

###### Transportation Problem

Basic Concept

 This problem involves the shipment of a homogeneous product from a number of supply locations to a number of demand locations.

Supply Locations Demand Locations

1 A

2 B

… …

m n

Problem: given needs at the demand locations how should I take limited supply at supply locations and move the goods to meet needs. Further suppose we wish to minimize cost.

Objective: Minimize cost

Variables Quantity of goods shipped from each supply point to each demand point

Restrictions: Non negative shipments

Supply availability at supply point

Demand need at a demand point

Transportation Problem

Formulating the Problem

Basic notation and the decision variable

Let us denote the supply locations as supplyi

Let us denote the demand locations as demandj

Let us define our fundamental decision variable as the set of individual shipment quantities from each supply location to each demand location and denote this variable algebraically as

Movesupplyi,demandj

Transportation Problem

Formulating the Problem

 The objective function:

We want to minimize total shipping cost so we need an expression for shipping cost

Let us define a data item giving the per unit cost of shipments from each supply location to each demand location as costsupplyi,demandj

 Our objective then becomes to minimize the sum of the shipment costs over all supplyi, demandj pairs or

 Minimize costsupplyi,demandjMovesupplyi,demandj

which is the per unit cost of moving from each supply location to each demand location times the amount shipped summed over all possible shipment routes

Transportation Problem

Formulating the Problem

There are three types of constraints:

1) Supply availability: limiting shipments from each supply

point to existing supply so that the sum of outgoing shipments from the supplyith supply point to all possible destinations (demandj) to not exceed supplysupplyi



2) Minimum demand: requiring shipments into the demandjth demand point be greater than or equal to demand at that point. Incoming shipments include shipments from all possible supply points supplyi to the demandjth demand point.



3) Nonnegative shipments:



###### Transportation Problem

Formulating the Problem

**Formulation and Example:**



Explicitly:



Transportation Problem

**Example:** **Shipping Goods**

**Three plants:** New York, Chicago, Los Angeles

**Four demand markets:** Miami, Houston, Minneapolis, Portland  
**Supply Available** **Demand Required**

New York 100 Miami 30

Chicago 75 Houston 75

Los Angeles 90 Minneapolis 90

Portland 50

**Distances:**

Miami Houston Minneapolis Portland

New York 30 7 6 23

Chicago 9 11 3 13

Los Angeles 17 6 13 7

**Transportation costs = 5 +5\*Distance:**

Miami Houston Minneapolis Portland

New York 20 40 35 120

Chicago 50 60 20 70

Los Angeles 90 35 70 40

###### Transportation Problem

**Example:** **Shipping Goods**



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Min | 20X11 | + | 40X12 | + | 35X13 | + | 120X14 | + | 50X21 | + | 60X22 | + | 20X23 | + | 70X24 | + | 90X31 | + | 35X32 | + | 70X33 | + | 40X34 |  |  |
| s.t. | X11 | + | X12 | + | X13 | + | X14 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ≤ | 100 |
|  |  |  |  |  |  |  |  |  | X21 | + | X22 | + | X23 | + | X24 |  |  |  |  |  |  |  |  | ≤ | 75 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | X31 | + | X32 | + | X33 | + | X34 | ≤ | 90 |
|  | X11 |  |  |  |  |  |  | + | X21 |  |  |  |  |  |  | + | X31 |  |  |  |  |  |  | ≥ | 30 |
|  |  |  | X12 |  |  |  |  |  |  | + | X22 |  |  |  |  |  |  | + | X32 |  |  |  |  | ≥ | 75 |
|  |  |  |  |  | X13 |  |  |  |  |  |  | + | X23 |  |  |  |  |  |  | + | X33 |  |  | ≥ | 90 |
|  |  |  |  |  |  |  | X14 |  |  |  |  |  |  | + | X24 |  |  |  |  |  |  | + | X34 | ≥ | 50 |
|  |  |  |  |  |  |  |  |  | Xij≥0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

**Transport Problem**

**Alternative Formulation Formats**



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| MOVE11 | MOVE12 | MOVE13 | MOVE14 | MOVE21 | MOVE22 | MOVE23 | MOVE24 | MOVE31 | MOVE32 | MOVE33 | MOVE34 |  | |
| 20 | 40 | 35 | 120 | 50 | 60 | 20 | 70 | 90 | 35 | 70 | 40 | Minimize | |
| 1 | 1 | 1 | 1 |  |  |  |  |  |  |  |  | ≤ | 100 |
|  |  |  |  | 1 | 1 | 1 | 1 |  |  |  |  | ≤ | 75 |
|  |  |  |  |  |  |  |  | 1 | 1 | 1 | 1 | ≤ | 90 |
| +1 |  |  |  | +1 |  |  |  | +1 |  |  |  | ≥ | +30 |
|  | +1 |  |  |  | +1 |  |  |  | +1 |  |  | ≥ | +75 |
|  |  | +1 |  |  |  | +1 |  |  |  | +1 |  | ≥ | +90 |
|  |  |  | +1 |  |  |  | +1 |  |  |  | +1 | ≥ | +50 |
| 1, | 1, | 1, | 1, | 1, | 1, | 1, | 1, | 1, | 1, | 1, | 1, | ≥ | 0 |

Transportation Problem

**Example:** **Shipping Goods – Solution**

- shadow price represents marginal values of the

resources

i.e. marginal value of additional units in Chicago = $15

- reduced cost represents marginal costs of forcing

non-basic variable into the solution

i.e. shipments from New York to Portland costs $75

- twenty units are left in New York

Optimal Solution: Objective value $7,425

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Table 5.8. Optimal Solution to the ABC Company Problem** | | | | | |
| Variable | Value | Reduced Cost | Equation | Slack | Shadow Price |
| Move11 | 30 | 0 | 1 | 20 | 0 |
| Move12 | 35 | 0 | 2 | 0 | -15 |
| Move13 | 15 | 0 | 3 | 0 | -5 |
| Move14 | 0 | 75 | 4 | 0 | 20 |
| Move21 | 0 | 45 | 5 | 0 | 40 |
| Move22 | 0 | 35 | 6 | 0 | 35 |
| Move23 | 75 | 0 | 7 | 0 | 45 |
| Move24 | 0 | 40 |  |  |  |
| Move31 | 0 | 75 |  |  |  |
| Move32 | 40 | 0 |  |  |  |
| Move33 | 0 | 40 |  |  |  |
| Move34 | 50 | 0 |  |  |  |

Optimal Shipping Patterns

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Origin | Destination | | | | | | | |
| Miami | | Houston | | Minneapolis | | Portland | |
| Units | Variable | Units | Variable | Units | Variable | Units | Variable |
| New York | 30 | Move11 | 35 | Move12 | 15 | Move13 |  |  |
| Chicago |  |  |  |  | 75 | Move23 |  |  |
| Los Angeles |  |  | 40 | Move32 |  |  | 50 | Move34 |

### Transport Problem

**Primal and Dual Algebra:**

# Primal



## Dual



### Transport Problem

Empirical Primal and Dual

Primal

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 20 | 40 | 35 | 120 | 50 | 60 | 20 | 70 | 90 | 35 | 70 | 40 | Minimize | |
| 1 | 1 | 1 | 1 |  |  |  |  |  |  |  |  | ≤ | 100 |
|  |  |  |  | 1 | 1 | 1 | 1 |  |  |  |  | ≤ | 75 |
|  |  |  |  |  |  |  |  | 1 | 1 | 1 | 1 | ≤ | 90 |
| +1 |  |  |  | +1 |  |  |  | +1 |  |  |  | ≥ | +30 |
|  | +1 |  |  |  | +1 |  |  |  | +1 |  |  | ≥ | +75 |
|  |  | +1 |  |  |  | +1 |  |  |  | +1 |  | ≥ | +90 |
|  |  |  | +1 |  |  |  | +1 |  |  |  | +1 | ≥ | +50 |

Dual

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | -100U1 | - | 75U2 | - | 90U3 | + | 30V1 | + | 75V2 | + | 90V3 | + | 50V4 |  |  |
| s.t. | -U1 |  |  |  |  | + | V1 |  |  |  |  |  |  | ≤ | 20 |
|  | -U1 |  |  |  |  |  |  | + | V2 |  |  |  |  | ≤ | 40 |
|  | -U1 |  |  |  |  |  |  |  |  | + | V3 |  |  | ≤ | 35 |
|  | -U1 |  |  |  |  |  |  |  |  |  |  | + | V4 | ≤ | 120 |
|  |  |  | -U2 |  |  | + | V1 |  |  |  |  |  |  | ≤ | 50 |
|  |  |  | -U2 |  |  |  |  | + | V2 |  |  |  |  | ≤ | 60 |
|  |  |  | -U2 |  |  |  |  |  |  | + | V3 |  |  | ≤ | 20 |
|  |  |  | -U2 |  |  |  |  |  |  |  |  | + | V4 | ≤ | 70 |
|  |  |  |  |  | -U3 | + | V1 |  |  |  |  |  |  | ≤ | 90 |
|  |  |  |  |  | -U3 |  |  | + | V2 |  |  |  |  | ≤ | 35 |
|  |  |  |  |  | -U3 |  |  |  |  | + | V3 |  |  | ≤ | 70 |
|  |  |  |  |  | -U3 |  |  |  |  |  |  | + | V4 | ≤ | 40 |

**Transport Problem**

**Empirical Dual and Solution**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | -100U1 | | - | 75U2 | - | | 90U3 | + | 30V1 | | + | 75V2 | + | | 90V3 | + | 50V4 | |  |  |
| s.t. | -U1 | |  |  |  | |  | + | V1 | |  |  |  | |  |  |  | | ≤ | 20 |
|  | -U1 | |  |  |  | |  |  |  | | + | V2 |  | |  |  |  | | ≤ | 40 |
|  | -U1 | |  |  |  | |  |  |  | |  |  | + | | V3 |  |  | | ≤ | 35 |
|  | -U1 | |  |  |  | |  |  |  | |  |  |  | |  | + | V4 | | ≤ | 120 |
|  |  | |  | -U2 |  | |  | + | V1 | |  |  |  | |  |  |  | | ≤ | 50 |
|  |  | |  | -U2 |  | |  |  |  | | + | V2 |  | |  |  |  | | ≤ | 60 |
|  |  | |  | -U2 |  | |  |  |  | |  |  | + | | V3 |  |  | | ≤ | 20 |
|  |  | |  | -U2 |  | |  |  |  | |  |  |  | |  | + | V4 | | ≤ | 70 |
|  |  | |  |  |  | | -U3 | + | V1 | |  |  |  | |  |  |  | | ≤ | 90 |
|  |  | |  |  |  | | -U3 |  |  | | + | V2 |  | |  |  |  | | ≤ | 35 |
|  |  | |  |  |  | | -U3 |  |  | |  |  | + | | V3 |  |  | | ≤ | 70 |
|  |  | |  |  |  | | -U3 |  |  | |  |  |  | |  | + | V4 | | ≤ | 40 |
| **Table 5.10. Optimal Dual Solution to the ABC Company Problem** | | | | | | | | | | | | | | | | | | | | | | |
| Variable | | | Value | | | | Reduced Cost | | | | Constraint | | | | Level | | | | Shadow Price | | | |
| U1 | | | 0 | | | | 20 | | | | 1 | | | | -20 | | | | 30 | | | |
| U2 | | | 15 | | | | 0 | | | | 2 | | | | 40 | | | | 35 | | | |
| U3 | | | 5 | | | | 0 | | | | 3 | | | | 35 | | | | 15 | | | |
| V1 | | | 20 | | | | 0 | | | | 4 | | | | 45 | | | | 0 | | | |
| V2 | | | 40 | | | | 0 | | | | 5 | | | | 5 | | | | 0 | | | |
| V3 | | | 35 | | | | 0 | | | | 6 | | | | 25 | | | | 0 | | | |
| V4 | | | 45 | | | | 0 | | | | 7 | | | | 20 | | | | 75 | | | |
|  | | |  | | | |  | | | | 8 | | | | 30 | | | | 0 | | | |
|  | | |  | | | |  | | | | 9 | | | | 15 | | | | 0 | | | |
|  | | |  | | | |  | | | | 10 | | | | 35 | | | | 40 | | | |
|  | | |  | | | |  | | | | 11 | | | | 30 | | | | 0 | | | |
|  | | |  | | | |  | | | | 12 | | | | 40 | | | | 50 | | | |

**Transport Problem**

**Primal/Dual Solution Comparison:**

Primal

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Variable | Value | Reduced Cost | Constraint | Level | Shadow Price |
| Move11 | 30 | 0 | 1 | 80 | 0 |
| Move12 | 35 | 0 | 2 | 75 | 15 |
| Move13 | 15 | 0 | 3 | 90 | 5 |
| Move14 | 0 | 75 | 4 | 30 | 20 |
| Move21 | 0 | 45 | 5 | 75 | 40 |
| Move22 | 0 | 35 | 6 | 90 | 35 |
| Move23 | 75 | 0 | 7 | 50 | 45 |
| Move24 | 0 | 40 |  |  |  |
| Move31 | 0 | 75 |  |  |  |
| Move32 | 40 | 0 |  |  |  |
| Move33 | 0 | 40 |  |  |  |
| Move34 | 50 | 0 |  |  |  |

Dual

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Variable | Value | Reduced Cost | Constraint | Level | Shadow Price |
| U1 | 0 | -20 | 1 | -20 | 30 |
| U2 | 15 | 0 | 2 | 40 | 35 |
| U3 | 5 | 0 | 3 | 35 | 15 |
| V1 | 20 | 0 | 4 | 45 | 0 |
| V2 | 40 | 0 | 5 | 5 | 0 |
| V3 | 35 | 0 | 6 | 25 | 0 |
| V4 | 45 | 0 | 7 | 20 | 75 |
|  |  |  | 8 | 30 | 0 |
|  |  |  | 9 | 15 | 0 |
|  |  |  | 10 | 35 | 40 |
|  |  |  | 11 | 30 | 0 |
|  |  |  | 12 | 40 | 50 |

**Transport Problem**

|  |
| --- |
| **Table 5.7. GAMS Statement of Transportation Example** |

1 SETS PLANT PLANT LOCATIONS

2 /NEWYORK, CHICAGO, LOSANGLS/

3 MARKET DEMAND MARKETS

4 /MIAMI, HOUSTON, MINEPLIS, PORTLAND/

5

6 PARAMETERS SUPPLY(PLANT) QUANT AVAILABLE AT EACH PLANT

7 /NEWYORK 100, CHICAGO 75, LOSANGLS 90/

8 DEMAND(MARKET) QUANT REQUIRED BY DEMAND MARKET

9 /MIAMI 30, HOUSTON 75,

10 MINEPLIS 90, PORTLAND 50/;

11

12 TABLE DISTANCE(PLANT,MARKET) DIST FROM PLANT TO MARKET

13

14 MIAMI HOUSTON MINEPLIS PORTLAND

15 NEWYORK 3 7 6 23

16 CHICAGO 9 11 3 13

17 LOSANGLS 17 6 13 7;

18

19

20 PARAMETER COST(PLANT,MARKET) CALC COST OF MOVING GOODS ;

21 COST(PLANT,MARKET) = 5 + 5 \* DISTANCE(PLANT,MARKET) ;

22

23 POSITIVE VARIABLES

24 SHIPMENTS(PLANT,MARKET) AMOUNT SHIPPED OVER A ROUTE;

25 VARIABLES

26 TCOST TOTAL COST OF SHIPPING OVER ALL ROUTES;

27 EQUATIONS

28 TCOSTEQ TOTAL COST ACCOUNTING EQUATION

29 SUPPLYEQ(PLANT) LIMIT ON SUPPLY AVAILABLE AT A PLANT

30 DEMANDEQ(MARKET) MIN REQUIREMENT AT A DEMAND MARKET;

31

32 TCOSTEQ..TCOST =E=

33 SUM((PLANT,MARKET),SHIPMENTS(PLANT,MARKET)

34 \* COST(PLANT,MARKET)) ;

35 SUPPLYEQ(PLANT)..SUM(MARKET,SHIPMENTS(PLANT,MARKET))

36 =L= SUPPLY(PLANT);

37 DEMANDEQ(MARKET)..SUM(PLANT,SHIPMENTS(PLANT,MARKET))

38 =G= DEMAND(MARKET);

39 MODEL TRANSPORT /ALL/;

40 SOLVE TRANSPORT USING LP MINIMIZING TCOST;

|  |
| --- |
|  |

###### Feed Mix Problem

Basic Concept

This problem involves composing a minimum cost diet from a set of available ingredients while maintaining nutritional characteristics within certain bounds.

Objective: Minimize total diet costs

Variables: how much of each feedstuff is used in the diet

Restrictions: Non negative feedstuff

Minimum requirements by nutrient

Maximum requirements by nutrient

Total volume of the diet

This problem requires two types of indices:

Type of feed ingredients available from which the diet can be

composed

ingredientj = {corn, soybeans, salt, etc.}

Type of nutritional characteristics which must fall within certain limits nutrient = {protein, calories, etc.}

###### Feed Mix Problem

Basic Concept

Variable -- Feed ingredientj amount of feedstuff

ingredientj fed to animal

Objective – total cost

We want to minimize total diet costs across all the feedstuffs so we need an expression for feedstuff costs.

Let us define a data item giving the per unit cost of ingredients as costingredientj.

Our objective then becomes to minimize the sum of the diet costs over all feed ingredients

Minimize  costingredientj Feedingredientj

which is the per unit cost of ingredients summed over the feed ingredients.

###### Feed Mix Problem

Basic Concept

Additional parameters representing how much of each nutrient is present in each feedstuff as well as the dietary minimum and maximum requirements for that nutrient are needed.

Let

1). anutrient,ingredientj be the amount of the

nutrientth nutrient present in one unit of the

ingredientjth feed ingredient

2). ULnutrient and LLnutrient be the maximum and

minimum amount of the nutrientth nutrient

in the diet

Then the nutrient constraints are formed by summing the nutrients generated from each feedstuff (anutrient,ingredientjFingredientj) and requiring these to exceed the dietary minimum and/or be less than the maximum.

Problem then focuses on how much of each feedstuff is used in the diet to maintain nutritional characteristics within certain bounds.

###### Feed Mix Problem

Formulating the Problem

There are four general types of constraints:

1) minimum nutrient requirements restricting the sum of the nutrients generated from each feedstuff (anutrient,ingredientjFingredientj) to meet the dietary minimum



2) maximum nutrient requirements restricting

the sum of the nutrients generated from each feedstuff

(anutrient,ingredientjFingredientj) to not exceed the dietary

maximum



3) total volume of the diet constraint requiring

the ingredients in the diet equal the required weight of

the diet. Suppose the weight of the formulated diet and

the feedstuffs are the same, then



4) nonnegative feedstuff



Feed Mix Problem

**Example:** **cattle feeding**

**Seven nutritional characteristics:**

energy, digestible protein, fat, vitamin A, calcium, salt, phosphorus

**Seven feed ingredient availability:**

corn, hay, soybeans, urea, dical phosphate, salt, vitamin A

**New product:** potato slurry

**Ingredient costs per kilogram (cingredientj)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Ingredient Costs for Diet Problem Example per kg** | | | | |
| Corn | | $0.133 | | |
| Dical | | $0.498 | | |
| Alfalfa hay | | $0.077 | | |
| Salt | | $0.110 | | |
| Soybeans | | $0.300 | | |
| Vitamin A | | $0.286 | | |
| Urea | | $0.332 | | |
| **Required Nutrient Characteristics per Kilogram** | | | | | | |
| Nutrient | | Unit | | Minimum Amount | Maximum Amount | |
| Net energy | | Mega calories | | 1.34351 | -- | |
| Digestible protein | | Kilograms | | 0.071 | 0.13 | |
| Fat | | Kilograms | | -- | 0.05 | |
| Vitamin A | | International Units | | 2200 | -- | |
| Salt | | Kilograms | | 0.015 | 0.02 | |
| Calcium | | Kilograms | | 0.0025 | 0.01 | |
| Phosphorus | | Kilograms | | 0.0035 | 0.012 | |
| Weight | | Kilograms | | 1 | 1 | |

###### Feed Mix Problem

**Example: cattle feeding**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 5.13. Nutrient Content per Kilogram of Feeds** | | | | | | | | |
| Characteristic | Corn | Hay | Soybean | Urea | Dical  Phosphate | Salt | Vitamin A  Concentrate | Potato  Slurry |
| Net energy | 1.48 | 0.49 | 1.29 |  |  |  |  | 1.39 |
| Digestible protein | 0.075 | 0.127 | 0.438 | 2.62 |  |  |  | 0.032 |
| Fat | 0.0357 | 0.022 | 0.013 |  |  |  |  | 0.009 |
| Vitamin A | 600 | 50880 | 80 |  |  |  | 2204600 |  |
| Salt |  |  |  |  |  | 1 |  |  |
| Calcium | 0.0002 | 0.0125 | 0.0036 |  | 0.2313 |  |  | 0.002 |
| Phosphorus | 0.0035 | 0.0023 | 0.0075 | 0.68 | 0.1865 |  |  | 0.0024 |

###### Feed Mix Problem

**Example: cattle feeding**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 5.14. Primal Formulation of Feed Problem** | | | | | | | | | | | | | | | | | |
|  | Corn |  | Hay |  | Soybean |  | Urea |  | Dical |  | Salt |  | Vitamin A |  | Slurry |  |  |
| Min | .133XC | + | .077XH | + | .3XSB | + | .332XUr | + | .498Xd | + | .110XSLT | + | .286XVA | + | PXSL |  |  |
| Protein | .075XC | + | .127XH | + | .438XSB | + | 2.62XUr |  |  |  |  |  |  | + | .032XSL | ≤ | .13 |
| Fat | .0357XC | + | .022XH | + | .013XSB |  |  |  |  |  |  |  |  | + | .009XSL | ≤ | .05 |
| Salt |  |  |  |  |  |  |  |  |  |  | XSLT |  |  |  |  | ≤ | .02 |
| Calcium | .0002XC | + | .0125XH | + | .0036XSB |  |  | + | .2313Xd |  |  |  |  | + | .002XSL | ≤ | .01 |
| Phosphorus | .0035XC | + | .0023XH | + | .0075XSB | + | .68XUr | + | .1865Xd |  |  |  |  | + | .0024XSL | ≤ | .012 |
| Energy | 1.48XC | + | .49XH | + | 1.29XSB |  |  |  |  |  |  |  |  | + | 1.39XSL | ≥ | 1.34351 |
| Protein | .075XC | + | .127XH | + | .438XSB | + | 2.62XUr |  |  |  |  |  |  | + | .032XSL | ≥ | .071 |
| Vit A | 600XC | + | 50880XH | + | 80XSB |  |  |  |  |  |  | + | 2204600XVA |  |  | ≥ | 2200 |
| Salt |  |  |  |  |  |  |  |  |  |  | XSLT |  |  |  |  | ≥ | .015 |
| Calcium | .0002XC | + | .0125XH | + | .0036XSB |  |  | + | .2313Xd |  |  |  |  | + | .002XSL | ≥ | .0025 |
| Phosphorus | .0035XC | + | .0023XH | + | .0075XSB | + | .68XUr | + | .1865Xd |  |  |  |  | + | .0024XSL | ≥ | .0035 |
| Volume | XC | + | XH | + | XSB | + | XUr | + | Xd | + | XSLT | + | XVA | + | XSL | = | 1 |

###### Feed Mix Problem

**Example:** **cattle feeding – Solution**

- least cost feed ration is 95.6% slurry, 0.1% vitamin A,

1.5% salt, 0.2% dicalcium phosphate, 1.4%urea,

1.1% soybeans, and 0.1% hay

- reduced costs of feeding corn is 0.95 cents.

- shadow prices: nonzero values indicate the binding constraints (the phosphorous maximum constraint along with the net energy, protein, salt, and calcium minimums and the weight constraint).

- If relaxing the energy minimum, we save $0.065.

Optimal Solution: Objective value = $0.021

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Table 5.16. Optimal Primal Solution to the Diet Example Problem** | | | | | |
| Variable | Value | Reduced Cost | Equation | Slack | Price |
| XC | 0 | 0.095 | Protein L Max | 0.059 | 0 |
| XH | 0.001 | 0 | Fat Max | 0.041 | 0 |
| XSB | 0.011 | 0 | Salt Max | 0.005 | 0 |
| XUr | 0.014 | 0 | Calcium Max | 0.007 | 0 |
| Xd | 0.002 | 0 | Phosphrs | 0.000 | -2.207 |
| XSLT | 0.015 | 0 | Net Engy Min | 0.000 | 0.065 |
| XVA | 0.001 | 0 | Protein Min | 0.000 | 0.741 |
| XSL | 0.956 | 0 | Vita Lim Min | 0.000 | 0 |
|  |  |  | Salt Lim Min | 0.000 | 0.218 |
|  |  |  | Calcium Min | .000 | 4.400 |
|  |  |  | Phosphrs | 0.008 | 0 |
|  |  |  | Weight | 0.000 | -0.108 |

Feed Mix Problem

**Primal and Dual Algebra:**

# Primal



## Dual



Feed Mix Problem

**Notes on Dual Algebra:**

Here we have an equality constraint. In fact a general LP is

Max cX

s.t. AX < b

DX > e

FX = g

X > 0

To write the dual we need to get in standard form and I will use max subject to less thans

First get rid of equality

Max cX

s.t. AX < b

DX > e

FX < g

FX > g

X > 0

Then covert all to less thans

Max cX

s.t. AX < b

-DX < -e

FX < g

-FX < -g

X > 0

Feed Mix Problem

**Notes on Dual Algebra:**

Max cX

s.t. AX < b

-DX < -e

FX < g

-FX < -g

X > 0

writing dual

Min ub -ve +wg - yg

s.t. uA -vD +wF - yF > c

u , v , w , y > 0

Then note one could make substitutions

v’=v and z= w-y

yielding

Min ub +v’e +zg

s.t. uA +v’D +zF > c

u > 0

v’ < 0

z 0

so when primal has **>** dual variable is negative

and when primal has **=** dual variable is unrestricted in sign

**Feed Mix Problem**

**Dual and Example**



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 5.17. Dual Formulation of Feed Mix Example Problem** | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | γ1 |  | γ 2 |  | γ 3 |  | γ 4 |  | γ 5 |  | β 1 |  | β 2 |  | β 3 |  | β 4 |  | β 5 |  | β 6 |  | α |  |  |
| Max | - .13 | - | .05 | - | .02 | - | .01 | - | .12 | + | 1.34351 | + | .071 | + | 2200 | + | .015 | + | .0025 | + | .0035 | + | 1 |  |  |
|  | - .075 | - | .0357 |  |  | - | .0002 | - | .0035 | + | 1.48 | + | .075 | + | 600 |  |  | + | .0002 | + | .0035 | + | 1 | ≤ | .133 |
|  | - .127 | - | .022 |  |  | - | .0125 | - | .0023 | + | .49 | + | .127 | + | 50880 |  |  | + | .0125 | + | .0023 | + | 1 | ≤ | .077 |
|  | - .438 | - | .013 |  |  | - | .0036 | - | .0075 | + | 1.29 | + | .438 | + | 80 |  |  | + | .0036 | + | .0075 | + | 1 | ≤ | .3 |
|  | -2.62 |  |  |  |  |  |  | - | .68 |  |  | + | 2.62 |  |  |  |  |  |  | + | .68 | + | 1 | ≤ | .332 |
|  |  |  |  |  |  | - | .2313 | - | .1865 |  |  |  |  |  |  |  |  | + | .2313 | + | .1865 | + | 1 | ≤ | .498 |
|  |  |  |  | - | 1 |  |  |  |  |  |  |  |  |  |  | + | 1 |  |  |  |  | + | 1 | ≤ | .110 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | + | 2204600 |  |  |  |  |  |  | + | 1 | ≤ | .286 |
|  | - .032 | - | .009 |  |  | - | .002 | - | .0024 | + | 1.39 | + | .032 |  |  |  |  | + | .002 | + | .0024 | + | 1 | ≤ | P |

**Feed Mix Problem**

**Primal / Dual Empirical Comparison:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 5.14. Primal Formulation of Feed Problem** | | | | | | | | | | | | | | | | | |
|  | Corn |  | Hay |  | Soybean |  | Urea |  | Dical |  | Salt |  | Vitamin A |  | Slurry |  |  |
| Min | .133XC | + | .077XH | + | .3XSB | + | .332XUr | + | .498Xd | + | .110XSLT | + | .286XVA | + | PXSL |  |  |
| s.t. | .075XC | + | .127XH | + | .438XSB | + | 2.62XUr |  |  |  |  |  |  | + | .032XSL | ≤ | .13 |
| Max | .0357XC | + | .022XH | + | .013XSB |  |  |  |  |  |  |  |  | + | .009XSL | ≤ | .05 |
| Nut |  |  |  |  |  |  |  |  |  |  | XSLT |  |  |  |  | ≤ | .02 |
|  | .0002XC | + | .0125XH | + | .0036XSB |  |  | + | .2313Xd |  |  |  |  | + | .002XSL | ≤ | .01 |
|  | .0035XC | + | .0023XH | + | .0075XSB | + | .68XUr | + | .1865Xd |  |  |  |  | + | .0024XSL | ≤ | .012 |
|  | 1.48XC | + | .49XH | + | 1.29XSB |  |  |  |  |  |  |  |  | + | 1.39XSL | ≥ | 1.34351 |
|  | .075XC | + | .127XH | + | .438XSB | + | 2.62XUr |  |  |  |  |  |  | + | .032XSL | ≥ | .071 |
| Min | 600XC | + | 50880XH | + | 80XSB |  |  |  |  |  |  | + | 2204600XVA |  |  | ≥ | 2200 |
| Nut |  |  |  |  |  |  |  |  |  |  | XSLT |  |  |  |  | ≥ | .015 |
|  | .0002XC | + | .0125XH | + | .0036XSB |  |  | + | .2313Xd |  |  |  |  | + | .002XSL | ≥ | .0025 |
|  | .0035XC | + | .0023XH | + | .0075XSB | + | .68XUr | + | .1865Xd |  |  |  |  | + | .0024XSL | ≥ | .0035 |
| Volume | XC | + | XH | + | XSB | + | XUr | + | Xd | + | XSLT | + | XVA | + | XSL | = | 1 |

**Feed Mix Problem**

**Primal / Dual Empirical Comparison:**

**≤**

**Dual:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 5.17. Dual Formulation of Feed Mix Example Problem** | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | γ1 |  | γ 2 |  | γ 3 |  | γ 4 |  | γ 5 |  | β 1 |  | β 2 |  | β 3 |  | β 4 |  | β 5 |  | β 6 |  | α |  |  |
| Max | - .13 | - | .05 | - | .02 | - | .01 | - | .12 | + | 1.34351 | + | .071 | + | 2200 | + | .015 | + | .0025 | + | .0035 | + | 1 |  |  |
|  | - .075 | - | .0357 |  |  | - | .0002 | - | .0035 | + | 1.48 | + | .075 | + | 600 |  |  | + | .0002 | + | .0035 | + | 1 | ≤ | .133 |
|  | - .127 | - | .022 |  |  | - | .0125 | - | .0023 | + | .49 | + | .127 | + | 50880 |  |  | + | .0125 | + | .0023 | + | 1 | ≤ | .077 |
|  | - .438 | - | .013 |  |  | - | .0036 | - | .0075 | + | 1.29 | + | .438 | + | 80 |  |  | + | .0036 | + | .0075 | + | 1 | ≤ | .3 |
|  | -2.62 |  |  |  |  |  |  | - | .68 |  |  | + | 2.62 |  |  |  |  |  |  | + | .68 | + | 1 | ≤ | .332 |
|  |  |  |  |  |  | - | .2313 | - | .1865 |  |  |  |  |  |  |  |  | + | .2313 | + | .1865 | + | 1 | ≤ | .498 |
|  |  |  |  | - | 1 |  |  |  |  |  |  |  |  |  |  | + | 1 |  |  |  |  | + | 1 | ≤ | .110 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | + | 2204600 |  |  |  |  |  |  | + | 1 | ≤ | .286 |
|  | - .032 | - | .009 |  |  | - | .002 | - | .0024 | + | 1.39 | + | .032 |  |  |  |  | + | .002 | + | .0024 | + | 1 | ≤ | P |

**Feed Mix Problem**

**Primal / Dual Solution Comparison**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 5.16. Optimal Primal Solution to the Diet Example Problem** | | | | | | | | | | | |
| Variable | | | Value | | Reduced Cost | | Constraint | | Level | | Price |
| XC | | | 0 | | 0.095 | | Protein L Max | | 0.071 | | 0 |
| XH | | | 0.001 | | 0 | | Fat Max | | 0.009 | | 0 |
| XSB | | | 0.011 | | 0 | | Salt Max | | 0.015 | | 0 |
| XUr | | | 0.014 | | 0 | | Calcium Max | | 0.002 | | 0 |
| Xd | | | 0.002 | | 0 | | Phosphrs | | 0.012 | | -2.207 |
| XSLT | | | 0.015 | | 0 | | Net Engy Min | | 1.344 | | 0.065 |
| XVA | | | 0.001 | | 0 | | Protein Min | | 0.071 | | 0.741 |
| XSL | | | 0.956 | | 0 | | Vita Lim Min | | 2200.0 | | 0 |
|  | | |  | |  | | Salt Lim Min | | 0.015 | | 0.218 |
|  | | |  | |  | | Calcium Min | | .002 | | 4.400 |
|  | | |  | |  | | Phosphrs | | 0.012 | | 0 |
|  | | |  | |  | | Weight | | 1 | | -0.108 |
| **Table 5.18. Optimal Solution to the Dual of the Feed Mix Problem** | | | | | | | | | | |
| Variable | | Value | Reduced Cost | | Constraint | | Level | | Shadow Price | |
| γ1 | | 0 | -0.059 | | Corn | | 0.038 | | 0 | |
| γ2 | | 0 | -0.041 | | Hay | | 0.077 | | 0.001 | |
| γ3 | | 0 | -0.005 | | Soybean | | 0.300 | | 0.011 | |
| γ4 | | 0 | -0.007 | | Orea | | 0.332 | | 0.014 | |
| γ5 | | 2.207 | 0 | | Dical | | 0.498 | | 0.002 | |
| β1 | | 0.065 | 0 | | Salt | | 0.110 | | 0.015 | |
| β2 | | 0.741 | 0 | | Vita | | 0.286 | | 0.001 | |
| β3 | | 0 | 0 | | Slurry | | 0.01 | | 0.956 | |
| β4 | | 0.218 | 0 | |  | |  | |  | |
| β5 | | 4.400 | 0 | |  | |  | |  | |
| β6 | | 0 | -0.008 | |  | |  | |  | |
| Α | | -0.108 | 0 | |  | |  | |  | |

**Feed Mix Problem**

**Cost Ranging Result**



**Feed Mix Problem**

|  |
| --- |
| **Table 5.15 GAMS Formulation of Diet Example** |

1 SET INGREDT NAMES OF THE AVAILABLE FEED INGREDIENTS

2 /CORN,HAY,SOYBEAN,UREA,DICAL,SALT,VITA,SLURRY/

3 NUTRIENT NUTRIENT REQUIREMENT CATEGORIES

4 /NETENGY,PROTEIN,FAT,VITALIM,SALTLIM,CALCIUM, PHOSPHRS/

5 LIMITS TYPES OF LIMITS IMPOSED ON NUTRIENTS /MIN,MAX/;

6 PARAMETER INGREDCOST(INGREDT) INGREDIENT COSTS PER KG BOUGHT

7 /CORN .133, HAY .077, SOYBEAN .300, UREA .332

8 , DICAL .498,SALT .110, VITA .286, SLURRY .01/;

9 TABLE NUTREQUIRE(NUTRIENT, LIMITS) NUTRIENT REQUIREMENTS

10 MIN MAX

11 NETENGY 1.34351

12 PROTEIN .071 .130

13 FAT 0 .05

14 VITALIM 2200

15 SALTLIM .015 .02

16 CALCIUM .0025 .0100

17 PHOSPHRS .0035 .0120;

18 TABLE CONTENT(NUTRIENT,INGREDT) NUTR CONTENTS PER KG OF FEED

19 CORN HAY SOYBEAN UREA DICAL SALT VITA SLURRY

20 NETENGY 1.48 .49 1.29 1.39

21 PROTEIN .075 .127 .438 2.62 0.032

22 FAT .0357 .022 .013 0.009

23 VITALIM 600 50880 80 2204600

24 SALTLIM 1

25 CALCIUM .0002 .0125 .0036 .2313 .002

26 PHOSPHRS 0035 .0023 .0075 .68 .1865 .0024;

27

28 POSITIVE VARIABLES

29 FEEDUSE(INGREDT) AMOUNT OF EACH INGREDIENT USED IN MIXING FEED;

30 VARIABLES

31 COST PER KG COST OF THE MIXED FEED;

32 EQUATIONS

33 OBJT OBJECTIVE FUNCTION ( TOTAL COST OF THE FEED )

34 MAXBD(NUTRIENT) MAX LIMITS ON EACH NUTRIENT IN THE BLENDED FEED

35 MINBD(NUTRIENT) MIN LIMITS ON EACH NUTRIENT IN THE BLENDED FEED

36 WEIGHT REQUIRED THAT EXACTLY ONE KG OF FEED BE PRODUCED;

37

38 OBJT..COST =E= SUM(INGREDT,INGREDCOST(INGREDT)\* FEEDUSE(INGREDT))

39 MAXBD(NUTRIENT)$NUTREQUIRE(NUTRIENT,"MAXIMUM")..

40 SUM(INGREDT,CONTENT(NUTRIENT , INGREDT) \* FEEDUSE(INGREDT))

41 =L= NUTREQUIRE(NUTRIENT, "MAXIMUM");

42 MINBD(NUTRIENT)$NUTREQUIRE(NUTRIENT,"MINIMUM")..

43 SUM(INGREDT,CONTENT(NUTRIENT, INGREDT) \* FEEDUSE(INGREDT))

44 =G= NUTREQUIRE(NUTRIENT, "MINIMUM");

45 WEIGHT.. SUM(INGREDT, FEEDUSE(INGREDT)) =E= 1. ;

46 MODEL FEEDING /ALL/;

47 SOLVE FEEDING USING LP MINIMIZING COST;

48

49 SET VARYPRICE PRICE SCENARIOS /1\*30/

50 PARAMETER SLURR(VARYPRICE,\*)

51 OPTION SOLPRINT = OFF;

52 LOOP (VARYPRICE,

53 INGREDCOST("SLURRY")= 0.01 + (ORD(VARYPRICE)‑1)\*0.005;

54 SOLVE FEEDING USING LP MINIMIZING COST;

55 SLURR(VARYPRICE,"SLURRY") = FEEDUSE.L("SLURRY");

56 SLURR(VARYPRICE,"PRICE") = INGREDCOST("SLURRY") ) ;

57 DISPLAY SLURR;

|  |
| --- |
|  |

###### Joint Products Problem

Formulation

Basic notation and the decision variable

Let us denote the set of

: the produced products as product

: the production possibilities as process

: the purchased inputs as input

: the available resources as resource

Let us define three fundamental decision variables as

: the set of produced product,

Salesproduct

: the set of production possibilities,

Productionprocess

: the set of purchased inputs,

BuyInputinput

Joint Products Problem

Formulation

To set up the joint products problem, four additional parameter values that give the composite relationship among product, process, input, and resource are needed.

These parameters are:

the quantity yielded by the production possibility, qproduct,process

the amount of the inputth input used by the processth production possibility, rinput,process

the amount of the resourceth resource used by the processth production possibility, sresource,process

the amount of resource availability or endowment by resource, bresource

###### Joint Products Problem

Formulating the Problem

The objective function:

We want to maximize total profits across all of the possible productions. To do so, three additional required parameters for sale price, input purchase cost, and other production costs associated with production are needed.

Let us define these parameters as

SalePriceproduct

InputCostinput

OtherCostprocess

Then the objective function becomes

Maximize







Joint Products Problem

Formulating the Problem

The constraints:

There are four general types of constraints:

1) demand and supply balance for which quantity sold of each product is less than or equal to the quantity yielded by production.



2) demand and supply balance for which quantity purchased of each fixed price input is greater than or equal to the quantity utilized by the production activities.



1. resource availability constraint insuring that the quantity used of each fixed quantity input does not exceed the resource endowments.



4) nonnegativity



Joint Products Problem

Formulating the Problem

Maximize







s.t.







Joint Products Problem

**Example: wheat production**

**Two products produced:** Wheat and wheat straw

**Three inputs:** Land, fertilizer, seed

**Seven production processes:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Outputs and Inputs Per Acre Process | | | | | | | |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Wheat yield in bu. | 30 | 50 | 65 | 75 | 80 | 80 | 75 |
| Wheat straw yield/bales | 10 | 17 | 22 | 26 | 29 | 31 | 32 |
| Fertilizer use in Kg. | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| Seed in pounds | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Land | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Wheat price = $4/bushel, wheat straw price =$0.5/bale

Fertilizer = $2 per kg, Seed = $0.2/lb.

$5 per acre production cost for each process

Land = 500 acres

###### Joint Products Problem

**Example:** **Wheat Production**

Maximize







s.t.







Demand(end)-SUPPLY(end) < supply (ex) – Demand(ex)

Demand(end)\_ Demand(ex) < +-SUPPLY(end)+supply (ex) Demand(ex)

###### Joint Products Problem

**Example:** **Wheat Production**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 4Sale1 | + | .5Sale2 | - | 5Y1 | - | 5Y2 | - | 5Y3 | - | 5Y4 | - | 5Y5 | - | 5Y6 | - | 5Y7 | - | 2Z1 | - | .2Z2 | MAX | |
| s.t. | Sale1 |  |  | - | 30Y1 | - | 50Y2 | - | 65Y3 | - | 75Y4 | - | 80Y5 | - | 80Y6 | - | 75Y7 |  |  |  |  | ≤ | 0 |
|  |  |  | Sale2 | - | 10Y1 | - | 17Y2 | - | 22Y3 | - | 26Y4 | - | 29Y5 | - | 31Y6 | - | 32Y7 |  |  |  |  | ≤ | 0 |
|  |  |  |  |  |  | + | 5Y2 | + | 10Y3 | + | 15Y4 | + | 20Y5 | + | 25Y6 | + | 30Y7 | - | Z1 |  |  | ≤ | 0 |
|  |  |  |  |  | 10Y1 | + | 10Y2 | + | 10Y3 | + | 10Y4 | + | 10Y5 | + | 10Y6 | + | 10Y7 |  |  | - | Z2 | ≤ | 0 |
|  |  |  |  |  | Y1 | + | Y2 | + | Y3 | + | Y4 | + | Y5 | + | Y6 | + | Y7 |  |  |  |  | ≤ | 500 |
|  | Sale1 | , | Sale2 | , | Y1 | , | Y2 | , | Y3 | , | Y4 | , | Y5 | , | Y6 | , | Y7 | , | Z1 | , | Z2 | ≥ | 0 |

Note that: Y refers to Productionprocess and Z refers to BuyInputinput

U1>= 4

U2 >\_ 0.5

-W1>= -2

###### Joint Products Problem

**Example:** **Wheat Production – Solution**

- 40,000 bushels of wheat and 14,500 bales of straw are produced by 500 acres of the fifth production possibility using 10,000 kilograms of fertilizer and 5,000 lbs. of seed

- reduced cost shows a $169.50 cost if the first production possibility is used.

- shadow prices are values of sales, purchase prices of the various outputs and inputs, and land values ($287.5).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Optimal Solution Value 143,750 | | | | | |
| Variable | Value | Reduced Cost CCostcCost | Equation | Slack | Shadow Price |
| Sale1 | 40,000 | 0 | Wheat | 0 | -4 |
| Sale2 | 14,500 | 0 | Straw | 0 | -0.5 |
| Process1 | 0 | -169.50 | Fertilizer | 0 | 2 |
| Process2 | 0 | -96.00 | Seed | 0 | 0.2 |
| Process3 | 0 | -43.50 | Land | 0 | 287.5 |
| Process4 | 0 | -11.50 |  |  |  |
| Process5 | 500 | 0 |  |  |  |
| Process6 | 0 | -9.00 |  |  |  |
| Process7 | 0 | -38.50 |  |  |  |
| Buyinputfert | 10,000 | 0 |  |  |  |
| Buyinputseet | 5,000 | 0 |  |  |  |

###### Joint Products Problem

**Example:** **Wheat Production – Dual**

****

**Dual objective: minimizes total marginal resource values**

U is the marginal resource value

V is the marginal product value

W is the marginal input cost

1st constraint (dual implication of a sales variable) insures that V is GE the sales price. A lower bound is imposed on the shadow price for the commodity sold.

2nd constraint (dual implication of a production variable) insures that the total value of the products yielded by a process is LE the value of the inputs used in production.

3rd constraint (dual implication of a purchase variable) insures that W is no more than its purchase price. An upper bound is imposed on the shadow price of the item which can be purchased.

###### Joint Products Problem

###### **Example:** **Wheat Production – Dual Empirical**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 5.22. Dual Formulation of Example Joint Products Problem** | | | | | | | | | | | |
| Min |  |  |  |  |  |  |  |  | 500U |  |  |
| s.t. | V1 |  |  |  |  |  |  |  |  | ≥ | 4 |
|  |  |  | V2 |  |  |  |  |  |  | ≥ | .5 |
|  | -30V1 | - | 10V2 |  |  | + | 10W2 | + | U | ≥ | -5 |
|  | -50V1 | - | 17V2 | + | 5W1 | + | 10W2 | + | U | ≥ | -5 |
|  | -65V1 | - | 22V2 | + | 10W1 | + | 10W2 | + | U | ≥ | -5 |
|  | -75V1 | - | 26V2 | + | 15W1 | + | 10W2 | + | U | ≥ | -5 |
|  | -80V1 | - | 29V2 | + | 20W1 | + | 10W2 | + | U | ≥ | -5 |
|  | -80V1 | - | 31V2 | + | 25W1 | + | 10W2 | + | U | ≥ | -5 |
|  | -75V1 | - | 32V2 | + | 30W1 | + | 10W2 | + | U | ≥ | -5 |
|  |  |  |  |  | -W1 |  |  |  |  | ≥ | -2 |
|  |  |  |  |  |  |  | -W2 |  |  | ≥ | -0.2 |
|  | V1 | , | V2 | , | W1 | , | W2 | , | U | ≥ | 0 |

+ 20W1 + 10W2 + U+5 ≥ +80V1 + 29V2

###### Joint Products Problem

**Example:** **Wheat Production – Dual**

The optimal solution to the dual problem corresponds exactly to the optimal primal solution where:

**Dual** **Primal**

Value Shadow Price

Reduced Cost Slack

Shadow Price Value

Slacks Reduced Cost

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Objective Function Value 143,750 | | | |  |  |
| Variable | Value | Reduced Cost | Equation | Slacks | Shadow |
| V1 | 4 | 0 | Wheat | 0 | 40,000 |
| V2 | 0.5 | 0 | Straw | 0 | 14,500 |
| W1 | 2 | 0 | Prod 1 | 169.5 | 0 |
| W2 | 0.2 | 0 | Prod 2 | 96 | 0 |
| U | 287.5 | 0 | Prod 3 | 43.5 | 0 |
|  |  |  | Prod 4 | 11.5 | 0 |
|  |  |  | Prod 5 | 0 | 500 |
|  |  |  | Prod 6 | 9 | 0 |
|  |  |  | Prod 7 | 38.5 | 0 |
|  |  |  | Fertilizer | 0 | 10,000 |
|  |  |  | Seed | 0 | 5,000 |

Joint Products Problem

**GAMS Formulation**

|  |
| --- |
| **Table 5.20. GAMS Formulation of the Joint Products Example** |

1 SET PRODUCTS LIST OF ALTERNATIVE PRODUCT /WHEAT, STRAW/

2 INPUTS PURCHASED INPUTS /SEED, FERT/

3 FIXED FIXED INPUTS /LAND/

4 PROCESS POSSIBLE INPUT COMBINATIONS /Y1\*Y7/;

5

6 PARAMETER PRICE(PRODUCTS) PRODUCT PRICES /WHEAT 4.00, STRAW 0.50/

7 COST(INPUTS) INPUT PRICES /SEED 0.20, FERT 2.00/

8 PRODCOST(PROCESS) PRODUCTION COSTS BY PROCESS

9 AVAILABLE(FIXED) FIXED INPUTS AVAILABLE / LAND 500 /;

10

11 PRODCOST(PROCESS) = 5;

12

13 TABLE YIELDS(PRODUCTS, PROCESS) PRODUCTION POSSIBILITIES YIELDS

14 Y1 Y2 Y3 Y4 Y5 Y6 Y7

15 WHEAT 30 50 65 75 80 80 75

16 STRAW 10 17 22 26 29 31 32;

17

18 TABLE USAGE(INPUTS,PROCESS) PURCHASED INPUT USAGE BY PRODUCTION POSSIBLIITIES

19 Y1 Y2 Y3 Y4 Y5 Y6 Y7

20 SEED 10 10 10 10 10 10 10

21 FERT 0 5 10 15 20 25 30;

22

23 TABLE FIXUSAGE(FIXED,PROCESS) FIXED INPUT USAGE BY PRODUCTION POSSIBLIITIES

24 Y1 Y2 Y3 Y4 Y5 Y6 Y7

25 LAND 1 1 1 1 1 1 1;

26

27 POSITIVE VARIABLES

28 SALES(PRODUCTS) AMOUNT OF EACH PRODUCT SOLD

29 PRODUCTION(PROCESS) LAND AREA GROWN WITH EACH INPUT PATTERN

30 BUY(INPUTS) AMOUNT OF EACH INPUT PURCHASED ;

31 VARIABLES

32 NETINCOME NET REVENUE (PROFIT);

33 EQUATIONS

34 OBJT OBJECTIVE FUNCTION (NET REVENUE)

35 YIELDBAL(PRODUCTS) BALANCES PRODUCT SALE WITH PRODUCTION

36 INPUTBAL(INPUTS) BALANCE INPUT PURCHASES WITH USAGE

37 AVAIL(FIXED) FIXED INPUT AVAILABILITY;

38

39 OBJT.. NETINCOME =E=

40 SUM(PRODUCTS , PRICE(PRODUCTS) \* SALES(PRODUCTS))

41 ‑ SUM(PROCESS , PRODCOST(PROCESS)

42 \* PRODUCTION(PROCESS))

43 ‑ SUM(INPUTS , COST(INPUTS)

44 \* BUY(INPUTS));

45 YIELDBAL(PRODUCTS)..

1. SUM(PROCESS, YIELDS(PRODUCTS,PROCESS)
2. \* PRODUCTION(PROCESS))

48 =G= SALES(PRODUCTS);

49 INPUTBAL(INPUTS)..

50 SUM(PROCESS, USAGE(INPUTS,PROCESS) \* PRODUCTION(PROCESS))

51 =L= BUY(INPUTS);

52 AVAIL(FIXED)..

53 SUM(PROCESS, FIXUSAGE(FIXED,PROCESS)\*PRODUCTION(PROCESS))

54 =L= AVAILABLE(FIXED);

55

56 MODEL JOINT /ALL/;

57 SOLVE JOINT USING LP MAXIMIZING NETINCOME;

Joiint Products Problem

**Alternative Formulations**

|  |
| --- |
| **Table 5.24 Alternative Computer Inputs for a Model** |
| Simple GAMS Input File |

POSITIVE VARIABLES X1 , X2, X3

VARIABLES Z

EQUATIONS OBJ, CONSTRAIN1 , CONSTRAIN2;

OBJ.. Z =E= 3 \* X1 + 2 \* X2 + 0.5\* X3;

CONSTRAIN1.. X1 + X2 +X3=L= 10;

CONSTRAIN2.. X1 ‑ X2 =L= 3;

MODEL PROBLEM /ALL/;

SOLVE PROBLEM USING LP MAXIMIZING Z;

|  |
| --- |
| Lindo Input |

MAX 3 \* X1 + 2 \* X2 + 0.5\* X3;

ST

X1 + X2 +X3 < 10

X1 ‑ X2 < 3

END

GO

|  |
| --- |
| Tableau Input File |

5 3

3. 2. 0.5 0. 0.

1. 1. 1. 1. 0. 10.

1. ‑1. 0. 0. 1. 3.

|  |
| --- |
| MPS Input File |

NAME CH2MPS

ROWS

N R1

L R2

L R3

COLUMNS

X1 R0 3. R1 1.

X1 R3 1.

X2 R0 2. R1 1.

X2 R1 ‑1.

X3 R0 0.5 R1 1.

RHS

RHS1 R1 10. R1 3.

ENDDATA

|  |
| --- |
| More Complex GAMS input file |

SET PROCESS TYPES OF PRODUCTION PROCESSES /X1,X2,X3/

RESOURCE TYPES OF RESOURCES /CONSTRAIN1,CONSTRAIN2/

PARAMETER

PRICE(PROCESS) PRODUCT PRICES BY PROCESS /X1 3,X2 2,X3 0.5/

PRODCOST(PROCESS) COST BY PROCESS /X1 0 ,X2 0, X3 0/

RESORAVAIL(RESOURCE) RESOURCE AVAILABLITY

/CONSTRAIN1 10 ,CONSTRAIN2 3/

TABLE RESOURUSE(RESOURCE,PROCESS) RESOURCE USAGE

X1 X2 X3

CONSTRAIN1 1 1 1

CONSTRAIN2 1 ‑1

POSITIVE VARIABLES PRODUCTION(PROCESS) ITEMS PRODUCED BY PROCESS;

VARIABLES PROFIT TOTALPROFIT;

EQUATIONS OBJT OBJECTIVE FUNCTION ( PROFIT )

AVAILABLE(RESOURCE) RESOURCES AVAILABLE ;

OBJT.. PROFIT=E= SUM(PROCESS,(PRICE(PROCESS)‑PRODCOST(PROCESS))\*

PRODUCTION(PROCESS)) ;

AVAILABLE(RESOURCE).. SUM(PROCESS,RESOURUSE(RESOURCE,PROCESS)

\*PRODUCTION(PROCESS)) =L= RESORAVAIL(RESOURCE);

MODEL RESALLOC /ALL/;

SOLVE RESALLOC USING LP MAXIMIZING PROFIT;

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