

### Homework Chapter III -IV

1. Farmer Brazos has a small farm on which he grows three types of crops: cotton, corn and hay. The farmer seeks to determine the amount of each crop he should grow so as to maximize net returns.

The scarce resources Joe must allocate are 640 acres of land and 900 acre ft of water

From previous experience, Joe has developed the following information per acre of crops grown:

	Cotton	Corn	Hay	
Water use ( acre feet per acre)		0.7	2.2	1.0
Yield per acre (crop units per acre)		0.8	100	2
Sale price (\$/ crop unit)		450	3.2	80
Cost of production (\$/acre)		320	200	25

Set up and solve this problem

- b. Interpret at least 2 elements from the numerical counterparts of each the following terms at optimality
- i.  $C_B B^{-1}$
  - ii.  $B^{-1}$
  - iii.  $B^{-1} b$
  - iv.  $C_B B^{-1} b$  (one element only)
  - v.  $C_B B^{-1} A_{NB} - C_{NB}$
- c. Do a cost range on
- i. Obj Coefficient of a the first basic variable
  - ii. Obj Coefficient of the first non basic variable
  - iii. Obj Coefficient of the first basic and the first non basic simultaneously  
(Where coefficients change by equal amounts)
- d. do a RHS range on
- i. 1st constraint
  - ii. 2nd constraint
  - iii. both constraints together  
(Where RHS coefficients change by equal amounts)

2. Solve the following

$$\begin{array}{rcll} \text{Max} & 4x_1 + 2x_2 & & \\ \text{s.t.} & 3x_1 + x_2 & \leq & 12 \\ & x_1 & \leq & 2 \\ & & x_2 & \leq 6 \\ & x_1, x_2 & \geq & 0 \end{array}$$

Explain your selections of variables to enter and leave the basis, the reduced costs and the shadow prices

3. Solve the following

$$\begin{array}{rcll} \text{Max} & 4x_1 + 2x_2 & & \\ \text{s.t.} & 2x_1 + x_2 & \leq & 12 \\ & x_1 & \leq & 11 \\ & & x_2 & \leq 6 \\ & x_1, x_2 & \geq & 0 \end{array}$$

Explain your selections of variables to enter and leave the basis and the reduced costs and variable values

4. Write the dual of

$$\begin{array}{rcll} \text{Max} & 4x_1 + 2x_2 - x_3 - 4x_4 + 2x_5 & & \\ \text{s.t.} & 3x_1 + x_2 - x_3 - x_4 + x_5 & \leq & 12 \\ & x_1 + x_2 + x_3 & \leq & 2 \\ & x_1, x_2, x_3, x_4, x_5 & \geq & 0 \end{array}$$

Explain the effect of the variables  $x_3$ ,  $x_4$  and  $x_5$  on the solution

5. Write and solve the dual of problem #1

Contrast the solution information (variable values, slacks, shadow prices, objective value, reduced costs for the two problems)

6. Demonstrate complementary slackness holds in the solution to problem 1

7. Explain the statement *all profits are allocated to resources* in the context of problem 1.