Notes on Crop Rotation Modeling in ASM

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During the course of a project done jointly with the NREL laboratory at Colorado State University and with Hagler Bailly, a set of features were included in the ASM model to depict rotations and input usage. These notes explain the modifications made.

The basic assumption regarding crop rotation is one of equilibrium. In a farm rotation model assuming equilibrium, the farm is assumed to operate in a steady state manner, repeatedly making identical decisions year after year. In such a case, the amount of land of each type of a crop carried over into the next phase of the rotation will be equal; thus, the initial acres and final acres may simply be set equal and their optimal levels determined by the model. This leads to a smaller model depicting a representative equilibrium year. Equilibrium solutions, however, do not portray growth situations or time paths of adjustment; only final equilibriums are created.

Equilibrium models are developed as follows: assume we are dealing with a continuous rotation that is used on a farm which lasts 4 periods. Also assume resource use, etc., equal to A_e, where e is the elapsed age of the rotation (0-3). Let us (assuming we start with zero initial acres in rotation) portray resource use over several periods.

		Begin Rotation in Year					
		1	2	3	4	5	6
Period 1	resource use	A_0					
Period 2	resource use	A_1	A_0				
Period 3	resource use	A_2	A_1	A_0			
Period 4	resource use	A_3	A_2	A_1	A_0		
Period 5	resource use		A_3	A_2	A_1		

Period 6	resource use	A_3	A_2	A_0		
Period 7	resource use		A_3	A_1	A_0	

Once the rotation pattern enters period 4 we have resource use by each age of the activity. In this and subsequent periods, resource use in any period may be written as

$$R_t = A_3 X_{t-3} + A_2 X_{t-2} + A_1 X_{t-1} + A_0 X_t$$

where X_t is the quantity of the activity begun in period t.

Under an equilibrium assumption assume we have equal acres in the rotation at all time periods with a continually repeated sequence. Thus, $X_t = X_{t-1} = X_{t-2} = X_{t-3}$ and the resource use may be rewritten as

$$R_t = A_3 X_t + A_2 X_t + A_1 X_t + A_0 X_t = (A_0 + A_1 + A_2 + A_3) X_t = X_t \sum_e A_e$$

In an equilibrium model, then, the resource use for the total rotation is the sum of its resource usages over its whole life. (At this juncture we should observe that these assumptions imply that carry over acres are equal.)

Thus, a general formulation of the equilibrium model with known life is

$$\begin{aligned} \text{Max} \quad & \sum_j C_j X_j \\ \text{s.t.} \quad & \sum_j A_{ij} X_j \leq b_i \quad \text{for all } i \\ & X_j \geq 0 \quad \text{for all } j \end{aligned}$$

where: X_j is the quantity of the jth rotation begun in a year.

C_j is the revenue per unit of X_j and equals the sum of the returns to the rotation over the periods (e) of its life;

A_{ij} is the use of resource i per unit of X_j and equals A_{ije} or the sum of the resource usages over the years of the rotation's life; and

b_i is the amount of resource i available.

This implies if we have a 4 year rotation and begin say 100 acres each year then typical resource usage is the sum of the resources used over all 4 years. Thus in terms of land the rotation would use 4 acres of land for each acre started since equal amounts of the rotation would have been begun 1, 2 and 3 years ago. We also account for yield from all four years in this year. We may simplify this model be dividing the resource use through by the years in the rotation, converting the coefficients to a representation of average resource use.

The above procedure is implemented in the AMS model through the addition of 2 variables. First we have rotation variables giving total acres in each rotation and the crops therein. Second we have individual variables giving the acres of the crops in the rotation and assume the inputs can vary. Algebraically this is portrayed as follows.

Suppose we have rotations 1 through k and crops 1 through j. Let us define R_k as the total acres in rotation k while U_{kj} are the acres and X_{kjm} as the acres of crop j in rotation K using input package m. Let us also assume that land is limited as are other resources and that we have L acres available along with a B_r units of resource r. Also assume crop j in rotation k when using input package m uses A_{rkjm} units of the rth resource and costs C_{kjm} units to produce wile yielding Y_{kjm} units of crop k. The conceptual version of the rotation model then is

$$
\begin{array}{llllll}
Max & +\sum_j P_j S_j & & -\sum_{jkm} C_{kjm} X_{kjm} & & \\
s.t. & & +\sum_k R_k & & = L & \\
 & & U_{kj} R_k & -\sum_m X_{kjm} & = 0 & for\ all\ kj \\
 & +S_j & & -\sum_{km} Y_{kjm} X_{kjm} & = 0 & for\ all\ j \\
 & & & +\sum_{kjm} A_{rkjm} X_{kjm} & \le B_r & for\ all\ r \\
 & S_j, & R_k, & X_{kjm} & \ge 0 &
\end{array}
$$

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In this model the first constraint limits land use across all the rotations to land available. The second constraint require the proportional mix of crops in the rotation to be present. The third balances total production with yields and the fourth accounts for total resource usage.

There is one important embodied assumption in this formulation and that is the input package can be chosen freely and does not affect the yields of the crops later in the rotation only the particular crop at hand.

A small example may be instructive. Suppose we wish to model 2 rotations. The first has 3 years of hay followed by 2 years of corn. The second alternates corn and sugar beets.   Suppose for simplicity that we are interested in varying inputs only for sugar beets and then only for one of 2 levels of irrigation. A tableau of a farm model of this is on the next page. The rotation feature shows that to use land you need to match an acre in the hay corn rotation with 0.6 acres of hay (3 years out of five) and 0.4 acres of corn. The sugar beet acres in rotation possibilities show how alternative input combinations are handled where the required beets can follow one of two irrigation regimes.

	Sell			Rotations		Crops in Rotation					
						Hay-corn		Corn-Sugarbeet			
	Hay	Corn	Sugarbeet	Hay-Corn	Corn-SBeet	Hay	Corn	Corn	SB Irr-1	SB Irr-2	
Profit	15	2	4			-110	-150	-155	-250	-300	
Land				1	1						#600
R1 Hay				-.6		1					=0
R1 Corn				-.4			1				=0
R2 Corn					-.5			1			=0
R2 Sbeet					-.5				1	1	=0
Avail Hay	1					-1.2					=0
Avail Corn		1					-180	-160			=0
Avail SB			1						-150	-200	=0
Water						1.5	2	2	1	2	#900

Actual Implementation

Several more comments are needed about the actual implementation. These involve the allowed rotations, and the ways Century data are used to form the rotation data. In terms of allowed rotations, four are modeled irrigated hay-corn with 3 years of each, a fallow-wheat with one year of each on dryland acres and monoculture rotations (included so we may examine water and fertilizer alternatives). Within each nitrogen fertilization alternatives are created. Also irrigation alternatives are included for the hay corn alternative.

Century is used in combination with ASM to parametize the rotation data. The Century data are used to form percentage changes from a base production system in terms of yield fertilizer use and irrigation water applied. The base ASM budgets are use to give data to which the Century derived percentage changes are applied. Thus the yields are always developed in terms of percentage changes from the ASM base yields for the region at hand in the 1992 crop year. Similarly, the irrigation and fertilizer levels are applied as percent changes from the base budgets.