How to formulate an applied LP problem

Notes for AGEC 622

Bruce McCarl

Regents Professor of Agricultural Economics

Texas A&M University

How to formulate an applied LP problem

McCarl and Spreen Chapter 6

Topics Covered

Tableau building
Identification of
- constraints
- variables
- relevant parameter values

We do this in the context of a problem

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Lets look at a problem

Suppose a farmer approaches you and wants to set up a problem. He wants to know given government program changes what should he grow. The first question is what can be grown

The farmer says

Corn Cotton Cattle

These are our first indication of variables

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		Corn	Cotton	Cattle			
Objective							

So we set up a tableau with variables across the top
And constraints down side plus an objective

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Now we ask what limits your plans
The farmer says Land and Labor

		Corn	Cotton	Cattle			
Objective							
Land							
Labor							

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Now we ask how much land and labor do you have
The farmer says 450 acres Land and a seasonal amount of Labor

		Corn	Cotton	Cattle			
Objective							
Land							**$\leq$ 450**
Spring Labor							
Summer Labor							
Fall Labor							

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Now we ask how much labor by season
Farmer works 4 days per week during 6 weeks of spring with a hired hand 8 hours per day and similar statements for other seasons

		Corn	Cotton	Cattle			
Objective							
Land							**≤ 450**
Spring Labor							**≤ 192**
Summer Labor							**≤ 245**
Fall Labor							**≤ 155**

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Now we ask how about land use
Farmer has 450 acres of which 300 are in pasture supporting 20 cows and 150 are planted to crops

		Corn	Cotton	Cattle			
Objective							
Crop land		**1**	**1**				**≤ 150**
Pasture land				**15**			**≤ 300**
Spring Labor							**≤ 192**
Summer Labor							**≤ 245**
Fall Labor							**≤ 155**

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Now we ask how about cows. Fasrmer says the cost of buying one is $50 plus a $50 production cost then sells them for 60 cents per pound with weight 800 pounds. The farm uses 60 hours of spring labor on the whole herd or 3 hours/animal (60/20)=3 and gives other data for the other periods. The cow herd is fed 800 bushels corn or 40 bu per head

		Corn	Cotton	Cattle			
Objective				**380**			
Crop land		**1**	**1**				**≤ 150**
Pasture land				**15**			**≤ 300**
Spring Labor				**3**			**≤ 192**
Summer Labor				**5**			**≤ 245**
Fall Labor				**3**			**≤ 155**
Corn on hand				**40**			**≤ 0**

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Now we ask how about corn. Farmer says yield is 100 bu per acre
Production cost is $100 per acre. 16 acres are planted in an 8 hour day in the spring
In the fall the farm can harvest 16 acres per day. Corn sells for $2.30 per bu

		Corn	Cotton	Cattle	Sell corn		
Objective		**-100**		**380**	**2.3**		
Crop land		**1**	**1**				**≤ 150**
Pasture land				**15**			**≤ 300**
Spring Labor		**0.5**		**3**			**≤ 192**
Summer Labor		**0.1**		**5**			**≤ 245**
Fall Labor		**0.5**		**3**			**≤ 155**
Corn on hand		**-100**		**40**	**1**		**≤ 0**

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Now we ask how about cotton. Yield is 1 bale per acre
Production cost is $200 per acre. The farm can get ready to and plant 16 acres in an 8 hour day in the spring and harvest 8 acres per day. Cotton sells for $325 per bale

		Corn	Cotton	Cattle	Sell Corn	Sell Cotton	
Objective		**-100**	**-200**	**380**	**2.3**	**325**	
Crop land		**1**	**1**				**≤ 150**
Pasture land				**15**			**≤ 300**
Spring Labor		**0.5**	**0.5**	**3**			**≤ 192**
Summer Labor		**0.1**	**0.2**	**5**			**≤ 245**
Fall Labor		**0.5**	**1**	**3**			**≤ 155**
Corn on hand		**-100**		**40**	**1**		**≤ 0**
Cotton on hand			**-1**			**1**	**≤ 0**

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Now we solve

		Corn	Cotton	Cows	Sell Corn	Sell Cotton		Slack	Shadow price
Objective		**-100**	**-200**	**380**	**2.3**	**325**		25260	
Crop land		**1**	**1**				**≤ 150**	0	130
Pasture land				**15**			**≤ 300**	0	19.2
Spring Labor		**0.5**	**0.5**	**3**			**≤ 192**	57	0
Summer Lab		**0.1**	**0.2**	**5**			**≤ 245**	130	0
Fall Labor		**0.5**	**0.75**	**3**			**≤ 155**	20	0
Corn on hand		**-100**		**40**	**1**		**≤ 0**	0	2.30
Cott. on hand			**-1**			**1**	**≤ 0**	0	325
Level		**150**	**0**	**20**	**14200**	**0**			
Reduced cost		**0**	**5**	**0**	**0**	**0**			

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Now a study lets see what it is worth to convert 100 acres pasture to crops

		Corn	Cotton	Cows	Sell Corn	Sell Cotton		Slack	Shadow price
Objective		**-100**	**-200**	**380**	**2.3**	**325**		35380	
Crop land		**1**	**1**				**≤ 250**	0	82
Pasture land				**15**			**≤ 200**	50	0
Spring Labor		**0.5**	**0.5**	**3**			**≤ 192**	37	0
Summer Labor		**0.1**	**0.2**	**5**			**≤ 245**	170	0
Fall Labor		**0.5**	**0.75**	**3**			**≤ 155**	0	96
Corn on hand		**-100**		**40**	**1**		**≤ 0**	0	2.30
Cotton on hand			**-1**			**1**	**≤ 0**	0	325
Level		**250**	**0**	**10**	**24600**	**0**			
Reduced cost		**0**	**53**	**0**	**0**	**0**			

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After solve

Obj 35,380

Before 25,260

Land development worth 10,120 per year

Application Thoughts

Why use LP

1. Have decision problem to resolve

2. Want to simulate the results of a change

Steps

1. Identify variables

2. Identify constraints

3. Identify coefficients

really an iterative process

Application Thoughts

Assumptions

1. Right OBJ, constraints, variables

2. Math
    1. Additive
    2. Proportional
    3. Certain
    4. Continuous

LP in Action - war stories

1. Repair man location
2. Hydropower scheduling
3. Machinery adequacy for growth
4. Portfolio selection

Toward Proper Modeling

Types of items in an applied LP problem

Constraints: $\mathbf{AX \leq b}$

- Types: technical, institutional, subjective.
- # of constraints affects # of non-zero variables
- carefully set up constraints – is this constraint necessary?
- What restriction should it be – LE, GE, EQ?
- When should it be relaxed?
- Unit consistency

Variables Identifications:

- Types: technical, accounting, convenience
- Unit consistency

Objective Function:

- Maximization/Minimization
- Determines optimal solution

Homogeneity of Units

$$
\begin{array}{lrcrcl}
\text{Max} & c_1X_1 & + & c_2X_2 & & \\
\text{s.t.} & a_{11}X_1 & + & a_{12}X_2 & \leq & b_1 \\
 & a_{21}X_1 & + & a_{22}X_2 & \leq & b_2
\end{array}
$$

Rules

1. All coefficients in a row have common numerators.

2. All coefficients in a column have common denominators.

Data Development

Good solutions do not arise from **bad** data **-Key considerations:**

- Time frame
  - objective function, technical coefficient (a_{ij}'s) and RHS data must be mutually consistent i.e. annual basis vs. monthly basis
- Uncertainty
  - how to incorporate data uncertainty
- Data sources
  - vary by problem + judgments (statistical estimation or deductive process)
- Consistency
  - homogeneity of units rules must hold
- Component specification
  - objective, RHS, technical coefficients