Ag Ec 641, 2009
Chapter XI, XII, XIII
Homework

1. Take the simple goal program

$$Max \begin{bmatrix} 3X_1 & + 3X_2 \\ 4X_1 & + X_2 \end{bmatrix}$$
$$X_1 + X_2 \leq 1$$
$$X_1 , X_2 \geq 0$$

a) Develop a weighted trade-off type objective function without targets.
b) Make it quadratic, incorporating diminishing preferences for goals as more of the goal is attained. Explain your formulation.

2. Develop a general multi-commodity spatial equilibrium problem in summation notation.

3. Suppose the Commodity Division of the USDA has asked you to conduct a study of the world sugar economy. Assume that the major sugar producing areas of the world are the United States, Europe, Latin America, Africa and Asia. The major consuming areas are the United States, Europe and Asia. Suppose the supply and demand relations for sugar in these areas are

Region		Supply		Demand
U.S.		$p^1 = 10{,}000 + 1/2Q^1$		$P_1 = 50{,}000 - 1/2Q_1$
Europe		$p^2 = 8{,}000 + Q^2$		$P_2 = 40{,}000 - Q_2$
Latin Amer.		$P^3 = 14{,}000 + 2Q^3$		
Africa		$p^4 = 7{,}000 + .8Q^4$		
Asia		$p^5 = 22{,}000 + Q^5$		$P_5 = 35{,}000 - 2Q_5$

Unit transport costs between regions are

		U.S.		Europe		Asia
U.S.		-		10		20
Europe		10		-		12
Latin Amer.		8		14		22
Africa		12		5		6
Asia		20		12		-

The problem is complicated by the fact that in the U.S., sugar has two primary uses. First, it is sold directly to consumers as granulated sugar, and second, it is used as an input in the production of soft drinks pastry, breakfast cereals, etc. In this second market it competes with corn fructose syrup for market share. Assume the demand for granulated sugar is as above, and the demand for sweetener in processed foods is

$$PN_1 = 27{,}000 - QN_1$$

where PN_1 is demand price and QN_1 is the quantity of sweetener used in the processed food market. The supply of corn fructose syrup in the U.S. is given by

$$PC^1 = 11{,}000 + 2QC^1$$

where PC^1 is supply price and QC^1 is quantity of syrup supplied. Corn fructose syrup and sugar substitute for each other at a 1 to 1 proportion.

Formulate a quadratic programming model of the world sugar economy incorporating the competition between sugar and corn fructose syrup in the U.S.

Tell how you would simulate the effect of an import quota into the U.S. on 2000 tons.

4. Suppose you have been hired by an international development agency to do an analysis of the effects of new technology on farming conditions in Jalava. Suppose that you have chosen to use a mathematical program to do this and have the following data:

Farms grow 4 crops: rice, maize, cassava and legumes. The year is divided into two seasons. Rice may be grown in three versions: year-long, wet season, and dry season; maize in the dry season, cassava in the dry season or year-long, and legumes in the dry season. The crops produce output, use land, labor, and miscellaneous outputs, and have prices as follows:

Crop		Wet Rice	Dry Rice	Year Long Rice	Maize	Dry Casava	Year Long Casava	Legume
Yield (Kg.)		4000	3500	4500	2800	12000	22000	1000
Labor Use (hours)	Wet Season	50	0	30	0	0	15	0
	Dry Season	0	50	25	30	20	10	18
Land Use (Hectares)	Wet Season	1	0	1	0	0	1	0
	Dry Season	0	1	1	1	1	1	1
Miscellaneous Input Use		4000	3800	4100	2000	100	150	1000

(rp/hectare)							
Price (rp/kg)	70	73	73	45	8	8	125
Calories/kg	3360	3360	3360	3490	1050	1050	4520

Assume the farm has 2 hectares and 75 man-days of labor in each period.
However, in addition to profits, the family is concerned about its diet. The family has 5 members. Assume that family members have a goal of consuming at least 1575 calories per day, with at least 210 from legumes, and no more than 400 calories from cassava. Further, assume the family would prefer to have at least 120 kg. annually of rice per member.

Formulate this as a multiple objective programming problem using a lexicographic for the diet and a profit max objective.

5. Give optimality conditions and tell whether (or how to find out whether) you have a maximum or a minimum, for the following problems:

a) $Z = 3X^2$

b) $Z = (4-X)^2 - 4X$

c) $Z = 3X^2 + 2Y^2 - 2XY$

d) opt $Z = 3X^2 + 2Y^2 - 2XY$

$X + Y = 2$

e) opt $Z = 3X^2 + 2Y^2 - 2XY$

$X + Y \leq 10$

$X, Y \geq 0$

6 **T**ake your earlier GAMS formulation

1. Add multiple objectives and solve for a lexicographic and 3 different weights settings
2. Add in a demand and supply curve and solve under perfect and imperfect competition