

## AGEC 641 - HOMEWORK

### Chapter 13 Price Endogenous Programming

1. Given the problem

$$\begin{aligned} \text{Max} \quad & CX + 0.5X'QX - dY \\ \text{s.t.} \quad & X \quad \quad \quad - EY \leq 0 \\ & \quad \quad \quad \quad \quad \quad \quad AY \leq b \\ & X \quad \quad \quad , \quad Y \geq 0 \end{aligned}$$

where  $X$  is a vector of quantities consumed

$C$  is the vector of intercepts of the demand equations for  $X$

$Q$  is the matrix of slopes of the demand equations for  $X$

$Y$  is a vector of production alternatives

$d$  is the vector of per unit costs of producing  $Y$

$E$  is the matrix of per unit yields of  $Y$

$A$  is a matrix of resource usages by  $Y$

$b$  is a vector of resource endowments

Do the following

- a. State the conditions under which a global optimal solution would arise.
- b. Explain the Kuhn Tucker conditions.
- c. Explain the significance of the 0.5 term which pre multiplies  $X'QX$ .

2. The potato growers have come to you with the following situation. Potatoes are produced during the first two quarters of a year. The following supply functions have been estimated:

Quarter	Function
1	$P_1 = 18,000 + 1/2X_1$
2	$P_2 = 40,000 + X_2$
3	$X_3 = 0$
4	$X_4 = 0$

where  $X_j$  is the quantity of potatoes supplied in quarter  $j$  in pounds.

After potatoes are harvested, they may be sold as fresh potatoes processed into frozen french fries. It takes 2 pounds of potatoes to produce 1 pound of frozen french fries, and the conversion costs are \$.10 per pound of frozen french fries produced. Processing may take place only during the harvest season (Quarters 1 and 2).

Fresh potatoes may be stored for sale in the future at a cost of \$.03 per pound for each quarter stored. Storage costs for frozen french fries are \$.05 per pound for each quarter stored.

The demand for fresh potatoes and frozen french fries is:

Quarter	Fresh Potato Demand	Frozen French Fry Demand
1	$P_{1F} = 10,000 - QF_1$	$P_{1Z} = 1000 - 1/2QZ_1$
2	$P_{2F} = 8,000 - 2QF_2$	$P_{2Z} = 2000 - QZ_2$
3	$P_{3F} = 15,000 - QF_3$	$P_{3Z} = 1500 - QZ_3$
4	$P_{4F} = 12,000 - QF_4$	$P_{4Z} = 3000 - QZ_4$

where  $P_{jF}$  = price of fresh potatoes in quarter 1,  
 $Q_{Fj}$  = quantity of fresh potatoes demanded in quarter 1,  
 $P_{jZ}$  = price of frozen french fries in quarter 1,  
 $Q_{Zj}$  = quantity of frozen french fries demanded in quarter 1.

a) Formulate a mathematical model which determines the competitive allocation of potatoes to the fresh and frozen markets and the optimal level of storage.

3. Given the following data for a marketing firm, develop, using simple calculus:

- a) perfectly competitive optimum
- b) monopolistic optimum
- c) monopsonistic optimum
- d) monopolistic monopsonistic optimum
- e) optimum when  $\alpha = \frac{1}{2}$  for all curves but the fourth where  $\alpha = 1$
- f) explain your results

Inverse demand curve for exports of the good  $P = 15 - .5q_1$

Inverse demand curve for domestic consumption of the good  $P = 18 - q_2$

Inverse supply curve for imports of the good  $P = 6 + q_3$

Inverse supply curve for domestic production of the good  $P = 3 + .5q_4$

4. Choose a problem for which you have a GAMS formulation from an earlier homework, add price endogenous to one demand condition and one supply, then solve it with GAMS.